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Issues and Challenges in 21st Century Mathematics Education
"Working toward Meaningful Teaching and Learning"

SEAMEO Regional Centre for QITEP IN MATHEMATICS

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The papers included have been reviewed and presented in the 4th International Symposium on Mathematics Education Innovation, on the 1-3 November 2016 hosted by Southeast Asian Ministers of Education Organization (SEAMEO) Quality Improvement for Teachers and Education Personnel (QITEP) in Mathematics.

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Foreword

This is with deep gratitude that I write this Foreword to the Proceedings of the 4th International Symposium on Mathematics Education Innovation (ISMEI).

SEAMEO Regional Centre for QITEP in Mathematics keeps maintaining its tradition to foster the exchange of innovative ideas and strategies for mathematics teaching and learning in modern classrooms and to encourage collaboration and partnership amongst mathematics educators. In this year's symposium, we have been pleased to provide mathematics educators with an intriguing theme on 'Issues and Challenges in 21st Century Mathematics Education'.

During the symposium, all paper presenters presented their works on several topics namely innovation in assessment and evaluation, curriculum issues, distance education, innovation in teaching and learning, learning environments, online learning, and teacher education. Their contributions helped the symposium as outstanding as it was. In addition to the contributed papers, we also invited five keynote speakers to give participants new insights about mathematics education. They were Assoc. Professor Allan White from the University of Western Sydney, Assist. Professor Maitree Inprasitha from the University of Khon Kaen, Professor Tom Lowrie of the University of Canberra, Professor Mohan Cinnapan from the University of South Australia, and Dr. Thien Lei Mee from our senior sister Center, SEAMEO RECSAM.

I believe that this proceeding will be a fresh impetus to stimulate further study and research in mathematics education.

Finally, we thank all authors and participants for their willingness to share their latest research and ideas. Without their effort, the symposium would not be possible. Keep up the good work and see you in 2018.

Prof. Subanar, Ph.D.
Centre Director

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Inner Space Exploration: The Impact of Brain Research upon Mathematics Education for the 21st Century

Allan Leslie White
University of Western Sydney (Retired), Australia.

Outer space exploration is an ongoing process of discovery and exploration of celestial bodies by means of developing space technology. Inner space exploration is an ongoing process of discovery and exploration of human bodies by means of developing brain technology. In 1924, Hans Berger succeeded in recording the first human electroencephalogram (EEG). With developments in technology, there are now a variety of approaches for examining the brain. Importantly, the implications of brain research for education are beginning to emerge. This paper will discuss some of these implications with special focus upon mathematics teaching and learning. In particular it will discuss a scale for assessing teaching strategies based upon their student learning outcomes centred around the construction of meaning. As the results of brain research are disseminated, teachers will be able to confirm or reject earlier research findings based only upon classroom observations. So now with this new data teachers will truly become inner space explorers.

Key Words: school mathematics, insight, instrumental understanding, rote memorization, relational understanding, brain research.

Introduction

Outer space exploration is an ongoing process of discovery and exploration of celestial bodies by means of developing space technology. The Hubble telescope, robotic probes, and human space flights are just some of the more recent examples that have resulted from the development of technology. However, in the beginning the study of space was carried out mainly by telescopes and the astronomers were forced to make inferences from their observations. Much later they were able to confirm or reject these inferences as technology provided more data.

Inner space exploration is an ongoing process of discovery and exploration of human bodies by means of developing brain technology. In 1924, Hans Berger succeeded in recording the first human electroencephalogram (EEG). Since then, there have been significant developments in technology, and now there are a variety of approaches for examining brain activity such as Magnetic resonance imaging (MRI), nuclear magnetic resonance imaging (NMRI), magnetic resonance tomography (MRT) and computed tomography (CT scans). These and other technologies are giving researchers the first glimpses of the vastness of our inner space just as super telescopes are mapping outer space. Brain research has impacted upon medicine and is being used to treat autism spectrum disorders, Alzheimer's disease, Parkinson's disease and other brain related conditions. Importantly for this paper, the implications of brain research for mathematics education are beginning to emerge.

In the beginning before technology, the study of the inner space of school students was carried out by education researchers mainly using classroom observations and student interviews. As the results of brain research are disseminated, teachers will be able to confirm or reject the inferences of this earlier research as the emerging technology provides further data. So teachers will truly become inner space explorers.

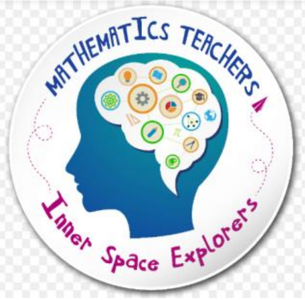

Figure 1. Mathematics teacher presentation badge

In the following sections I will attempt to briefly present a small sample of the findings that have relevance for school mathematics education in the twenty-first century with a special focus upon the importance of understanding or the construction of meaning.

Thinking, Learning and Understanding

To teach a mathematical idea, a representation is needed. This representation can involve spoken language (conversation or songs), written symbols (numbers, algebra, etc.), pictures (photos, graphs, etc.), video, dynamic images or physical objects. For students to receive this representation they must think, and from a cognitive science viewpoint, this thinking produces an inner space representation. Thus a problem arises for all mathematics teachers in assessing the quality of this inner space representation of the student.

This problem has been tackled by many researchers across many disciplines such as philosophy and psychology. The early Behavourists (see Skinner, 1953) rejected the idea of inner space representation because it could not be observed. They were like the early astronomers whose basic telescope was not powerful enough to see beyond the moon. For other groups, such as the Constructivists, there were various strategies used for making inferences about the quality of these inner space representations (or constructions) yet none of these strategies have actually involved directly observing the inner space representation or what happens in the brains of the students when they are thinking. Some research studies asked the students about their thinking but as the students are often unable to articulate their thinking these inferences were always subject to doubt.

> We all have systems of concepts that we use in thinking, but we cannot consciously inspect our conceptual inventory. We all draw conclusions instantly in conversation, but we cannot consciously look at each inference and our own inference-drawing mechanisms while we are in the act of inferring on a massive scale second by second. We all speak in a language that has a grammar, but we do not consciously put sentences together word by word, checking consciously that we are following the grammatical rules of our language. To us, it seems easy: We just talk, and listen, and draw inferences without effort. But what goes on in our minds behind the scenes is enormously complex and largely unavailable to us (Lakoff & Nunez, 2000, p.27).

From a theoretical viewpoint, cognitive science makes two assumptions in the communication process between the teacher's external representation and the student's inner space construction of that representation. First they assume there is a relationship between the external and inner space representations (the student's construction), and secondly the internal representation can be connected to other representations in useful ways (connected knowledge or schemas).

From a brain research viewpoint, the key concept has been termed 'brain plasticity' or 'neuroplasticity', which refers to the ability of the brain to change. Research has shown that the brain can reorganise itself in remarkable ways as a result of a change in stimuli. The process of learning begins when neurons form networks that fire together. The more an individual uses the networks the more developed they become until eventually they become automatic or compressed (compression will be discussed later in the paper). Our brains form distributed networks, and when students work mathematically, different areas of the brain communicate with each other. Conversely, when there is less use then the networks decay and eventually become lost. So learning and forgetting is essentially a process of rewiring the brain by forming or strengthening new connections and allowing old connections to decay. This has amazing implications for teacher's beliefs around mathematics ability as brain researchers have shown that:

> Children are not always stuck with mental abilities they are born with; that the damaged brain can often reorganise itself so that when one part fails, another can often substitute; … One of these scientists even showed that thinking, learning, and acting can turn our genes on and off, thus shaping our brain anatomy and our behaviour (Doidge, 2008, p. xv).

The brain researchers disprove the traditional belief that some children are born with the ability to do mathematics while others are not. They suggest that it is how the child's inner space is developed which determines mathematical ability.

> … scientists now know that any brain differences present at birth are eclipsed by the learning experiences we have from birth onward (Boaler, 2016, p. 5).

Children are not born knowing mathematics. They are born with the potential to learn mathematics. How this potential is nurtured, encouraged, and challenged is the responsibility of parents and teachers.

> Students can grasp high-level ideas, but they will not develop the brain connections that allow them to do so if they are given low-level work and negative messages about their own potential. (Boaler, 2015, p. xvii)

If a student is challenged and scaffolded and encouraged to struggle and persist then they may reach their potential. Unfortunately not all teaching strategies produce this outcome where students reach their maximum potential, and some strategies are even harmful.

The Importance Of Challenge And Struggle

In the current move to make mathematics learning joyful and fun, brain research has shown that we should not remove struggle and challenge. While it is probably true that a laughing child learns easier, brain research has shown the brain improves through concentration and challenge. Electronic games provide good examples of how children will struggle to overcome challenges while having fun (will be discussed more fully later in the paper). So it is possible to have both joyful and challenging learning.

Research shows that when students struggle and make mistakes, synapses fire and the brain grows (Boaler, 2015). This also has implications for what is known as instructional scaffolding. Scaffolding should be a learning process designed to promote a deeper level of learning. Scaffolding first introduced in the late 1950s by Jerome Bruner is regarded as the support given during the learning process which is tailored to the needs of the student with the intention of helping the student achieve certain learning goals. Scaffolding should help the student face and overcome challenges through struggle, not remove the challenge and the struggle to learn by making mistakes. Brain research has revealed the importance of mistakes,

> Educators have long known that students who experience 'cognitive conflict' learn deeply and that struggling with a new idea or concept is very productive for learning (Piaget, 1970). But recent research on the brain has produced what I believe to be a stunning new result. Moser and colleagues (2007) showed that when students make mistakes in mathematics, brain activity happens that does not happen when students get work correct. For people with a growth mindset the act of making a mistake results in particularly significant brain growth. (Boaler, 2014, p.17)

The amount of scaffolding given by the teacher varies with each individual student, and should avoid the process of cognitive emptying (colloquially know an dumbing down) where a teacher provides so much scaffolding that it empties a task of its cognitive challenge and the student answers just a series of relatively simple questions (Brousseau, 1984).

As mentioned earlier, a good example can be found by examining the development of electronic games. The programmers of these games have adapted educational principles to their work. The game sets a challenge (usually in the form of a level) and the child has to overcome this challenge. Based upon the child's responses the game individually adjusts so that the challenge is not too great. In the quest to reach the next level the child makes lots of mistakes, but the colour, visual stimulation, sound and excitement of the game are highly motivating. What brain research has also shown is that when a child overcomes a challenge then they receive a reward through a dopamine hit. Dopamine is a neurotransmitter that acts in a number of ways. When a child achieves the goal the game celebrates with the child causing an excess in dopamine which flows to other parts of the brain causing feelings of pleasure. The brain seeks opportunities to repeat the pleasure. Hence the difficulty faced by parents and teachers trying to stop the child from playing because they are stopping the child from self-stimulating in a natural way. As well, the intense concentration for short bursts demanded by the game grows the child's brain particularly around visualisation. What is encouraging for teachers is that some of the best programmers are now developing games involving mathematics. They are tapping into brain research that showed that when students work on mathematics their brain activity is distributed among many different networks including visual processing.

Attitudes, Beliefs And Mindsets

When a child learns a new concept, in their inner space an electric signal ignites, cutting across synapses and connecting different parts of the brain. If the child learns a concept deeply, then the synaptic activity creates lasting connections in the child's brain, whereas surface learning quickly decays. How this decay occurs was outlined by Sousa (2008) who stated that scientists currently believe there are two types of temporary memory. Firstly, *immediate memory* is the place where the brain stores information briefly until the learner decides what to do with it. Information remains here for about 30 seconds after which it is lost from the memory as unimportant. Secondly, the *working memory* is the place where the brain stores information for a limited time of 10 to 20 minutes usually but sometimes longer as it is being processed. The transfer from immediate memory to working memory occurs when the learner makes a judgement that the information makes sense or is relevant. If the information either makes sense or is relevant then it is likely to be transferred to the working memory, and if it has both then it is almost certain to be transferred to the long-term memory.

How students make this judgement of relevance is influenced by the students' attitudes and beliefs around learning and how they regard their own ability. Mathematics teachers are expected to teach the curriculum while inculcating positive attitudes towards mathematics and by engaging and motivating their students to work mathematically. Psychologist Barbara Dweck (2006) and her research team collected data over a number of years and concluded that everyone held a core belief about their learning and their brain. They made a distinction between what they labelled as a fixed mindset and a growth mindset. (In outer space terms, is the universe stationary or expanding?) Someone with a fixed mindset believes that while they can learn things, they cannot change their intelligence level. Whereas someone with a growth mindset believes that the brain can be changed through hard work and the more a person struggles the smarter they become. Studies have shown that learning through challenge and struggle can grow the brain. There is an obvious connection here between growth mindset and brain plasticity. Professor Jo Boaler (2016) in her new book provides a wealth of research evidence involving mathematics learning that supports Dweck's work.

> It turns out that even believing you are smart - one of the fixed mindset messages - is damaging, as students with this fixed mindset are less willing to try more challenging work or subjects because they are afraid of slipping up and no longer being seen as smart. Students with a growth mindset take on hard work, and they view mistakes as a challenge and motivation to do more (Boaler, 2016, p. 7)

Boaler and her team have developed a website (Youcubed), produced many short videos (search for Jo Boaler on Youtube for a selection), and published considerable material on how to promote growth mindsets in the classroom. Boaler and her team list seven positive norms for teachers to promote in their classrooms that are claimed to arise from brain research (Boaler, 2016, pp. 269-277). They are: Everyone can learn mathematics to the highest level; Mistakes are valuable; Questions are really important; Mathematics is about creativity and making sense; Mathematics is about connections and communicating; mathematics is about learning and performing; and the final norm is that depth is more important than speed.

My aim for the rest of this paper is not to replicate or summarise Boaler's material, as the reader can get access to it through the links I have mentioned. Instead I want to continue considering the implications of the Scale For Teaching For Understanding that allows a classification of teaching strategies according to the student learning outcomes and current brain research for school mathematics teaching and learning. In some areas, the paper will overlap or resonate with Boaler's work. So in the following section I will briefly provide an overview of the scale for teaching for understanding before elaborating upon mathematical insight.

Scale For Teaching For Understanding

Mathematics teachers have many teaching strategies from which to choose. Some strategies produce good student learning outcomes and some poor outcomes. In my earlier papers (White, 2011, 2013) the negative effects or poor outcomes of strategies using behaviourism, rote memorisation and skills based teaching have been discussed in some detail. These are a few of the many strategies regarded as ineffective or even harmful to the development of mathematical understanding. Why is understanding or constructing meaning so important? Constructing inner space meaning determines the possibility that information will be learned and retained in the long term memory, the goal of all mathematics teaching and learning. As mentioned earlier, making sense or meaning is a crucial consideration of the learner in moving information from the immediate memory to both the working and long term memories.

> Students may diligently follow the teacher's instructions to memorize facts or perform a sequence of tasks repeatedly, and may even get the correct answers. But if they have not found meaning by the end of the learning episode, there is little likelihood of long-term storage (Sousa, 2008, p. 56).

It appears that the process of making inner sense, meaning or understanding does not have a single end point but is actually and ongoing process of the increasing accumulation of input, connections, compression and insight. Mathematics teaching research literature contains a number of duopolies such as Skemp (1976, 1977, 1979, 1986, 1989, 1992) who proposed the terms instrumental and relational understanding. Instrumental mathematical understanding is described as acquiring 'rules without reasons' or knowing 'how or what' to do to get an answer, whereas relational understanding is concerned with meaning and knowing how, what and why it gives an answer. Skemp (1976, 1977) discusses the developing of schemas as evidence of the construction of relational understanding and this resonates very strongly with the inner space structure of the neural connections within the brain and with the research literature on ‘connected knowledge’.

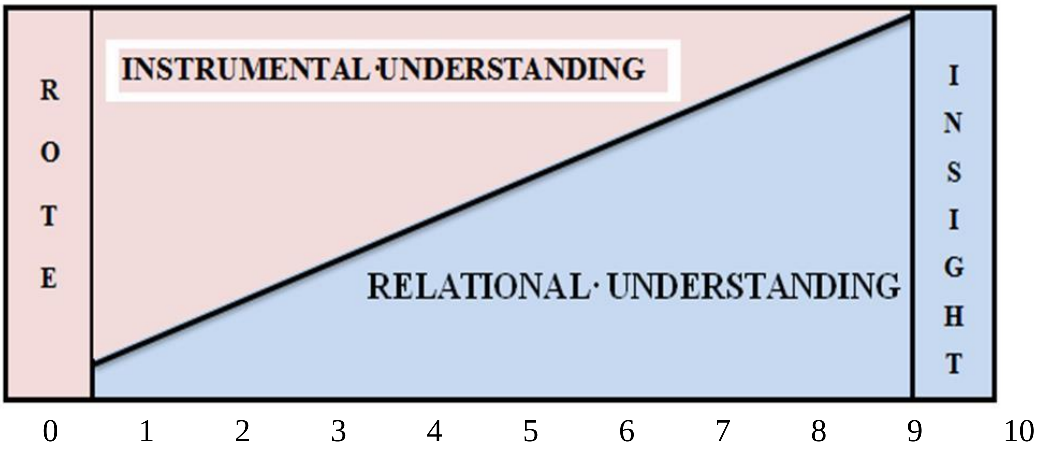

Figure 2. Teaching For Understanding

The scale of teaching for understanding was constructed as a continuum (see Figure 2) based on the assumptions that all teaching strategies can be classified according to their aims and student learning outcomes using Skemp’s types of understanding, and that the struggle to assist learners to understand is the same struggle to make sense or meaning. This scale has been discussed more fully elsewhere (White, 2014).

Briefly, the left end of the scale (score 0) is the most extreme end of instrumental teaching strategies which is rote memorization, where there is no attempt to assist students to understand or connect what they are memorizing with what they already know.

Sousa (2008) contrasts two kinds of practice as rote and elaborative rehearsal regarding their effects on the brain. Rote rehearsal is a process of learning information in a fixed way without meaning and is easily forgotten. Elaborative rehearsal encourages learners to form links between new and prior learning, to detect patterns and relationships and construct meaning. The construction of meaning involves the building of cognitive schemas that will assist long term memory. Elaborative rehearsal leads to meaningful, long-term learning. Of course there are a range of elaborative rehearsal teaching strategies that differ in success. The scale shows the majority of teaching strategies score from 1 to 9, indicating they are a combination of varying degrees of instrumental, and relational aims and outcomes. Each approach would include various memory and elaborative rehearsal strategies that are all important in the process of building more sophisticated concepts that are meaningful to the learner.

When we consider the time allocated to practice or rehearsal then there is another distinction made in the literature between *massed practice* and *distributed practice* (Sousa, 2008). Cramming, which usually occurs in a brief intense time period just before an examination, is an example of massed practice where material is crammed into the working memory, but is quickly forgotten without further sustained practice. There is no sense making and so it never makes it into the long term memory. Distributed practice on the other hand is sustained practice over time, building understanding and resulting in long-term storage and maybe compression and insight. Distributive practice resonates very strongly with the East Asian Repetitive Learning which is continuous practice with increasing variation as a route to understanding (Leung, 2014), and this is often misunderstood as a form of rote memorisation. It is not rote memorisation, as it seeks to build understanding through increasing the complexity, challenge and the connections with prior knowledge.

The interplay of the instrumental and relational aspects of understanding leads to what is often termed compression which is also sometimes confused with rote memorisation.

> Mathematics is amazingly compressible:-you may struggle a long time, step by step, to work through the same process or idea from several approaches. But once you really understand it and have the mental perspective to see it as a whole, there is often a tremendous mental compression. You can file it away, recall it quickly and completely when you need it, and use it as just one step in some other mental process. The insight that goes with this compression is one of the real joys of mathematics (Thurston, 1990, p. 847).

Thurston (1990) states the process of building understanding leads to insight, which resonates with Cognitive Science’s second assumption that an internal representation can be connected to other representations in useful ways. Insight was apparently derived from a Dutch word for ‘seeing inside’ and is loosely defined as the process within the mind of a learner who when exposed to new information enables the learner to grasp the core or essential features of a known problem or phenomena. An insight seems to result from a connective process within the brain or a quick restructuring that produces new understanding that is a compression of the connected information. Think of the compression of the mathematics we learned as children in the primary (elementary) school that allows us to easily perform four digit by three digit multiplication calculations. So encouraging student insight is a goal in the process of teaching for understanding.

Researchers draw strong connections between insight, creativity and exceptional abilities, with any significant and exceptional intellectual accomplishment almost always involving intellectual insights (Sternberg, 1985). Insights can occur as a result of the conscious and unconscious mind. The unconscious mind can continue to operate when the conscious mind is otherwise distracted, hence the large number of cases of mathematics students claiming to have gone to bed with an unsolved mathematics problem only to wake the next morning with an insight into the solution.

> Perhaps the most fundamental, and initially the most startling, result in cognitive science is that most of our thought is unconscious that is, fundamentally inaccessible to our direct, conscious introspection. Most everyday thinking occurs too fast and at too low a level in the mind to be thus accessible. Most cognition happens backstage. That includes mathematical cognition (Lakoff & Nunez, 2000, p.27).

An insight is not an end in itself but can contribute to further understanding and further insight. Thus a student in year 7 may develop an understanding of Pythagoras’ theorem involving the area of squares on the sides of a right triangle. In year 8 the student might have an insight involving an understanding that the area of semi-circles drawn on the triangle with sides as diameters also obeys the theorem. Later in year 10, the same student may develop a further insight where Pythagoras’ theorem is seen as one example (one angle equal to 90 degrees) of the more general rule known as the cosine rule (for any angles) where $c^2 = a^2 + b^2 – 2abcosC$. The student grasps that when C = 90 degrees then the subtraction term disappears as cos 90 is zero and we have the theorem. Mathematics teaching that leads to the production of multiple insights in the learner is postulated as a desirable goal for the teacher.

It is the accumulation of insights that leads to the desired compression of mathematical understanding. This compression provides the mathematical tools to efficiently tackle more sophisticated and complicated mathematical problems.

Conclusion

This paper has sought to discuss some of the findings that brain research is providing to the teaching and learning of school mathematics. The paper aims to motivate mathematics teachers to develop strategies that challenge students to struggle with problems while encouraging the students to joyfully build their mathematical understanding and develop links and connections within their knowledge, while developing positive growth mindsets towards their mathematical learning and knowledge.

For outer space we have the term *astronaut*, for inner space I do not want to call mathematics teachers *neuronauts.* I think the title of *inner space explorers* is a better term for the very complex job that mathematics teachers must face each day. They seek to develop the inner space of the nation's youth to go beyond current thinking and reach far distant intellectual goals. The future of the nation is in the hands of these *inner space explorers.*

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Lesson Study as an Innovation for Teacher Professional Development: A Decade of Thailand Experience

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Abstract

Before entering the 21st century, most of affluent countries have been preparing new skills for their children while most of non-affluent countries do not recognize this shift. Report on TIMSS and PISA is a good evidence for the difference. However, during the last decade before entering the 21st century, all of the countries across the world have become recognized the necessity of 21st century skills and attempting to improve their school education to serve these skill demand for their citizen. If taking Thailand as a case, it has been undergoing an educational reform movement since 1999, after the first Educational Act was launched. A number of educational institutions have been attempting to respond to the national agenda, to reform learning process, and respond to the new demand of 2001 curriculum which emphasis on the integration of three major components of the curriculum: subject matters, skills and learning processes, and desirable characters. This paper describes a decade of Thailand experience attempting to implement Japanese Lesson Study as an adaptive innovation for teacher professional development through the project running by Center for Research in Mathematics Education (CRME), Khon Kaen University since 2002.

1. Status Quo of School Education in Thailand

School education in Thailand is an underrepresented field; no research in the field. It is associated to teacher education program in a sense that those who graduate from preparation program have been recruited and work at schools. However, in various parts of the world, the need for better-qualified teachers has been a critical issue in the minds of parents and educators. There are historical questions in this field which are relevant to the present time: What are the essential characteristics of a professional program for teachers? Should a program for teachers differ from a liberal arts program, and if so, what should be the distinctive features of the treatment of subject matter in each type of program? What types of courses in professional education should be required of prospective teachers? (Gibb, 2003).

Much of the research (Lewis, 2002; Gibb, 2003) supports the idea that teacher preparation is important, and that knowledge and skills are built over time, in a coherent program of study. The National Council for Accreditation of Teacher Education (2010) suggests that (1) high quality educator preparation makes a difference in student learning, (2) teacher preparation increases teacher retention, and (3) teacher preparation helps candidates acquire essential knowledge and skills.

Thailand has encountered many problems in establishing programs for teacher education, particularly in science and mathematics. Not until the last decade, have the 36 teachers' colleges and 8 universities of education located across the country gained high respectability in the provision of teachers to elementary and secondary schools (Inprasitha, 2006; Thailand Education Deans Council, 2010). The students who entered teachers' colleges and universities of education at that time were high-achieving students from various schools. However, during 1960s – 1970s the universities of education were changed to be more comprehensive, and in 1992 the teachers' colleges were changed to be Rajabhat Institutes and then to Rajabhat universities, with faculties of education at these universities. These education faculties have become to be regarded as *'second-class'* faculties in terms of their profile (Inprasitha, 2006). The graduates feel inferior to graduates from other programs and often have negative attitudes towards their career. This is a crucial problem for most current teacher education programs.

After the 1999 Educational Act was enacted, Thailand began on an educational reform movement. Since then, most school teachers have been attempting to improve their teaching practice. Unfortunately, they lack innovation to improve their everyday work. Most teachers still use a traditional teaching style focusing on the coverage of contents. Often this approach neglects the emphasis of students' learning processes and their attitudes toward learning with understanding. More importantly, a number of teachers view themselves as the reformers group (*e.g.*, master teachers, initiative teachers etc.,) but in effect still view professional development from the old paradigm. Similarly, the educational reform movement in many countries report similar issues such as professional development of teachers being a central issue, and that teachers need to learn how to capture students' learning processes and to examine their own practice. There was also a lack of clarity about how to best design initiatives that involve the examination of practice (*cf*. Ball, 1996; Lampert, 1999; Shulman, 1992; Fernandez, Cannon & Chokshi, 2003).

The new 2001 Basic Education Curriculum has been implemented in response to the 1999 Educational Act. This new curriculum demands that high school graduates possess content or subject matters, learning process and skills, and desirable characters as shown in the Figure 1. Teaching mathematics by integrating these three components is a challenge for most mathematics teachers in Thailand. One initiative by Center for Research in Mathematics Education, Khon Kaen University is an example of this challenge. Creating problem solving classroom by integrating Open Approach and Lesson Study is an implementation for this challenge.

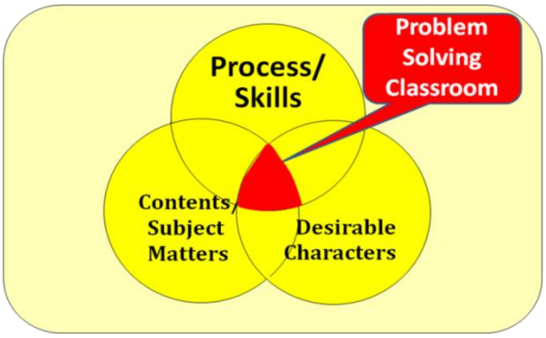

Figure 1 An integration of three components in a problem solving classroom (Inprasitha, 2003)

2. The long-term plan for implementing Lesson Study

There are many attempts in implementing new innovations in Thailand. But, most of the attempts often failed. One cause came from the lack of context preparation for using those innovations or approaches. Lesson Study as a Japanese teaching professional development has been developed and used in Japan for more than 130 years (Shimizu, 2006). Now, it is expanded throughout the world for improving teacher's profession. However, in order to implement in Thailand, the following context preparations have been implemented.

2002	- Incubation of Ideas with 15 student teachers and the first two schools
2002-2005	- Experimentation in a number of schools
2006-2008	- Starting Whole School Approach in 4 schools in the Northeast of Thailand
2009-2011	- Expansion in 22 schools in the Northeast and North of Thailand
2011 – 2011	- Attempting to expand nationwide

2.1 Incubation of Ideas in two schools in 2002

Traditionally, most of mathematics classroom in Thailand depend heavily on following the national textbooks. Textbook style starts with introducing some definitions, principles, rules, or formula follow by some examples, and close with some assigned exercises. Unfortunately, most of exercises are kinds of closed problems, which have one and only one correct answer. This style of textbook has influenced the teacher's style of teaching, or even become teaching method for most of them. The script of teaching of mathematics teachers will look like this; starting with explaining new contents, some examples, and closing with giving the students some assigned exercises. This method of teaching has formed the classroom culture where the students cannot initiate their own learning. They become passive learners in this kind of mathematics classrooms.

In order to change this kind of mathematics classroom, the author challenges the idea of using open-ended problems in the classrooms of 15 student teachers in 2002 academic year. In the summer of 2001 academic year during March and May, 15 student teachers had been coached by the author how to make lesson plan using open-ended problems as a focal mathematics activity. They were trained to organize teaching sequence in a new way, which starting with posing prepared open-ended problems to the students, allowing time for the students to think by themselves within their small-group working. While the students were doing their small-group working, the student teachers had to make some notes for collecting students' ideas emerged in the classroom and bringing for whole class discussion. On every Friday, all 15 student teachers came back to the university to discuss about the teaching problems with the author. In the first half of the semester, all of student teachers had pressure with many aspects. For examples, the classroom teachers reminded them that they should go directly to teach to cover the contents, otherwise their students would run into trouble when having mid-term test. Even the students in the classroom also complained that the student teacher did not teaching anything according to the textbook. They were afraid that they did not learn what other students in other classes learned. The student teachers presented these problems in Friday discussion. They all faced the same problem in every class and in every school.

However, after the first half of the semester has passed, the situation changed. The students in every class showed a good sign of responses to the class. They like to present their ideas in the whole discussion session. One typical example in every school is most of incapable students in traditional classroom play an important role in group-working activities. Hands-on activities provided them a chance to use their ability. At the end of semester, there had great changes in the classroom of all student teachers and the student teachers also changed their worldview about teaching and learning mathematics. They perceived that learning mathematics is more than just coverage of contents. This change influences their career path after they graduated. For the author, this one semester gave an idea on how to integrate Open Approach which emphasis on using open-ended problems in the traditional mathematics classroom to challenge mathematics teachers to change the way they teach mathematics. He provided much training for teachers during 2002 -2005.

2.2 Experimentation in Some Schools during 2002-2005

Since the second half of 2002, the author introduced how to use some 5-6 open-ended problems in the classroom through short training program in many schools. During 2002-2005, more than 800 teachers had been trained to use open-ended problems in their lesson plan while emphasis on changing their role of teaching in the classes by posing the open-ended problems and allowed time for students to think by their own in group-working activities, following by whole class discussion. There has a great change in those teachers as in the following survey (Inprasitha, Loipha, and Silanoi, 2006).   The great impact of changing classroom after using open-ended problems was apparent to teachers. However, most of the teachers were still being worried about how to cover the contents because they used only some 5-6 open-ended problems in their classroom. Almost of teachers who had been trained to use open-ended problem called for open-ended problems which covering all of the contents in every grade.

The Center for Research in Mathematics Education does not respond to this demand. The author then decided to recommend teachers to create open-ended problems by themselves. This made them to be perceived that it was very difficult for them to do that. Then, it is timely to prepare to introduce a tool for creating “open-ended problems” by school teachers, rather asking the Center to provide for them. The tool for creating “open-ended problems” is that they have to collaboratively working together according to the three basic steps of lesson study; collaboratively plan, do, and see for making lesson plan which emphasis on using “open-ended problems” in the mathematical activities in the lesson plan. This idea had been implemented in the first two lab schools in 2006.

2.3 Lesson Study with Whole School Approach

As mentioned in the earlier session, the idea of integrating Open Approach into the three steps of lesson study; collaboratively plan the lesson together for creating lesson plans emphasis on how to creating “open-ended problems” in terms of 3-4 short instructions, teachers have to anticipate the students’ responses to their instructions, and coming to discussion in the end of the week. In the first stage of implementing lesson study, the author did not focus in the detailed research lesson like those of Japanese teachers but still emphasis on how to help the teachers to reconceptualize "what does it mean 'mathematics classroom' for them?" In terms of research, they were trained to use classroom as a unit of analysis for improving their daily practices. The author proposed a new approach for teaching which integrating both the idea of teaching and research for the tool of the teachers.

In this model, teachers had been encouraged to focus on how to investigate the "students' ideas" emerged in the classroom. Their focus is on how to bring "those students' ideas" to be discussed in the whole class session in order that all or most of the students in the class being engaed in such aspects of the problem, which is very important for them to view problems from many angles. This is the central issue for using open-ended problems in these classrooms. As we might expect, most of the teachers have limited tools to collect students' ideas. Even though we recommended them to use short note to collect the students' ideas but they usually ignore what is important for the students. They are not familiar with the students' natural ways of thinking. This also returns back to the step that they have a difficulty in anticipating the students' ideas when making lesson plan.

Positively, teachers come to realize that "classroom" is not just a simple idea. It is a culture which the teacher and students in that class have created in the long-term tradition. Parts of this classroom culture are classroom norm, belief system, attitudes, and values. All of these has formed the complicated aspects of that classroom culture. Thus, changing the classroom is not just providing teachers with new method of teaching and expecting that the changing will occur in the classroom. Creating new teaching practices by integrating Open Approach into Lesson Study is a promising context for changing teachers' roles in the mathematics classroom. This idea has been implemented in the following two lab schools since 2006. Khoo Khum Pittayasan, a typical expansion school in the remote area of Khon Kaen province 30 kilometers far from Khon Kaen city and Chumchon Ban Chonnabot, an elementary school in the typical country side of Thailand in some 50 kilometers far from Khon Kaen city voluntarily participated in the project.

The most difficult part of implementing lesson study in schools in Thailand is how to form lesson study team. We do not have senior or expert teachers in schools like those of Japan. We also lack of outside knowledgeable persons to support the schools. In order to have effective lesson study team in the project school, Faculty of Education, Khon Kaen university has trained our graduate students in master degree program in mathematics education, which first offered in 2003 to take part in the workshop organized by the Faculty during 2003-2005. We organized our workshop into small group mixing both teachers or school principals and supervisors. The graduate students take their roles to observe group-working and then come to reflect what they observed after the representative people of the group presented their work.

The roles of graduate students provided a chance for school teachers to reflect on their traditional roles. In 2006, when we started to fully implement the idea of lesson study and Open Approach, our graduate students have been assigned to be members of lesson study team working closely with school teachers where as they used the school they participated as their research site. Thus, each lesson study team will look like this; three classroom teachers from grade 1, 2, and 3 added with one graduate student, one teacher from other grade (option), the principal (mostly attended in reflection session). A team for grades 4, 5 and 6 or for grades junior 1, 2, and 3 will do the same way to form their lesson study team. Three steps of lesson study have been practiced as follows: Monday or Tuesday was set for collaboratively plan the lesson for each team. One teacher went to teach according to usual time table in a week. Then, all teachers in that school with the school principal took leadership in reflection session in the end of the week, may be Thursday or Friday. The author adapted many steps of lesson study by putting revision step into yearly cycle. This made the three step lesson study could be practiced as a weekly cycle. Thus, we can plan to do lesson study every week for still covering all content teachers have to teach. This adaptive version relaxed teacher to be comfort to use innovations like lesson study and Open Approach in their classroom. They feel like they have outside knowledgeable persons to help them improve the classroom, rather feeling that they had to have more extra work than they used to do. Figure 3 shows three steps of lesson study adapted to be used in mathematics classroom in the project schools in Thailand in two schools mentioned earlier.

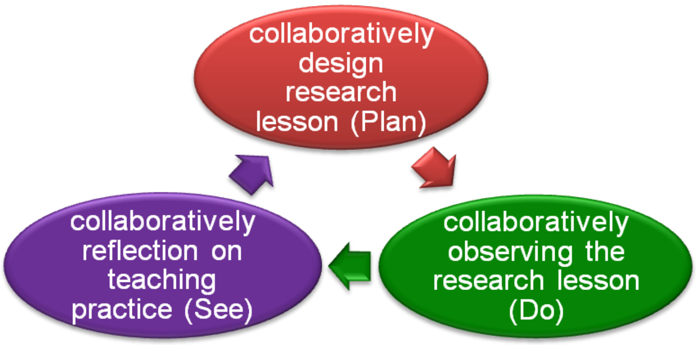

Figure 3 Adaptive Lesson study in Thailand (Inprasitha, 2006)

2.4 Expansion in 22 schools during 2009-2011

Since 2009, for sustainable lesson study in Thai context, the model was fully developed for using in 22 schools as an expansion project for three years 2009 – 2011. The model consisted of collaboration from Office of the Basic Education Commission, Office of Knowledge Management and Development, Graduate Study in Mathematics Education and Center for Research in Mathematics Education (CRME), Khon Kaen University, as well as collaboration from the international cooperation. And also we adapted the Japanese Mathematics Textbook to 22 schools.

By this year we start to implement Japanese mathematics textbook in the project schools, Plianram & Inprasitha (2009) compared the features of Thai and Japanese Mathematics Textbook. The results are as follow; the features of Thai and Japanese Mathematics Textbook were different, the teachers' understanding of Textbooks also different, teachers had difficulties to use, to understand the content in the Japanese Mathematics textbooks when they were adapting. Boonlerts & Inprasitha (2010) investigated the characteristics of textbook and instruction in "Multiplication" in Japan, Singapore, and Thailand by using a comparative study in mathematics textbooks and teacher guide books for grade 1-3.

The LS project thus became very much school-based with teachers taking a leading role. This concurs with the observation of Fernandez & Yoshida (2004) that one feature of Lesson Study is that it often takes the form of "Konaikenshu" (in school training).

The educational reform in Thailand following the National Educational Act 1999 placed curriculum demands on teachers requiring them to improve their own teaching. Unfortunately, perhaps due to a lack of innovation for improving their routine work, most teachers still use traditional teaching emphasizing the content, often overlooking the importance of the students' learning process and attitude toward learning with understanding.

In order to create a 'problem solving classroom', some features of the Open Approach were incorporated into the adaptive LS as reported in this paper. It is noted that the Open Approach is used as the teaching approach within the LS process (see as fig. 4.)

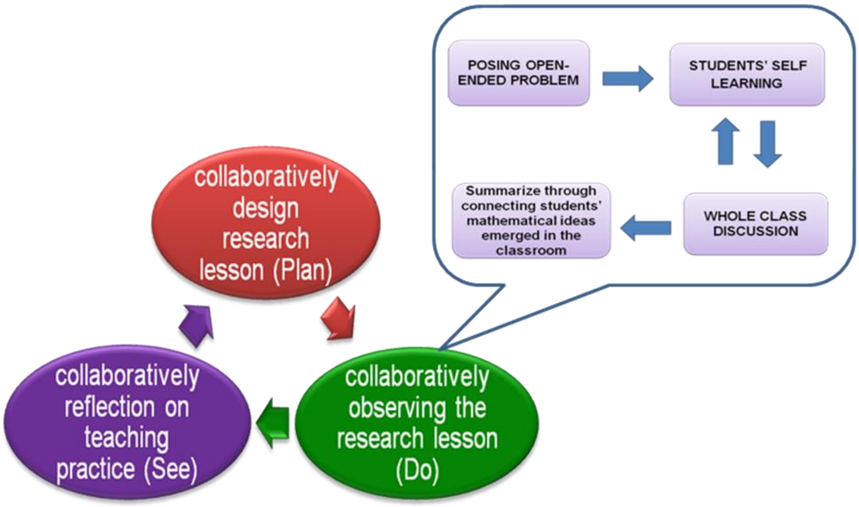

Figure 4 Four phases of the Open Approach incorporated into Lesson Study (Inprasitha, 2011).

2.5 Attempt to Nation Wide since 2011

Since 2011, we are attempting to expand LS nationwide. The model of Lesson Study Team for expansion will look like this.

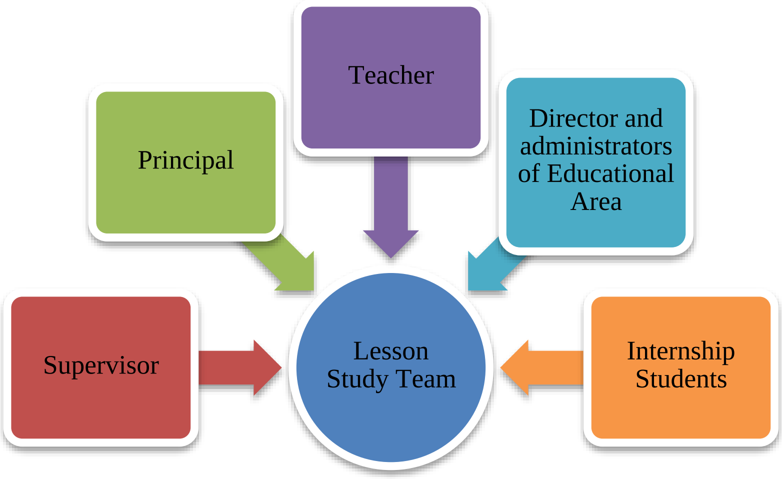

Figure 5 Model of Lesson Study Team

3. Concluding remark

Thailand has been undergoing an educational reform movement since 1999. Unfortunately, in the opinion of the author, it was not until the introduction of the LS process and the Open Approach into schools, that there appeared evidence of a major change in the teacher classroom response to this educational reform. Introducing LS into Thai schools has influenced not only the improvement of teaching practice, but also improvement of the system of teacher education. A number of major changes have occurred during the last ten years since the introduction of LS and the Open Approach in 2002 and further research will probably be needed to provide sound evidence to support such claims. This paper has summarized a decade of Thailand experience to adapt Lesson Study and Open Approach to Teacher Professional Development in Thailand.

The experiences described in this paper were shared among APEC member economies in the Human Resource Development Working Group. However, this change is small in comparison with the long history of more than 100 years of LS in Japan. There is a need for Thailand to continue to adapt Lesson Study to fit the many contexts of the local schools in Thailand.

Acknowledgments

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Understanding knowledge and knowledge use during teaching: The case of geometry

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4th ISMEI Keynote paper, Yogjakarta

Abstract

A significant number of recent research studies have attempted to examine mathematics teaching by considering knowledge that teachers need for their work. Interest in mathematics teacher knowledge that could support better learning has generated several lines of useful inquiry with a great deal of focus on the nature of this knowledge, ways to measure this knowledge and, the use of that information to support the on-going professional development of mathematics teachers. While the specification of knowledge strands for teaching constitutes an important development, the field needs to move forward in identifying how that knowledge is used during in practice. It would seem reasonable to suggest that teachers with rich and sophisticated mathematical and pedagogical knowledge would demonstrate a high level of flexibility in the way their knowledge is exploited in engaging their students. In this talk, I will attempt to sketch the structure and composition of teachers' geometric content knowledge and teaching of that content knowledge. In so doing, I will be drawing on *schemas* and *representations* as the theoretical lens to examine knowledge transactions between the teacher and student.

Introduction

Student performances in high-stakes mathematics assessment programs such as TIMSS and PISA has been gaining increasing currency among school leadership and educational policy makers (Australian Council for Educational Research, 2011). Interest in student performances are important and necessary but discussion also need to examine the question of participation and engagement. Equally, what are factors that would support higher levels of students' involvement in learning mathematics and using mathematics to solve every-day problems and problems that emerge in their professional career? Arguments presented in this paper are aimed at exploring one major factor, namely, the role of teaching and teachers of mathematics in inducting and supporting students into the world and wonders of mathematics. In so doing, I make the assumption that teachers are a powerful force in moulding the thinking underlying mathematics knowledge acquisition and utilisation both in concrete and abstract contexts.

Teacher knowledge and teaching mathematics

Current reforms and debate about improving the quality of mathematical learning are increasingly concerned with the kind of learning experiences teachers could provide for their students (Australian Curriculum, Assessment and Reporting Authority, 2010; Australian Association of Mathematics Teachers, 2009). The quality of these learning experiences in turn depends on teachers' own knowledge and experiences, and how that set of knowledge is utilised in the course of teaching. In the past decade, there has been a surge of interest in examining what constitutes teacher knowledge and how this knowledge impacts on teachers' decisions about content and strategies that are used to engage learners. An OCED report on Education in Indonesia identified teachers as a primary resource in enhancing the quality of mathematics teachers at levels of schools and the urgency in identifying and developing appropriate professional development needs of Indonesian mathematics teachers (OECD/Asian Development Bank, 2015).

The stream of research on teacher knowledge that drives their practice was pioneered by the work of Shulman (1986) who alerted the community to that fundamental to teachers' work in the classroom is their knowledge for teaching mathematics. Specifically, Shulman conceptualised that teachers' knowledge of subject-matter has to be translated into forms that are more learner-friendly. This latter knowledge was referred to as Pedagogical Content Knowledge (PCK) as opposed to Subject-Matter Knowledge (SMK). A review of the discussions of the quality of teaching and of the research literature on experienced and beginning teachers of mathematics (e.g., Lampert, 2001; Leinhardt et al., 1991) clearly indicates that in order for teachers to be able to perform the diverse roles during their teaching, they need to be able to access and exploit the following types of knowledge both before and during teaching: (1) appropriate mathematical content, (2) appropriate knowledge about the learner (s), and (3) appropriate knowledge about how to teach the mathematical content to the learner(s). Let's examine these constructs in some detail now.

<u>Mathematical content</u>: The quality and the quantity of the mathematical content accessed and exploited are influenced by at least two factors. First, it depends to a large extent on the organizational quality of the teachers' repertoires of subject-matter knowledge and their reflective awareness of that knowledge (Schoenfeld, 1985). Included in the teachers' repertoires of subject-matter knowledge are: (1) substantive mathematical knowledge such as

facts, ideas, theorems, mathematical explanations, concepts, processes (and connections between these elements) (2) understanding of knowledge about the nature and discourse of mathematics, (3) knowledge about mathematics in culture and society, and (4) dispositions towards the subject Second, it is also influenced by the teachers' dispositional orientation towards mathematics. Support for this assertion comes from recent research by Lawson and Chinnappan (2015) that found that the successful accessing and exploitation of knowledge is influenced by the organizational quality of the person's knowledge base.

Knowledge about the learner: This includes information that the teachers hold about the learners: (1) repertoires of conceptual and procedural knowledge about the topic (2) misconceptions (3) motivations and, (4) conceptions about learning of the subject matter (Leinhardt et al., 1991). A review of the research literature indicates that the quantity and the quality of the knowledge about the learners accessed and exploited by teachers is based on the quality of the teachers' observations of the learners before and during teaching and the teachers' pedagogical content knowledge perceptions about the learners and perceptions about their roles as a teacher. Pedagogical content knowledge includes components such as (1) understanding the central topics in each subject-matter as it is generally taught to children of a particular grade level; (2) knowing the core concepts, processes and skills that a topic has the potential of conveying to the students; (3) knowing what aspects of a topic are most difficult for the students to learn; (4) knowing what representations (e.g., analogies, metaphors, exemplars, demonstrations, simulations, and manipulations) are most effective in communicating the appropriate understandings or attitudes of a topic to students of particular backgrounds, and (5) knowing what student misconceptions are likely to get in the way of learning.

Knowledge about how to teach mathematics: Leinhardt et al. (1991) have viewed this type of knowledge as being embedded within the teachers' repertoires of *curriculum scripts*. A curriculum script consists of a loosely ordered set of goals and actions that a teacher has built up over time for teaching a particular topic. It contains layers of accumulated knowledge about how to teach the topic, including sequences of ideas or steps to be introduced, strategies for explaining, representations to be used, and markers for concepts or procedures that are likely to cause difficulties for the students (Leinhardt et al., 1991: 89). Curriculum scripts vary in their flexibility and the extent to which they incorporate alternative routes for content presentation or directions a lesson may take based on student input. The curriculum scripts accessed and exploited by teachers before and during teaching are influenced by the teacher's pedagogical content knowledge and the teacher's perceptions about students

Emerging questions for professional development teachers

The review of research on teacher knowledge, actions and student learning led to the following questions:

- What is the quality and the quantity of the mathematical content accessed and exploited by teachers both before and during teaching?
- How can we characterise the above knowledge?
- How does the organizational quality of the teachers' repertoires of substantive mathematical knowledge and their reflective awareness of that knowledge impact on their accessibility and use during teaching?

Theoretical considerations

Schematised knowledge: Schemas are knowledge structures or chunks of meaningful mathematical information. Cognitive psychologists and mathematics educators, in describing knowledge acquisition and utilisation, have adopted the framework of schemas. Mayer (1975) advocated the identification of schema during mathematical learning. It is suggested that learning involves the acquisition and integration of disparate information into schemas. Knowledge that is organised into schemas has been shown to be easier to access than that which is not because, in processing the connected knowledge, less of the limited working memory capacity is used thus freeing up mental resources to analyse the more difficult parts of problems. Sweller (2008) suggested the study of schema or schematised mathematical knowledge would provide useful insights into students' algebraic and geometric knowledge. Chinnappan (1998) used the framework of schemas to analyse Year 10 students' understanding of the properties of two- and three-dimensional figures and theorems that captured relations among angles and sides of such figures. Invariably, connectedness forms an important facet of schema-based examination and analyses of quality of mathematical knowledge. We will be measuring connectedness by considering the type of relations between two or more units of knowledge and the number of such relations. This procedure will be an extension of the scoring rubric that was developed by Chinnappan and Lawson (2005).

Mathematical understanding

While schema analysis is expected to generate data that can be used to make judgments about quality of concepts, we aim to provide a second learning platform for students to exhibit the sophistication of their concepts. Developments in cognitive psychology have given currency to the view that the understanding of a mathematical concept is reflected by the representation of that concept in problems (Anderson, 2009). Thus, one useful way to examine students' understanding of geometry concepts is to analyse the nature of problem representations that they are able to construct with relevant sub-concepts. Accordingly, students will be expected to develop multiple representations for a given set of problems, and articulate differences and similarities among the representations of these problems. In doing so, students can be expected to reveal shades of meaning that they may have developed with not only geometry concepts but also provide evidence of links to and accessing of, say, algebraic knowledge. This ensuing matrix of relations that are internal and external to geometry can provide insights into the richness of students' schemas. Further, the quality of reasoning that students could produce in their justification of links among the representations constitutes a second layer of index of their mathematical understanding (Barmby, Harries, Higgins & Suggate, 2009)

In a review of research on mathematical problem solving, Lesh et al., (2007) argued the need to reconceptualise problem solving as *representational fluency* in ways that would reflect students having to function in high-performance work environments such as engineering and design. He argued that research that provides students with opportunities to reveal their thoughts via model-eliciting activities is needed in order to better examine the quality of mathematics concepts student have acquired. Under the PF condition, students are afforded such opportunities via their interactions with other members of the group. Each member is participating in cycles of information generation, revision and communication, within problem space. He/she is also forced to take responsibilities to access prior knowledge in order to construct representations of the given problem.

Mathematical Knowledge for Teaching

The pioneering work of Shulman led Ball and her associates (Ball, 2000; Ball, Thames & Phelps, 2008) to focus on mathematics teachers and fine tune the knowledge strands that are necessary for teaching mathematics effectively. The outcome of this work was the development of a number of new knowledge clusters for mathematics practice collectively referred to as *Mathematics Knowledge for Teaching* (MKT). We regard MKT as providing a theoretical umbrella for understanding and describing teacher knowledge that is critical to their work. Within MKT, there are two main categories of knowledge: Content (Subject-matter) Knowledge and Pedagogical Content Knowledge. The Content Knowledge category was decomposed into Common Content Knowledge (knowledge of mathematics common to most educated adults), Specialised Content Knowledge (specific and detailed knowledge of mathematics required to teach it) and Knowledge at the Mathematics Horizon. In our attempts to better understand teacher knowledge necessary for supporting high school mathematics, we have been inspired by the above dimensions of teacher knowledge for teaching mathematics proposed by Ball and colleagues.

The model of teacher knowledge (Figure 1), as proposed by Ball, Thames and Phelps (2008) provides a powerful theoretical lens within which to examine and describe teacher knowledge (Figure 1) that can support current curriculum reform movements in mathematics pedagogy.

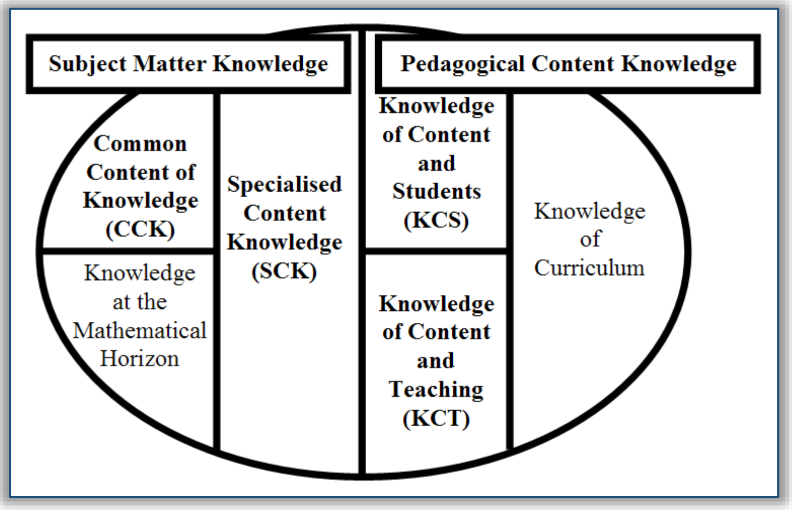

Figure 1: Schematic representation of mathematical knowledge for teaching (Ball, Thames & Phelps, 2008)

Ball *et al.*,'s conceptualisation of MKT led researchers to develop tasks to help identify and measure the various components. However, most of these efforts have been invested in conceptualising and measuring MKT in the context of primary mathematics and Ball (personal communication) suggests there is a need to analyse the character of MKT for secondary mathematics. We attempt to fill this void by focussing on investigating two strands, namely, SCK and PCK of secondary mathematics teachers.

SCK of geometry

In discussions about SCK in geometry, we are concerned with geometric content unique to teaching. This knowledge base includes basic geometric concepts, explanations about the key attributes of these concepts and connections between them, different ways of representing these concepts and how they may be contextualised in human activities, and applications of geometric concepts in the solution of routine and non-routine problems. One example of SCK is the concept of symmetry in 2-D objects. In teaching this core concept, teachers could invoke a range of knowledge fragments including an informal definition of symmetry, a formal definition of symmetry, conditions that need to be satisfied in order for an object to be judged as symmetric, symmetry of a number of 2-D shapes, reasons as to why some objects have a symmetric property while other do not, extensions of symmetry to coordinate geometry and algebra, relationship between symmetry and tessellations, symmetry in arts and so on. Throughout these instances, there is a common knowledge strand about symmetry relevant to teaching its multiple meanings and associations. In their analysis of teacher knowledge for teaching, Chinnappan and Lawson (2005) provided evidence of SCK that teachers have built around the concept of a square. Teachers with a robust body of SCK in geometry can be expected to activate a wide range of SCK. My assumption is that the greater the range and depth of such knowledge the greater will be teachers' ability to flexibly extend this knowledge to their PCK.

PCK of geometry

Pedagogical content knowledge of geometry includes components such as (1) understanding the central geometric topics as generally taught to students of a particular grade level, (2) knowing the core concepts, processes and skills to be conveyed to students in geometry, (3) knowing what aspects of geometry are most difficult for students to learn, (4) knowing what representations (e.g., analogies, metaphors, exemplars, demonstrations, simulations, and manipulations) are most effective in communicating the appropriate understandings or attitudes of a geometry topic to students of particular backgrounds, and (5) knowing related misconceptions likely to get in the way of student learning. Teacher's knowledge about why students experience difficulty with proving tessellations involving 2-D shapes is an example of their PCK.

For example, a teacher's knowledge about the concept of symmetry could include a formal and informal definition, properties of 2-D shapes that lend themselves for symmetry and strategies for determining symmetry. Such bits of information, we argue, can be visually captured within the framework of representations. In analysing SCK, we are interested in identifying teaching tasks that have a high density and cognitively demanding. In their analysis of SCK, Ball, et al., (2008; 400), argued that 'teachers must hold unpacked mathematical knowledge because teaching involves making features of particular content visible to and learnable by students'. In order to make content 'visible and learnable by students' teachers need to draw on complex knowledge structures, namely, representations of geometry.

SCK and PCK in the context of geometric constructions – preliminary data

We are involved in a long-term study about teachers' knowledge strands that girds their practice in the domain of geometry. Figure 2 shows a response from a student who was asked to construct an angle that is exactly 30^0 in size by using a rule and compass only by one of the participating teachers.

We can examine teachers' knowledge from the perspective of a) why would a teacher pose such a problem and b) how she would make judgements about the students' response and explore future learning opportunities as suggested by Sullivan et al., (2009). Let us consider the first perspective. By limiting the students to use ruler and compass, she would like students to access knowledge of properties of equilateral triangle and conceptual basis of bisecting angle. The latter involved drawing arcs one the segment that originate from a vertex, using the cut-off points on the segment and draw another set of arcs, and finally joining the vertex to the point at which the arcs cross each other. Here one notes evidence of multiple facets of teacher's SCK. If we approach the analysis from the second angle, she could be expected to arrive at the conclusion that this student had used the knowledge that all sides of an equilateral triangle are equal in length and bisecting an angle of 60 degrees will yield the solution outcome (30^0). Again, there is evidence of SCK that is relevant to and played out during the course construction.

But what are potential actions of the teachers that could constitute PCK? We are currently generating data to answer this question. We anticipate strands of PCK in this context would emerge from the kind of questions, models and other scaffolds the teacher could provide in assisting students who struggle with the construction task as well as extending students who were successful such as the one in Figure 2.

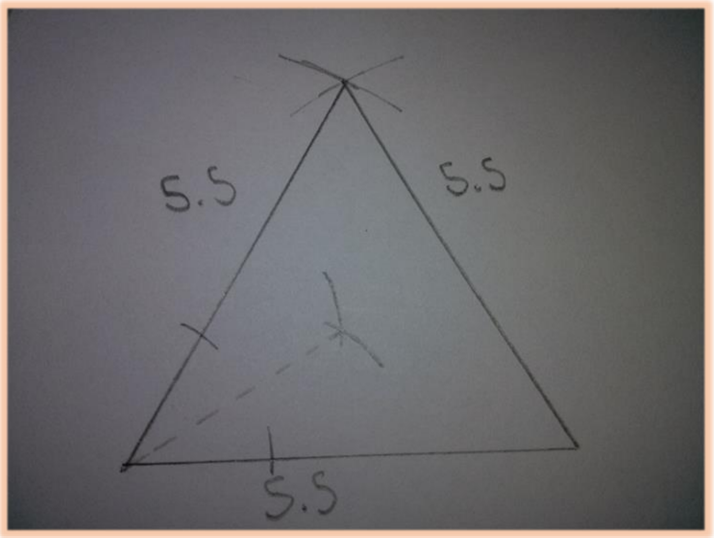

Figure 2

Discussion and conclusion

Teachers and teaching have been found to be the major factors related to students' engagement with and achievement in mathematics. According to the NCTM (2000), 'Effective teaching requires knowing and understanding mathematics, students as learners, and pedagogical strategies' (p. 17). In the current era of globalization and information, teachers' knowledge in teaching mathematics is becoming more complex and dynamic. The unpacking of this knowledge in supporting effective learning has been the aim of a number of studies (Beswick, 2014; Sullivan, 2011). Since the advent of PCK by Shulman (1986), the field has been active in developing other constructs to capture content and pedagogy relevant to mathematics. The question of the relative nature and roles of content vs pedagogy in teaching mathematics is an issue of major concern to mathematics teachers and educators.

In this paper, I have attempted extend the field of teacher knowledge for teaching mathematics by untangling relations between the content and pedagogical knowledge that is relevant to the teaching of geometry. In doing so, our results are expected to address three important problems: a) contribute to the theory building efforts of Herbst and colleagues (Herbst & Kosko, 2014) towards the framework of Mathematics Knowledge for Teaching from a geometry perspective, b) help improve the quality of geometric knowledge that high school geometry teachers need in order to lift the achievement and participation of young Australian (and c) develop the notion of *knowledge connectedness* (Lawson and Chinnappan, 2015) that is relevant to the teaching of geometry.

What are the potential implications for the professional development of mathematics teachers, in general, and of Indonesian teachers in particular? Firstly, there is a need to better understand the complexity of geometry and teaching geometry, and the kind support that Indonesian students need in supporting and developing their spatial reasoning. The work of Lowrie and colleagues at the University of Canberra are generating useful data that are relevant to this issue. My knowledge analysis here shows that current models of professional development programs may not be directing their resources in assisting teachers understand their own *learning needs* that will empower them become on-going learners in enhancing their practices. I would suggest that Indonesian teachers' *learning needs* could be explored by 'unpacking' and mapping their knowledge base that drive their current practices, and following the growth of that knowledge (Chinnappan, 2015) as teachers develop more sophisticated and interconnected geometric knowledge for teaching geometry.

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PLC : Creating a school-eco-system to learning communities

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Abstract

This explores how knowledge can be built and shared in teachers' learning communities. I believe deeply in the power of professional learning communities as a catalyst for ongoing change, improvement and innovation in education; as such, i also believe it is incumbent on all of us in the field of education to be professionals, to be learners, and to work as members of communities or teams. Because teachers need to be knowledgeable in ever-changing contexts, ongoing professional learning simply must be part-and-parcel of their work. But how teacher learning is conceived and practiced constructs the relationship between teachers and knowledge. Should teachers be passive recipients of others' expertise? Should they be researchers, scholars, theorizers? The author suggests teacher learning communities offer the opportunity to recapture a Deweyan approach to teacher professionalism, one that involves systematic observations and analyses of classrooms and student work and ongoing collegial dialogue. The community building literature,combined with an ecological perspective, is used to evaluate the strategies adopted to build a learning community across pre-service teacher, in-service teacher and system level interactions. Both pre-service and in-service teachers were interviewed at the beginning and end of the project and field notes made about the community building strategies employed. The learning community was found to operate like any ecosystem, change was slow and the effects were felt in ways remote from the original intention, however, there is evidence that building a teacher education community resulted in gains in professional learning at all levels with some strategies being more effective than others.At the heart of the author's argument is a vision of teachers not only as users of pedagogical knowledge, but also as creators, disseminators, and preservers of it.

Keywords: *Ecosystem; Learning communities; Professional learning;*

The Implementation of Learning Model Using Inquiry Learning and Discovery Learning To Learning Achievement Viewed From Spatial Intelligence

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Abstract

The goals of this research was to know the effect of the learning models on the learning achievement viewed from the spatial quotient. The learning models compared were Inquiry learning, Discovery learning and Classical model. The type of this study was a quasi-experimental study with a 3 x 3 factorial design. The study population was all grade VIII students of Junior High School in Solo city. Sample was collected by stratified cluster random sampling and consisted of 260 students, divided into three groups, namely: 86 students in Experimental Group 1, 86 students in Experimental Group 2, and 88 students in Control Group . The instruments of the research include mathematics achievement test and spatial intelligence test. The technique of analyze data used the two-way multivariate analysis of variance with unequal cell. The results of this research could be concluded as follows : (1) Inquiry Learning and Discovery Learning gave a better learning achievement than the Classical learning, besides that Inquiry Learning and Discovery Learning gave the same learning achievement; (2) the students with high spatial intelligence gave the same learning achievement with middle spatial intelligence. In addition, students who had high and middle spatial intelligence have better learning achievement than students who had low spatial intelligence.; (3) in each learning model, the students with the high and middle spatial intelligence had a better learning achievement than students who had low spatial intelligence, and the students with the high and middle spatial intelligence had the same learning achievement; (4) in each of the spatial intelligence, the cooperative learning model of Inquiry Learning and Discovery Learning gave a better learning achievement than the Classical model, and the cooperative learning model of Inquiry Learning and Discovery Learning gave the same learning achievement.

Key words: *Inquiry Learning, Discovery Learning, Spatial Quotient, Learning Achievement, Communication Mathematics.*

Introduction

Civilization of a country will progress rapidly if citizens can learn, understand, develop and apply the rules of mathematical science in various aspects of life well. Classic problems which still continue to be pursued for improvement is still low mathematics achievement of students in various elementary to secondary education.

One of the cities in Indonesia that learners have low mathematics achievement is the city of Surakarta. Based on data from 2014 showing off show that mastery of math national exams SMP / MTS in the academic year 2013/2014 is still low, particularly in understanding the nature and elements of geometry and use them in problem solving. Absorptive capacity of learners in mastering this subject at 59.92% is still below the national average of 60.58%.

Learners who have low mathematics achievement, especially in understanding the material geometry or geometry due to the process of learning that takes place not running optimally and appropriate. This indicates that students still have not mastered the overall properties of the geometry flat side, so that in its application to construction problems, combined with the calculation of volume, area and perimeter geometry flat sides learners are still difficulties in resolving issues relating to the concept.

The low achievement of students' mathematics learning is influenced by many factors related to the learning process in schools as learning materials are too abstract and less attractive, teaching methods always centered on teachers so that students tend to be passive and not have the opportunity to think mathematically (Slameto, 2003) , With these conditions, the paradigm of the learning process that is more focused on the results slowly began amended to direct the learning process quality and meaningful. The role of teachers and learners should be actionable on an ongoing basis and have a positive feedback.

In addition to learning achievement, in the learning process also includes communication activities. Communication is a fairly important part of mathematics and mathematics education (Hirschfeld, 2008: 4) in Dona (2013). Learners through communication can convey ideas that are owned and clarify understanding, concerning what is being studied. Learners can develop the ability to communicate with the express language, both written and verbal connection with matters mathematical, so communication can be defined mathematically as a way of delivering ideas with language and mathematical symbols.

Most of the learning model used by teachers in the learning process is still not optimal to improve the understanding or the potential of the self-learners that mathematics achievement obtained are still low. The trend in the implementation model of learning, the teacher is still a central role in the teaching process and the involvement of students still have not seen optimally.

One of the success factors in improving learning achievement of learners is to directly involve their contribution in the teaching process with the use of appropriate learning models. Successful teachers are teachers who can implement learning models that match the characteristics of learners and can provide a positive impact through the role of learners increasingly widespread in the educational process.

In connection with the retention of material geometry learners are still low, then the learning model that is suitable is constructivism learning model. Learning constructivism is a learning process that explains how the knowledge compiled in man (Ali Hamza & Muhlisrarini, 2014). Based on the understanding of constructivism, in the learning process, the teacher does not necessarily transfer knowledge to students in the form of the all perfect. In other words, students must build a knowledge based on their experiences. In other words, a learning experience can be effective and meaningful if in the process of teaching and learning activities that learners can create learning experiences that can be, as expressed Dheeraj and Kumari (2013: 1) that:

> *The constructivists proposed several instructional strategies, among the cooperative learning, collaborative learning, problem based learning etc. are prominent. Therefore, to keep pace with the changing circumstances, we should not keep ourselves aloof from new experiments in the field of education to make learning more effective and enjoyable experience for pupils.*

There are several constructivism learning model that has been developed by experts and practitioners, including the discovery based learning model is done cooperatively by learners. Through studies conducted by Wilcox (Slavin, 1977) in Hosnan (2014) on learning-based discovery, learners are able to actively participate and contribute to the learning process through experiments that have been conducted cooperatively so that learners can discover principles and concepts independent and formed a new knowledge within them.

Discovery-based learning model that has been developed by educational experts of which is inquiry learning and discovery learning. Both the learning model has been proven to enhance the participation of learners in the learning process, so that the learning achievement can be maximized.

Inquiry learning model (inquiry learning) emphasizes the process of seeking and finding. The role of learners in this method is to seek and find themselves the subject matter, while educators act as a facilitator and mentor learners to learn. Inquiry learning is a series of learning activities that emphasize critical thinking and analytical processes to seek and find their own answer to the problem in question.

In addition to inquiry learning models, appropriate learning models used in the study to discuss the material geometry flat side is a model of discovery learning and also a learning model that was developed based on the views of constructivism. As a model of learning, discovery learning has the same principles as the inquiry learning. There is no difference of principle on these two terms, in discovery learning more emphasis on the discovery of concepts or principles that were previously unknown. Differences discovery and inquiry can be seen on the problem given by the teacher. On discovery problems that confronted the students is a problem that is engineered by the teacher. While the problem is not the result of engineering inquiry, so that learners have to put all your mind and skill to get the findings in the matter through the trial process.

Application of inquiry model of learning, discovery learning with, or classical on mathematic learning is one way so that learners can solve problems related to mathematics, especially in the writing of these learners can master the concepts of geometry well.

The learning activities geared to empower all the potential of the learners so that they can have the competencies expected through efforts to grow and expand; attitude / attitude, knowledge / knowledge, and skills / skill (Hosnan, 2014). Another quality that developed the curriculum and should be realized in the learning process of which is a double boost intelligence (multiple intelligences) owned by each learner. Double intelligence which is on each individual learner plays an important role in optimizing their potential and learning achievement.

One type of multiple intelligences are visual-spatial intelligence. According Yaumi (2012) visual-spatial intelligence, or so-called spatial intelligence is the ability to understand images and forms including the ability to interpret spatial dimensions can not be seen. Spatial intelligence is largely dependent on the ability to draw shapes and space of an object, an ability to think of the form.

Related to spatial intelligence, research that has been done by Harmony and Theis (2012) states that there is interaction between the learning outcomes of spatial intelligence mathematics students of class VII. Siti Marliah Tambunan (2006) states that there is a relationship between the spatial total, topology and Euclidean mathematics learning achievement, but there is no relationship between the projective spatial intelligence and academic achievement in mathematics. Hoong and Khoh (2012) states that the level of spatial intelligence have a significant effect on mathematics achievement.

Achievement of learning and communication skills of learners will be seen in the learning process with the application of inquiry learning and discovery learning with one of the multiple intelligences owned by learners ie spatial intelligence. Therefore, researchers wanted to know the effect of the application of models of inquiry learning and discovery learning in terms of spatial intelligence to learn and communicate mathematical achievement of learners.

RESEARCH METHODS

This research is quasi-experimental research design research factor 3 x 3. The study population was all students in grade VIII SMP Negeri Surakarta throughout the sampling using stratified cluster random sampling. Samples obtained is class VIII SMP Negeri 7 Surakarta for the high school category, SMP Negeri 14 Surakarta for medium school category, and SMP Negeri 20 Surakarta to lower school category.

The dependent variable in this study is the learning achievement and mathematical communication, while the independent variables the model of learning and spatial learners. Data collection techniques used in this research is the method of documentation and testing. Methods used to collect data documentation capabilities early learners is drawn from the final exam scores odd semester academic year 2014/2015 as initial data learning achievement. The test method used to measure student achievement, communication skills mathematical and spatial intelligence. The tests used to measure learning achievement and mathematical communication after the treatment using material cubes and blocks, while tests to measure mathematical communication scratch using materials circle, while tests for spatial intelligence using questions multiple choice obtained from about psychological test and other sources that can be used as a reference to measure the spatial learners.

Before the treatment of samples, test the balance using multivariate analysis of variance test a different cell and concluded that all three samples had the same initial capability or balanced.

Test instruments that will be used in research conducted prior validation of the content, internal consistency test, test distinguishing features, and reliability testing to determine the feasibility of test items. Test the hypothesis of the study conducted by multivariate analysis of variance test two different cell, for prerequisite test before hypothesis test include univariate and multivariate normality test, then test the homogeneity of variance and variance covariance matrix equality test. After testing the hypothesis test berkutnya step further by analysis of variance of two different cell, to determine the final conclusions followed by the multiple comparison test Scheffe method '.

RESULTS AND DISCUSSION

Based on data from the study, performed the prerequisite test first and then test the hypothesis with the test Wilks'. From the results of univariate and multivariate normality test can be concluded that the population of univariate and multivariate normal distribution. To test the homogeneity of variance and variance covariance matrix equality test research data shows that the population has the variance and the variance covariance matrix of the same. Summary of multivariate analysis of variance test two roads with different cells can be seen in Table 1.

Table 1. Multivariate Analysis of Variance Summary of Two Way with Sel Not The Same

Sumber Variasi	Matriks SSCP	*Dk*	*Λ*	F_{obs}	F_{α}	Kep Uji
Faktor A	$\boldsymbol{H_A} = \begin{bmatrix} 17588,43 & 17270,1 \\ 17270,1 & 17771,28 \end{bmatrix}$	2	0,706	11,23	2,389	H_0A rejected
Faktor B	$\boldsymbol{H_B} = \begin{bmatrix} 7593,37 & 7959,03 \\ 7959,03 & 8465,17 \end{bmatrix}$	2	0,851	23,46	2,389	H_0B rejected
Interaction	$\boldsymbol{H_{AB}} = \begin{bmatrix} 3895,20 & 2733,87 \\ 2733,87 & 2230,51 \end{bmatrix}$	4	0,953	0,981	1,957	H_0 Accept
Galat	$\boldsymbol{H_E} = \begin{bmatrix} 55715,66 & 43534,71 \\ 43534,71 & 51907,83 \end{bmatrix}$	251	-	-	-	-
Total	$\boldsymbol{T} = \begin{bmatrix} 84792,67 & 71497,72 \\ 71497,72 & 80374,79 \end{bmatrix}$	260	-	-	-	-

Based on Table 1, it can be concluded that: (1) there are differences in effects between learning model of learning achievement and mathematical communication, (2) there are differences in effects between the spatial intelligence of learning achievement and mathematical communication, and (3) there is no interaction between the learning model and spatial intelligence to the student achievement and mathematical communication. Table 1 shows that H_0A rejected, H_0B rejected, and H_0AB accepted. Thus, the need for further testing on each dependent variable learning achievements (X1) and mathematical communication (X2) with variance analysis test two different cell. The following summary of the analysis of variance indicated two different cell in Table 2.

Table 2. Univariate Analysis of Variance Summary Two Way Cells Not The Same

Sumber Variasi	Matriks SSCP	*Dk*	*Λ*	F_{obs}	F_{α}	Kep Uji
Faktor A	$\boldsymbol{H_A} = \begin{bmatrix} 17588,43 & 17270,1 \\ 17270,1 & 17771,28 \end{bmatrix}$	2	0,706	11,23	2,389	H_0A rejected
Faktor B	$\boldsymbol{H_B} = \begin{bmatrix} 7593,37 & 7959,03 \\ 7959,03 & 8465,17 \end{bmatrix}$	2	0,851	23,46	2,389	H_0B rejected
Interaction	$\boldsymbol{H_{AB}} = \begin{bmatrix} 3895,20 & 2733,87 \\ 2733,87 & 2230,51 \end{bmatrix}$	4	0,953	0,981	1,957	H_0 accepted
Galat	$\boldsymbol{H_E} = \begin{bmatrix} 55715,66 & 43534,71 \\ 43534,71 & 51907,83 \end{bmatrix}$	251	-	-	-	-
Total	$\boldsymbol{T} = \begin{bmatrix} 84792,67 & 71497,72 \\ 71497,72 & 80374,79 \end{bmatrix}$	260	-	-	-	-

Based on the results of the univariate analysis of variance of two different cell known that the learning model and spatial intelligence on each dependent variable Fobs> F α , consequently F_obs∈DK so H0A and H0B rejected, while the interaction effect ⩽ F α Fobs, consequently F_obs so H0AB accepted DK , then with a significance level of 5% can be concluded that: (1) there are differences in effects between learning model of learning achievement and ability of mathematical communication, (2) there are differences in effects between the spatial intelligence of learning achievement and ability of mathematical communication, and (3) do not No interaction between the learning model and spatial intelligence to the student achievement and mathematical communication skills.

Having obtained the results of univariate two different cell stating that H0A and H0B rejected, then to know which treatment is significantly different from the other multiple comparison test conducted on each dependent variable. In this study, the multiple comparison test used is the Scheffe method. Before going into multiple comparison test, the mean marginal presented in each cell in Table 3.

Table 3. Average Data Research

Learning Model (Faktor *A*)		Spatial Intelligence (Faktor *B*)			Average Marginal Low
		High	Medium	Low	
Inquiry Learning	Learning Achievement(X_1)	73,826	70,826	50	61,942
	Mathematics Communication (X_2)	73,098	70,036	48,214	60,667
Discovery Learning	Learning Achievemen (X_1)	72,727	71,277	58,560	63,47
	Mathematics Communication (X_2)	70,682	62,628	59,00	63,64
Clasical	Learning Achievemen (X_1)	63,900	69,820	45,920	51,707
	Mathematics Communication (X_2)	57,875	61,644	42,45	49,048
Average Marginal Column	Learning Achievemen (X_1)	71,151	70,477	51,490	64,372
	Mathematics Communication (X_2)	66,30	65,020	50,580	60,633

The next stage presented a summary of the multiple comparison test between lines by Scheffe method 'on each of the dependent variables in Table 4.

Table 4. Summary Comparison Average Interlinear

The dependent variable: learning achievement

H_0	F_{obs}	$(a\text{-}1)F_\alpha$	Decision
$\mu_{11.} = \mu_{12.}$	0,307	6,064	H_0 accepted

$\mu_{11.} = \mu_{13.}$	13,945	6,064	H_0 rejected
$\mu_{12.} = \mu_{13.}$	18,419	6,064	H_0 rejected

The dependent variable: mathematical communication

H_0	F_{obs}	$(a\text{-}1)F_\alpha$	Decision
$\mu_{21.} = \mu_{22.}$	2,048	6,064	H_0 accepted
$\mu_{21.} = \mu_{23.}$	14,837	6,064	H_0 rejected
$\mu_{22.} = \mu_{23.}$	27,998	6,064	H_0 rejected

Based on Table 3 and Table 4, obtained the following conclusions: (1) inquiry learning model learning and discovery learning achievements of learners acquire better than classical learning model and the model of inquiry learning and discovery learning achievements of learners acquire the same; (2) The inquiry learning model of learning and discovery learning acquire mathematical communication ability learners are better than classical learning model and the model of inquiry learning and discovery learning acquire the same mathematical communication skills.

For multiple comparison test between columns on each dependent variable that knowledge and skills are presented in Table 5.

Table 5. Comparison of Mean Inter Summary ColumnVariable Bound: Learning Achievement

H_0	F_{obs}	$(b\text{-}1)F_\alpha$	Decision
$\mu_{1.1} = \mu_{1.2}$	2,312	6,064	H_0 accepted
$\mu_{1.1} = \mu_{1.3}$	42,23	6,064	H_0 rejected
$\mu_{1.2} = \mu_{1.3}$	32,39	6,064	H_0 rejected

Variable Bound: Mathematical Communications

H_0	F_{obs}	$(b\text{-}1)F_\alpha$	Decision
$\mu_{2.1} = \mu_{2.2}$	3,91	6,064	H_0 accepted
$\mu_{2.1} = \mu_{2.3}$	27,25	6,064	H_0 rejected
$\mu_{2.2} = \mu_{2.3}$	14,45	6,064	H_0 rejected

Based on Table 3 and Table 5, obtained the following conclusions: (1) the achievement of learners with high spatial intelligence and is better than students with low spatial intelligence, and learning achievements of learners with high spatial intelligence and is the same; (2) mathematical communication kemampuna learners with high spatial intelligence and is better than students with low spatial intelligence, mathematical and communication skills of learners with high spatial intelligence and is the same.

There is conformity between the hypotheses of the study and research which states that the model of Inquiry Learning and Discovery Learning produces learning achievement and communication skills mathematically better than learning model classical, but the learning model inqury and learning model of Discovery, the achievements of learning and communication skills mathematical same. These findings were influenced by the characteristics of Inquiry and Discovery learning model that directs learners to think critically, creatively and inofativ in order to find new things relating to the subject matter being studied. Learners become the subject of education and central role, and can construct their own knowledge that they will get through the learning process. Teaching based on inquiry centered on learners where they have a duty to enter into an issue or to seek answers to the question contents through a procedure outlined clearly and can work with both groups (Kourilsky in Oemar, 2001). Learning discovery learning model that is used in a constructivist approach that aims to guide the students more active role in the learning process. Cranton (Devi, 2014) explains that in active learning, learners are immersed in an experience where they are involved in the manufacture, investigation, action, imagination, discovery, interaction, hypotheses and personal reflection.

In this study, using a review of spatial intelligence shows that the achievement of learning and communication skills mathematical learners with intelligence high spatial better than on the learner with the level of spatial intelligence is low and learners with spatial intelligence is producing learning achievement and communication skills mathematically better than learners with low spatial intelligence. These findings have caused a significant difference between the level of spatial intelligence possessed learners closely related to learning outcomes.

The results also showed that the absence of the effect of interaction between the learning model and spatial intelligence, so that in view of connectivity in every learning model with spatial intelligence refers to the first and second hypothesis. On learning model Inquiry Learning, learners who possesses high spatial acquire learning achievements and abilities mathematical communication are better than the spatial intelligence is low and learners who have spatial intelligence was mengasilkan achievement of learning and communication skills mathematically better than the intellect while low spatial learners who have high spatial intelligence and is producing learning achievement and the same mathematical communication skills. On learning model Discovery Learning, learners who possesses high spatial acquire learning achievements and abilities mathematical communication are better than the spatial intelligence is low and learners who have spatial intelligence was mengasilkan achievement of learning and communication skills mathematically better than the intellect while low spatial learners who have high spatial intelligence and is producing learning achievement and the same mathematical communication skills. While learning the classical, learners who possesses high spatial acquire learning achievements and abilities mathematical communication are better than the spatial intelligence is low and learners who have spatial intelligence was mengasilkan achievement of learning and communication skills mathematically better than the spatial low whereas students who have high spatial intelligence and is producing learning achievement and the same mathematical communication skills.

In addition, it also stated that there is no interaction effect between each spatial intelligence and learning model, so in view of connectivity in every spatial learning model refers to the first and second hypothesis. For students who have spatial intelligence high, school performance and communication skills mathematically derived the same if given the learning model Inquiry Learning and Discovery Learning, and achievement of learning and communication skills mathematical learners obtained will be better if given learning model Inquiry Learning and Discovery learning compared to classical learning model. For students who have spatial intelligence being, academic achievement and communication skills mathematically derived the same if given the learning model Inquiry Learning and Discovery Learning, and achievement of learning and communication skills mathematically obtained will be better if given learning model Inquiry Learning and Discovery learning compared to classical learning model. As for the students who have spatial intelligence is low, learning achievement and communication skills mathematically derived the same if given the learning model Inquiry Learning and Discovery Learning and learning achievement and communication skills mathematically obtained will be better if given learning model Inquiry Learning and Discovery learning compared to classical learning model.

CONCLUSIONS AND RECOMMENDATIONS

Based on the study of theory and the results of the discussion and formulation of the problem which has been described earlier, then compiled the conclusions in this study: (1) The achievement of learning and communication skills mathematically derived learners in a learning-based discovery, on the model of Inquiry Learning and model of Discovery Learning better than the classical model of learning and teaching model of Inquiry as well as Discovery learning model. (2). Learners with high spatial intelligence acquire learning achievement and mathematical communication skills are better than learners who have low spatial intelligence. Learners with spatial intelligence is gaining learning achievements and communication skills mathematically better than learners who have spatial intelligence is low, whereas learners who have intelligence high spatial acquire learning achievements and communication skills mathematically as good as the learners that has spatial intelligence being. (3). Learning on the model of Inquiry Learning, Discovery Learning, and classical, learners with spatial intelligence high achievement of learning and communication skills mathematically better than the students with spatial intelligence is low, and learners with spatial intelligence are having learning achievement and communication skills mathematically better than the students with low spatial intelligence, whereas spatial learners with learning achievement and the ability to produce the same mathematical communication with students with moderate spatial intelligence. (4). Learners who have spatial intelligence high, medium, and low achievement of learning and communication skills mathematically better if given learning-based discovery learning model inquiry and discovery than the learning model classical, and achievement of learning and communication mathematical subject learning model inquiry as well as students who are subject to discovery learning model.

In this study gives an idea associated with increased learning achievement and communication skills mathematically, it is advisable Teachers should apply the learning model of inquiry and discovery as a variation and as one of the references in the classroom and all elements of education are expected to provide facilities and learning resources adequate, necessary in the learning process to support creativity and improve learning achievement, in addition to the authors hope that researchers or prospective researchers can continue or expand research using the model of inquiry learning and discovery learning with the reviews that others, including the multiple intelligence eg mathematical intelligence logical, intrapersonal intelligence, interpersonal intelligence, and so forth.

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A Learning Trajectory of Indonesian 12-Year-Olds' Understanding about Division on Fraction

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Abstract

The purpose of this study is to describe the mathematical-hands-on-activities that can support students to gain their better understanding in dividing fractions. This study is started by testing, analyzing, and refining the initial hypothetical learning trajectory (HLT) (preliminary research phase), implementing the revised HLT (pilot experiment phase), and ended by developing a learning trajectory of 12-year-olds students in understanding division on fraction (teaching experiment phase). In developing the trajectory, a design research is employed by using four contextual-based learning series (biscuit sharing, sharing the remaining chocolate bar, arranging bedroom mats, and run surrounding ceremony flag field), including providing some concrete materials or pictorial models as its manipulative tools. Seven mathematics experts and twenty-five 12-year-olds students are involved during the research. Among four designed learning goals, they are determining the quotient of division of integer by proper fraction, proper fraction by integer, two proper fractions, and two fractions, students succeed to demonstrate their understanding and stated that "16 : 1/2 = 32", "25/36 : 5 = 5/36", "24/64 : 1/2 = 6/8", and "15/2 : 3/4 = 10" respectively in the end of each designed activity.

***Keywords**: division on fraction, learning trajectory, understanding*

The concepts of fraction have been known by Indonesian students informally through their daily life activities and are also introduced formally at the primary and secondary levels. Although students have learned the concepts of fraction, their lack of understanding on particular concepts of fraction sometimes happens in their learning process (Rifa'at, Parvati, & Tambelu, 1996). Students' lack of opportunities to solve various daily life problems related to the concepts of fraction (Meng & Sivasubramaniam, 2009) causes difficulty in assisting students to develop their formal thinking. This situation can lead to students' perception about how difficult to learn the concept of division on fraction. Consequently, students' performance on tasks involving division on fraction is typically very poor (Gregg & Gregg, 2007).

Students' difficulties in learning the concepts of division on fraction may be caused by the transition from "*teaching by telling*" to "*students constructing*" (Gravemeijer, 2004a) is not fully implemented. Under these circumstances, there is a belief that contextual. In designing a contextual -based classroom learning activity for 12-year-old students, teacher should know well about the four Piaget's stages of students' cognitive. Valanides (1998) stated that 7-12 years old students are generally classified into concrete operation stage. In other words, 12-year-old students will understand the construction of certain concepts by the help of concrete materials (Saondi, 2011). The use of suitable concrete materials and pictorial models are believed can support students to understand the concepts of division on fraction. The use of concrete materials and pictorial models can deepen students' understanding on the concepts of fraction and support students in constructing procedures on division on fraction (Cramer, et al., 2010).

THEORETICAL FRAMEWORK

Division on Fraction

Division on fraction is a division that involves fractions. Sinicrope, Mick, and Kolb (2002) generally classified five possible interpretations of division on fraction. Those interpretations are measurement division, partitive division, the determination of a unit rate, the inverse of multiplication, and the inverse of a Cartesian product. By using measurement division interpretation, division “4 : 1/2” can be interpreted by answering question ”*How many halves in 4*?” while by using partitive division interpretation, division “8/3 : 4” can be interpreted by distributing eight thirds of pie among four people. The determination of a unit rate is focused on finding the unit rate. For example, ”*Andi can swim 20 m in 2½ minutes. How many meters that can be reached by Andi in 1 minute*?”. In this case, the unit rate can be found by dividing 20 with 2½. By using inverse of multiplication interpretation, dividing p/q by r/s has the same meaning with multiplying p/q by the inverse multiplication of r/s. The inverse of Cartesian product interpretation is focused on the process of finding the missing length of rectangle, which its area and one of its side’s length are given.

Students’ Understanding

Students can be said understood if they can relate the new knowledge with the existing concepts in students’ scheme (Skemp, 1987). Their representation of thinking becomes an indicator that they have understood particular concepts. Students can take advantage of the graphical and symbolic languages to explore their thinking and allow teachers a window on children's understandings (Worthington, 2005). Knowledge and the importance of visual representations (pictorial models) are needed since pictorial model is a powerful cultural tool that supports learning and has an important role in students’ developing understanding (Worthington, 2005). At this point, students’ verbal communication, representation of their thinking, and solution can be the indicator of their understanding.

Learning Trajectory

Learning trajectory is a description of the learning activities route that can be understood by students in building their understanding. Simon (as cited in Armanto, 2002) defined the learning trajectory as Hypothetical Learning Trajectory (HLT). HLT is made by considering three main components, namely learning goals, learning activities that can be used to support learning, and conjectured learning and thinking (Simon & Tzur, 2004). The conjectured learning and thinking can be the result of thinking and learning process created by students during learning certain concepts (Winarti, 2011).

METHOD

Design research becomes the focus of this study. Design research is implemented in three phases, they are preliminary design, teaching experiment, and retrospective analysis (Gravemeijer, 2004b). Preliminary design is started by clarifying the learning objectives and listing the anticipation actions during the learning process to form initial HLT. Teaching Experiment emphasizes on the testing of initial HLT to form revised HLT while retrospective analysis is aimed to compare the revised HLT with the actual learning process.

Student Selection

This study is divided into two parts, namely pilot and teaching experiment. Pilot experiment is conducted based on the initial HLT and teaching experiment is implemented based on the revised HLT. Both pilot and teaching experiments are conducted in SD Laboratorium Universitas Negeri Surabaya (UNESA), Surabaya. In the pilot experiment, six 12-year-old students were involved. After doing several improvement, the revised HLT will be implemented to the whole class in the “real” teaching experiment. Twenty 12-year-old students out of the ones that involved in pilot experiment were involved in this experiment.

Data Collection

The procedures in collecting the data during the study are pre- and post-test, observation, interview, and questionnaire. The collected data will be analyzed retrospectively by using the HLT as its guide. All the learning processes are recorded and transcribed. The process of data analysis involves some mathematics experts to have a different interpretation. In order to check the validity and reliability of the study, data triangulation and HLT as means to support validity are used to measure the validity of the result of this study while cross interpretation and trackability are techniques in measuring the reliability.

Teaching Episodes

This study emphasizes on how students learn the four concepts of division on fraction. The summarized teaching episodes (see Table 1), including its tasks, had been checked its validity by five mathematics experts from UNESA.

Table 1. Teaching Episodes

No	Activity	Objective	Description
1	Biscuit Sharing	Finding the quotient of integer by proper fraction	This activity is designed to support students in distributing halves of biscuits among several students. Students are asked to find the number of students to get half of biscuit.
2	Sharing the Remaining Chocolate Bar	Finding the quotient of proper fraction by integer	This activity is designed to support students in distributing the remaining chocolate bar among some students. Students are asked to find the fraction received by each students.
3	Arranging Bedroom Mats	Finding the quotient of two proper fractions	This activity is designed to support students in finding the missing length of the bed if the area and the width of the bed are given. Students are asked to find the length of the bed.
4	Run Surrounding Ceremony Flag Field	Finding the quotient of two fractions	This activity is designed to support students in determining the needed duration to run 1/4 round and one full round of the field.

RESULTS

Designing Initial and Revised HLT

An initial HLT is formed in preliminary design phase. This initial HLT consists of three main components, namely learning objectives, the planned learning activities including its media, and conjectured learning and thinking. Two mathematics experts and a mathematics teacher are involved at this phase. The research was continued by the testing of initial HLT in pilot experiment phase. The information and experiences that are obtained from this phase become a consideration in redesigning the initial set of learning activities that have been developed and to find other new conjectured learning and thinking of students' mental activities. Three groups consisting of 2 students each are involved during this pilot experiment in order to form Revised HLT (see Appendix A).

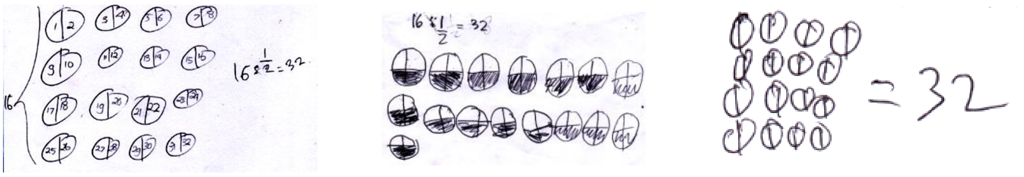
Teaching Experiment Phase

Activity 1

Students' interpretation about division on fraction as measurement division became the mathematical idea in conducting this activity. According to students' answer in figuring out the given prerequisite problems, students succeeded in showing how to represent the given pictorial model in its appropriate fraction or to visualize the given fraction in its appropriate pictorial model. The following figure shows the problem and students' written works during the activity 1.

Problem 1

Andi has 16 biscuits. He will share those biscuits among his friends such that each person has a half of biscuit. How many friends is to get Andi's biscuit?



(a) Group 3 (b) Group 4 (c) Group 5

Figure 2 Problem and Student's Written Works in Activity 1

By considering Figure 2, three groups drew sixteen representations of biscuit, drew line in each pictorial model as divider, and counted the number of halves. Students in group 5 left the pictorial models unshaded (see Figure 2.c) while students in group 4 shaded one part of halves on each pictorial model (see Figure 2.b). Both groups also counted the number of halves mentally. Students in group 3 also left the pictorial models unshaded or uncolored, but they wrote number 1 to 32 as counter in determining the number of halves (see Figure 2.a).

Students in group 3 succeeded in interpreting division on fraction as a measurement division. It could be seen when during the interview students stated that finding the quotient of "16 : 1/2" was similar to find the number of halves in sixteen. Students in group 4 did more advance in interpreting division on fraction. Instead of using measurement to interpret "16 : 1/2", they 4 also interpreted "16 : 1/2" as determination of unit rate. This unexpected idea could be seen when students stated in interview that if each student would get one biscuit, so there were sixteen students who get one biscuit. Students also succeeded in determining the number of students who got the biscuit if there were two biscuits to be given each. Finally, by using this interpretation, students succeeded in showing that the quotient of "16 : 1/2" was 32. In revised HLT, students also expected to do the real cutting of biscuits. This expected action was shown when students in group 1 and group 2 tried to figure out the given problem by cutting the given biscuits.

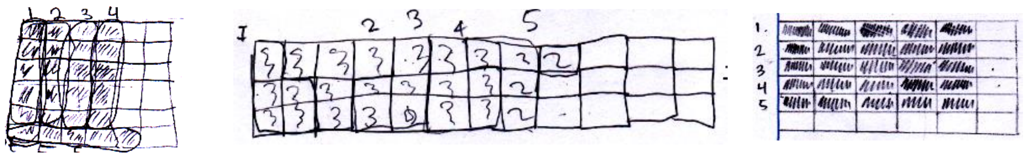
Activity 2

Students' interpretation about division on fraction as partitive division became the mathematical idea in conducting this activity. According to students' answer in figuring out the given prerequisite problems, instead of using circle, square, or other 2-dimensional shapes, students used rectangle in representing fraction in pictorial model. The following figure shows the problem and students' written works during the activity 2.

In representing the remaining blocks, students knew that those remaining blocks could be represented as fraction 25/36. Students also succeeded in expressing the fraction in its appropriate pictorial model. Although students could sketch the pictorial model of fraction 25/36, in sketching the pictorial model of the remaining chocolate bar, students were used several dimensions. Only students in group 4 who sketched the pictorial model exactly the same with the dimension of the chocolate bar (see Figure 3.d).

Problem 2

Teacher brings a chocolate bar. The chocolate will be distributed eventually as a gift to five students who earn higher score. Unfortunately, some blocks have already eaten. If the remaining is 25 of 36 blocks, how many part of chocolate bar got by each student? Express it in fraction.



(a)
Group 1

(b)
Group 2

(c)
Group 3

(d)
Group 4

(e)
Group 5

Figure 3 Problem and Student's Written Works in Activity 2

The activity continued by asking students to determine the number of blocks got by each student. Students stated that each student would get five blocks. As expected in revised HLT, in determining the blocks got by each student, grouping became students' main idea. Some students grouped each five blocks in each row (see Figure 3.c), some students grouped each five blocks in each column (see Figure 3.e), and some students grouped each five blocks in each row or column (see Figure 3.d). The rest of students grouped each five adjacent blocks randomly. The next activity was asking students to form the blocks got by student in fraction. Most of students succeeded in forming the five blocks as fraction 5/36 by their selves, although some students needed to be guided to express it into fraction 5/36.

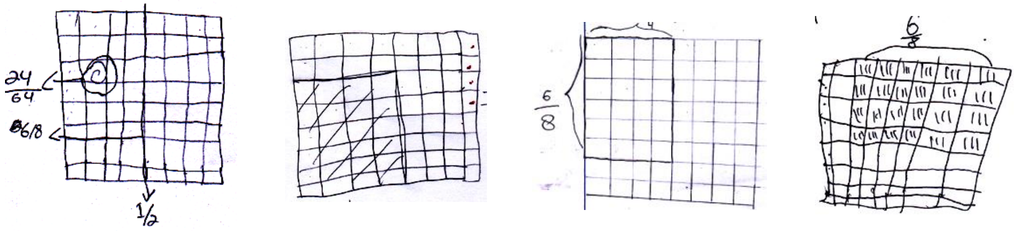
The interesting finding was occurred when students in group 1 and group 4 wrote statement "25/36 : 5/1 = 5/36" on their sheets. After conducting interview, students stated that numerator of the quotient "5" could be found by dividing the numerator of the dividend by the numerator of divisor (25 ÷ 5) and the denominator of the quotient "36" could be found by dividing the denominator of dividend by the denominator of divisor (36 ÷ 1).

Activity 3

Students' interpretation about division on fraction as the inverse of a Cartesian product became the mathematical idea in conducting this activity. According to students' answer in figuring out the given prerequisite problems, students have shown their understanding in determining the certain length on each side of square and determining the area of rectangle. The following figure shows the problem and students' written works during the activity 3.

Problem 3

The shape of Arya's bedroom is square. 64 puzzle mats already installed on its floor. Since the new Arya's bed arrives, then some mats must be removed and will be replaced by the bed. The bed will be placed in one corner of the room. If the width of the bed is a half of the length side of the room and the bed will cover 24/64 of the whole bedroom floor, the length of bed is ....



(a) Group 1 (b) Group 2 (a) Group 3 (d) Group 5

Figure 4 Problem and Student's Written Works in Activity 3

In the beginning of the lesson, students were expected to sketch the bedroom and the bed. In order to support students in sketching the bedroom and the bed, some models of mats puzzle were provided to each group. According to the collected data, students succeeded in forming the given models of mat to form the tiles on the bedroom. The problem occurred when students in group 4 were difficult to arrange the tiles. It was because of students' cooperation did not work properly. One student arranged the tiles from top to bottom, while another student arranged the tiles from left to right. Therefore, instead of arranging bedroom 8 × 8, the arranged bedroom's tile was 8 × 9. After several discussions, students in group 4 agreed to form 8 tiles from left to the right and 8 tiles from the top to the bottom.

It was also expected that students would choose on which corner that the bed would be placed. Four groups did what was expected in revised HLT by sketching the pictorial model of bedroom and the bed (see Figure 4). Only students in group 4 who did not sketch the pictorial model of the bedroom and the bed on their sheet. Instead of sketching the arranged models of mat to their answer sheet, students chose to let the arrangement un-sketched.

By considering Figure 4, since the width of the bed compared with the length of the bedroom was 1/2, so students divided the length of the bedroom by 2 and took one part as the width of the bed. As expected in revised HLT, it was possible for students to write "1/2" as the width of the bed on their pictorial models (see Figure 4.a), although students also expected to leave the width unmarked (see Figure 4.b). Since the area of the bed was 24/64, students removed 24 models of mat. Unfortunately, the similar problem that occurred in pilot experiment also happened in this activity. Students in group 3 removed 24 of 64 mats without considering the given condition first. The similar anticipation also used to support students in figuring out the problem.

The next step was determining the length of the bed compared with the length of the bedroom. It was expected that students would state 3/4 length side of the bedroom became the length of the bed. Unfortunately, no students simplified the length of the bed. Students used 6/8 instead of 3/4 (see Figure 4a, Figure 4.c, and Figure 4.d). As stated before, students in group 4 did different way in answering the given problem. Instead of sketching the arrangement of the models of mat to be a pictorial model, students explained their answer in written. The interesting finding was that the consistency of this group in expressing the given questions into division on fraction. Students in group 4 ended their answer on Activity 1, 2, and 3 respectively as "16 : 1/2 = 32", "25/36 : 5/1 = 5/36", and "24/64 : 1/2 = 6/8".

Activity 4

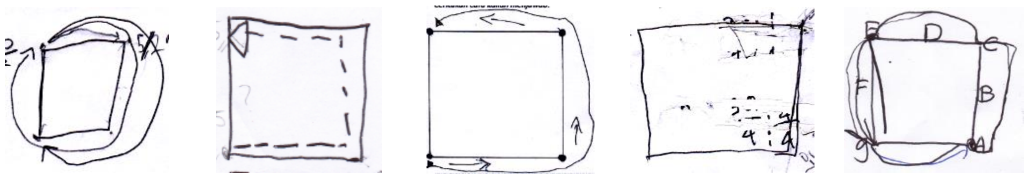
Students' interpretation about division on fraction as the determination of a unit rate became the mathematical idea in conducting this activity. According to students' answer in figuring out the given prerequisite problems, students have shown their understanding in determining the certain length on each side of square and determining the area of rectangle. The following figure shows the problem and students' written works during the activity 4.

Problem 4

Before doing some physical exercises, Andi must run one full round surrounding his school square-shaped flag ceremony field as a warming up activity. Andi can reach 3/4 round of the field in 15/2 minutes.

(a) How long does it take to reach 1/4 round of the field?

(b) How long does it take to reach one full round of the field?



(a) Group 1 (b) Group 2 (a) Group 3 (d) Group 4 (e) Group 5

Figure 5 Problem and Student's Written Works in Activity 4

In the beginning of the lesson, students succeeded in sketching the ceremony flag field (see Figure 5). Most of students inserted the given route on their pictorial models. Unfortunately, no students wrote "3/4" as the route traveled by Andi and "15/2" as the duration to travel the route. According to the route that sketched on the pictorial model, actually students knew that 3/4 round of the field had the meaning to travel from the bottom left to the top left of the field or it reserve (consider the arrow made by students on Figure 5.c) except students in group 4 (see Figure 5.d). Students in group 4 understood 3/4 round of the field by interpreting the fraction 3/4 as part as whole. The whole route of the running had a meaning travel on all four sides of the field. Since, it was known that Andi only travels 3/4 round of the field, so only 3 of 4 sides which traveled by Andi.

According to the collected data, students succeeded in determining the duration needed by Andi to travel 1/4 round of the field. It was need 5/2 minutes to travel 1/4 round of the field. In determining the 5/2 minutes, the expected action occurred when some students wrote "15/2 : 3 = 5/2" or "15/2 : 3/1 = 5/2" on their sheets. Students said that since 3/4 had a meaning there were three quarters, so to find the duration of a quarter, they needed to divide the 15/2 by 3. The interesting finding occurred when students in group 2 stated "15 : 3 = 5 = 5/2" on their sheet. It was known that students tried to simplify "15/2" by 15 and kept the "1/2" on their mind. After finding the quotient of 15 by 3, they wrote back the "1/2" to form "5/2" as its answer. Although this idea was not expected in revised HLT, it could be said that the students' idea still adjust with the expected actions.

The activity continued by asking students to answer the second question about determining the duration needed by Andi to travel one full round of the field. In determining the duration, it was expected that students add the duration for 3/4 round, that was 15/2 minutes, and the duration for 1/4 round, that was 5/2 minutes to form 10 minutes. This expectation occurred when students in group 1, group 2, and group 5 answered the question. Students in group 1 knew that it was need another 1/4 round of the field to form one full round of the field. Therefore, instead of just wrote the duration to travel one full round of the field, that was 20/2 minutes, students in group 1 also sketch the route of the given situation. The interesting findings occurred when students in group 2 and group 5 proposed their answers. Similar to the idea of students in group 1, students in group 2 and group 5 knew that it was need another 1/4 round of the field to form one full round of the field. Therefore they added the durations. But, instead of adding “15/2” and “5/2”, students wrote “15 + 5 = 20”. After conducting some interviews, it was knew that students tried to simplify “15/2” and “5/2” by 15 and 5 respectively and kept the “1/2” on their mind. After finding the sum of 15 by 5, then they wrote back the “1/2” to form “20/2” as its answer.

In determining the duration of one full round of the field, it was also expected that students multiplied the duration for 1/4 round, that was 5/2 minutes, by 4 to form 10 minutes. This expectation occurred when students in group 3 answered the question. Students in group 3 knew that running one full round of the field had a meaning running on four sides of square. Since it was needed 5/2 minutes to travel on one side of the field, to travel on all four sides of the field students just multiplied the needed duration to travel on one side by 4 to from 20/2 minutes.

CONCLUSION

Dividing Integer by Proper Fraction

The “Biscuit Sharing” was meant to support students in understanding the division of integer by proper fraction. By interpreting the division on fraction as measurement division, students succeeded in showing their understanding about how to find the quotient of “12 : 1/2” or “16 : 1/2”. By proposing a contextual problem in finding the number of Andi’s friend that will get a half of biscuit, students interpreted “12 : 1/2” as finding the number of halves in twelve biscuits or “16 : 1/2” as finding the number of halves in sixteen biscuits. This finding is in line with the study conducted by Holisin (2002) that the use of concrete approach that associated with the use of concrete material is effective to gain students’ better understanding about the division on fraction.

Dividing Proper Fraction by Integer

The “Sharing the Remaining Chocolate Bar” was meant to support students in understanding the division of proper fraction by integer. By interpreting the division on fraction as partitive division, students succeeded in showing their understanding about how to find the quotient of “25/36 : 5”. By proposing a contextual distributing the remaining chocolate bar, that is 25/36, among five students, students succeeded in interpreting “25/36 : 5” as distributing 25 numbers of 1/36 among five students and calculating the number of 1/36’s got by each students as the quotient of “25/36 : 5”.

Dividing Two Proper Fractions

The “Arranging Bedroom Mats” was meant to support students in understanding the division of two proper fractions. In this activity, instead of using the real puzzle mats, researcher used the model of puzzle mats made by paper. By interpreting the division on fraction as the inverse of Cartesian product, students succeeded in showing their understanding about how to find the quotient of “8/64 : 1/4” or “24/64 : 1/2”. By proposing a contextual problem finding the length of the bed if the area and the width of the bed is 8/64 and 1/4 respectively, students succeeded in showing their understanding that the length of the bed can be found by dividing the area and the width of the bed.

Dividing Two Fractions

The “Run Surrounding Ceremony Flag Field” was meant to support students in understanding the division of two fractions. This activity totally designed to avoid the use of concrete material. Instead of using concrete material that can be manipulated, students used the satellite image of students’ ceremony flag field as the learning media. By interpreting the division on fraction as determination of unit rate, students succeeded in showing their understanding about how to find the quotient of “15/2 : 3/4”. By proposing a contextual problem finding the duration to run one full round of the field if it is given that the duration to run 3/4 round of the field is 15/2 minutes, students succeeded in showing their understanding in finding the duration to run one full round of the field by dividing 15/2 by 3/4.

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APPENDIX A

CONJECTURED LEARNING AND THINKING (CLT)

Biscuit Sharing

Students are expected to
1.1 represent 16 biscuits into 16 2-dimensional figures, such as circle, rectangle, square, or other 2-dimensional figures.
1.2 use line in dividing the pictorial models into 2 parts.
1.3 shade or color the pictorial models.
1.4 write 1/2 in each part of pictorial model.
1.5 find the total amount of halves in 16 pictorial models.
1.6 write numbers 1 to 32 in each half as a counter, named each part by students' name, draw 32 characters who will receive the halves, do the real cutting of biscuits, or calculate mentally in determining the number of halves.
1.7 write 16 : 1/2 to simplify the given situation.

Sharing the Remaining Chocolate Bar

Students are expected to
2.1 visualize the remaining blocks into its pictorial model.
2.2 use 2-dimensional figures to create the pictorial model.
2.3 shade or color the regions that represent the eaten blocks.
2.4 express fractions 25/36 as 25 times of 1/36.
2.5 write numbers 1 to 25 on each tiles as a counter or do the calculation mentally in determining the number of 1/36.
2.6 distribute 25 pictorial models of fractions 1/36 eventually among 5 students.
2.7 draw 5 characters of students in representing 5 students.
2.8 draw all pictorial models of fractions 1/36 received by each student, group every 5 regions, or use some arrows in showing the distribution.
2.9 represent those 5 pictorial models of fractions 1/36 received by each students as pictorial model of fractions 5/36.

Arranging Bedroom Mats

Students are expected to
3.1 sketch the bed in the room and determine the width of bed is 1/2 length side of the room.
3.2 sketch a square having 64 tiles.
3.3 place the pictorial model of bed horizontally or vertically in one of the corners of the room.
3.4 use other concrete materials to represent the bed.
3.5 write 1/2 as the width of bed.
3.6 sketch the bed on pictorial model whose area is 24/64 of the area of the room.
3.7 shade, color, or write symbols such as dots on the tiles that will be replaced by the bed.
3.8 answer the length of the bed is 6 units.
3.9 determine the length of the bed as 6/8 or 3/4 length side of the room.

Run Surrounding Ceremony Flag Field

Students are expected to
4.1 create pictorial model of the field and insert the route traveled by Andi (3/4 round of the field) in 15/2 minutes on it.
4.2 determine the duration needed to reach 1/4 round of the field, that is 5/2 minutes.
4.3 determine the position of 1/4 round of the field by dividing the route into 4 and take 1 part.
4.4 divide the duration needed to travel 3/4 round of the field by 3.
4.5 do mathematical operation 15/2 × 1/3 = 5/2.
4.6 determine that one full round of the field can be reached in 10 minutes.
4.7 add 15/2 (duration for 3/4 round) and 5/2 (duration of 1/4 round), 15/2 + 5/2 = 20/2 = 10 minutes, multiply 5/2 (duration of 1/4 round) by 4, 5/2 × 4 = 20/2 = 10 minutes, or do partitive addition, 5/2 + 5/2 + 5/2 + 5/2 = 20/2 = 10 minutes determining the duration to reach one full round of the field.
4.8 find the algorithm in finding the quotient of 15/2 : 3/4 = 10.

Inconsistency Between Beliefs, Knowledge and Teaching Practice Regarding Mathematical Problem Solving: A Case Study of a Primary Teacher

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Abstract

This is a case study investigating a primary teacher's beliefs, knowledge, and instructional practice about mathematical problem solving. Data were collected through interviewing one primary teacher regarding his beliefs in the nature of mathematics, teaching mathematics, and learning mathematics as well as knowledge about content and pedagogical problem solving. It was also observed on his instructional practice which focus on the way he helped his students solved several different mathematical problems in class based on Polya's problem solving process: understand the problem, devising a plan, carrying out the plan, and looking back. Findings of this study point out that while the teacher's beliefs, which is closely related to problem solving view, are consistent with his sufficient knowledge of problem solving, there is a gap between such beliefs and knowledge with his teaching practice. The gap appeared primarily on the directive teaching which corresponds to instrumental view he held in most of Polya's process during his teaching practice, which is not consistent with beliefs and knowledge he professed during the interview. Some possible causes related to several associate factors such as immediate classroom situation and teaching practice experience are discussed to explain such inconsistency.

Keywords: *inconsistency, problem solving, teacher beliefs, teacher knowledge, teaching practice*

Introduction

Developing problem-solving skills in mathematics classroom is known as important task for teacher. However, there is evidence that many teachers have not met requirements to implement problem solving approach in their classroom (Hollingsworth et al, 2003). To that, some consideration have been given by many scholars regarding the factor of teacher's quality in implementing problem solving (Ernest, 1989; Fennema, Carpenter, & Peterson, 1989; Raymond, 1997). Such factors include teacher beliefs, teacher knowledge, and teacher's teaching practice itself. Bray's study results (2011) suggested that teacher's beliefs seemed most likely influence on how teacher structure class discussion whereas teacher's knowledge appeared to drive quality of teachers' responses within classroom discussion.

In attempt to look for the relationship between teacher’s teaching practice and teacher beliefs regarding mathematics instruction, some tried to conceptualise and develop the models of such relationship (see for example Anderson et al, 2005; Fennema et al., 1989; Raymond, 1997) while others reported how such models were confirmed into practical interest (see Purnomo et al, 2016; Siswono et al, 2015). For instance, conceptually, Fennema and his colleagues emphasizes that classroom instruction is determined by teachers' decisions, which in turn are influenced by the interaction between knowledge and beliefs. Regarding mathematical problem solving, specifically, such interaction were then studied by Siswono et al (2016) on three secondary mathematics teachers finding that teachers’ beliefs have a strong relationship with teachers’ knowledge about problem solving. In particular, the instrumental teacher’s beliefs in their study were consistent with his insufficient knowledge about problem solving, while both platonist and problem-solving teacher’s beliefs were consistent with their sufficient knowledge of either content or pedagogical problem solving. Thus, this study show consistency between teacher beliefs and knowledge. With regards to primary teacher, how such relationship holds? How consistent is it to teacher’s teaching practice about mathematical problem solving?

Hence, this present study aims to examine the relationship between the beliefs, knowledge, and practice regarding mathematical problem solving of a primary teacher.

Theoretical Background

Beliefs about mathematical problem solving

Beliefs can be defined as one’s knowledge, theories and conceptions and include whatever one considers as true knowledge, although he or she can not provide convincing evidence (Pehkonen & Pietilä, 2003). It is noted that beliefs in mathematical problem solving is closely related with beliefs about the nature of mathematics as well as teaching and learning mathematics. (Viholainen, Asikainen, & Hirvonen, 2014) explained that beliefs about the nature of mathematics influence beliefs concerning mathematical problem-solving or vice versa, and that beliefs concerning the learning of mathematics also imply beliefs about the teaching of mathematics. Meanwhile, Ernest (1989) stated that teachers' view on the nature of mathematics affect on how they play role in classroom teaching and learning. To that, he presents three different philosophical views of the nature of mathematics: instrumental, platonist, and problem solving. In attempt to simplify these views, Beswick (2005) summarized connections among the nature of mathematics, mathematics learning, and mathematics teaching (see table 1).

Table 1. Summary of beliefs about mathematics, mathematics teaching, and mathematics learning

Beliefs about the nature of mathematics	Beliefs about mathematics teaching	Beliefs about mathematics learning
Instrumentalist	Content focussed with an emphasis on performance	Skill mastery, passive reception of knowledge
Platonist	Content focussed with an emphasis on understanding	Active construction of understanding
Problem solving	Learner focussed	Autonomous exploration of own interests

Knowledge about mathematical problem solving

Understanding knowledge for teaching problem solving, Chapman (2015) summarised three types of knowledge for teaching problem solving: problem solving content knowledge, pedagogical problem solving knowledge, and affective factors and beliefs. (see table 1).

Table 2**.** Knowledge needed in understanding problem-solving

Type of knowledge	**Knowledge**	**Description**
Problem-solving content knowledge	Mathematical problem-solving proficiency	Understanding what is needed for successful mathematical problem-solving
	Mathematical problems	Understanding of the nature of meaningful problems; structure and purpose of different types of problems; impact of problem characteristics on learners
	Mathematical problem solving	Being proficient in problem solving Understanding of mathematical problem solving as a way of thinking; problem solving models and the meaning and use of heuristics; how to interpreting students' unusual solutions; and implications of students' different approaches
	Problem posing	Understanding of problem posing before, during and after problem-solving
Pedagogical problem-solving knowledge	Students as mathematical problem solvers	Understanding what a student knows, can do, and is disposed to do (e.g., students' difficulties with problem-solving; characteristics of good problem solvers; students' problem-solving thinking)
	Instructional practices for problem-solving	Understanding how and what it means to help students to become better problem solvers
Affective factors and beliefs		Understanding nature and impact of productive and unproductive affective factors and beliefs on learning and teaching problem solving and teaching

Teaching practice on mathematical problem solving

On understanding teaching practice which regards to problem solving, many scholars have discussed on what best instructional process which teachers require to hold problem-solving instruction. For example, Franke, Kazemi, & Battey (2007) explained that within the problem-solving instruction, teachers need to orchestrate class discussion so that students share multiple problem-solving strategies, analyze relations among strategies, and explore contradictions in students' ideas to provide greater insight into the mathematical focus. Additionally, the four Polya's stages of solving a problem, i.e. understanding the problem, making a plan, carrying out the plan, looking back, also imply how te teach problem-solving (Ontario Ministry of Education, 2008). However, it is noted that those phases should be used flexibly. For instance, when encountering students who have devised a plan to solve a problem, a teacher should realize them that while carrying out the plan that it may need revision and will try something different.

Method

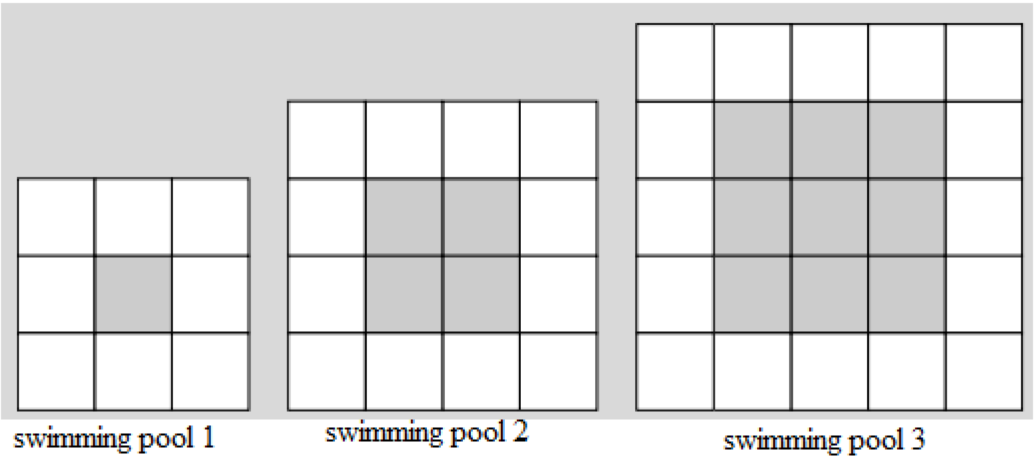
This is a case study which describes qualitatively one Indonesian primary teacher's beliefs, knowledge, and teaching practice regarding problem-solving. . The teacher was selected from the result of interviewing 10 primary teachers purposively chosen from Surabaya city using a set of belief-related task discussing the nature of mathematics, learning mathematics, and teaching mathematics. Of all the teachers, we selected Irul as our further interest of participant since the significant beliefs alligned with problem solving view he professed during interview.

Such tasks were arranged in the form of incomplete statements. We provided three options for each incomplete statement illustrating the description of each of beliefs. As illustration, when asked about the what people should learn from mathematics, Irul was asked to complete the statement by selecting one of options: (1) to have skills in calculating and applying mathematical formulae or procedures when solving daily life problem, (2) to be proficient in understanding topics in mathematics, such as algebra, statistics, probability, geometry, and the interrelationships among those topics entirely, and (3) to have thinking skills such as understanding regularities of phenomena, being critical and creative in solving any problems. Then, he explained why he selected that option. Meanwhile, to collect data about teacher' knowledge, our questions were primarily inspired by Chapman's category of problem-solving knowledge for teaching described in table 2. Table 3 and 4 summarizes respectively such interview for beliefs and knowledge examined to the teachers.

In describing Irul's teaching practice, we employed several lesson observations on all the three teachers. Due to the need of investigating how teachers helped their students to learn to solve mathematical problems, we provided 10 problems for upper primary school students,

DESIGNING SWIMMING POOL

Mr Tony will design several square-shaped swimming pools. To cover the centre of each of swimming pool, he plans to use grey-colored tiles, while to cover the circumference of the swiming pool, he plans to use white-colored tiles. His design is given as follows.



How many grey-colored tiles and white-colored tiles would be in the swimming pool 10? Explain your reason.

Figure 1. Examples of problems used in lesson observation

To analyze teachers' practice, we were guided by framework we developed according to Polya's model of solving problem as described in table 1. We focused on the significant feature of how each of the teachers guided students in each of problem-solving stages. Such feature was then confirmed to whether it reflects on directive teaching, the teaching in which the teacher explains a concept, gives an example of applying the concept and finally offers the students some exercises for practicing problems (Antonius et al, 2007), or consultative teaching, the teaching in which teacher emphasizes guiding students to actively and independently construct new knowledge by using their prior knowledge and experiences (Blum and Ferri, 2009).

Table 3. Framework of describing teachers' teaching practice

Problem-solving stages	**Questions**

Teacher guides students to figure out the problem by thinking and expressing their opinions on the problem posed *(Understand the problem)*	How does teacher engage students in understanding the problem situation?
	How does teacher encourage students to understand the problem by associating the problem with their prior knowledge?
	How does teacher make sure students understand the problem?
	How does the teacher ask the students to identify the required information for the purpose of solving the problem?
Teacher guides students to select a/an appropriate strategies/model of finding solution and draw attention that it is very likely to change during the process of solving the problem. *(Devising a Plan)*	How does teacher guide students to discover the relationship between the information that has been presented with information that does not exist but had been identified in the previous step in preparing the plan for settlement?
	How does teacher give students opportunity to determine their own strategy that is used, rather than directing students on a particular strategy?
	How does teacher give students opportunity to convey the idea of a strategy that was selected by the students / groups?
	How does teacher provide reinforcement or provide guidance to improve the model/ strategies that have been determined?
	How does teacher make sure that every student/group has determined the strategy/plan of each?
Teacher guides students to implement what is planned *(Carrying out the Plan)*	How does teacher guide students to implement the determined strategy/plan?
	How does teacher ensure every student/group has arrived at a solution, regardless whether it is correct, partially correct, or incorrect?
Teacher guides students to evaluate their the process and the solution they resulted in the previous process *(Looking back)*	How does teacher ask the students to look back on the process and the solution they found?
	How does teacher ask students to identify the strengths and weaknesses of the solution they found?
	How does teacher ask difficulties experienced by students during solving the problem?
	How does teacher give students opportunity to present their solution/strategies to other students?
	How does teacher ask students to rethink other strategies/solutions may be more likely effective/better to answer the problem?
	How does teacher guide students to consider the various solutions discussed?

	How does teacher guide students to make generalizations of the processes and solutions obtained in a more general/complex problem?
	How does teacher guide students to identify possible changes of solutions if the situation/context of the initial problem is changed?
	How does teacher guide student draw on conclusions based on the discussion with the emphasis on the main ideas as a reflection for the further problem-solving experience?

Analysis of data was carried out by firstly reducing data, displaying data, and finally drawing conclusion and verification (Miles & Huberman (1994). The conclusion was sought to understand the characteristics of belief and knowledge espoused by teachers and how such two variables interact each other shaping their idea of mathematical problem solving.

Results and Discussion

Irul's beliefs about mathematical problem solving

In most cases examined about the nature of mathematics, Irul holds problem solving view, except when he expressed his ideas about the successful of completing strategy he made when solving mathematics problem. He said, "I believe that the completion strategy I planned will always manage to find solution to the problem, as long as the concepts and mathematical procedures that I have used are correct, because in mathematics, it has actually been in accordance with the agreement ". He also agreed that mathematics is about thinking and capturing real-world phenomena. He argued, "by studying mathematics correctly, we will be critical in using various types of problems existing in this world."

Irul's belief in teaching mathematics is also in line with the view of problem solving. Irul shows a view consistent in placing the learner as the focus of learning in instructional practices. He said, "I think our students have a great potential so that we need not to act as if they were unable to learn. Rather, we simply provoke their potential so that they can flourish, hence, the teacher's role is as facilitator in learning mathematics. He said, "to clarify the errors made by student within a learning activity, Irul agreed to give students an opportunity to discuss the ideas that emerged from their thinking and determine which one is best to clarify their misunderstandings. According to him, "... sometimes students can be more creative when giving reasons why her other job is right or wrong, for example such as when there are children who miscalculated in front of the class, there are also some children who give their opinions about how the calculation should be done." However, in motivating students to learn mathematics, he also concurred with platonist view. He said, "I agree that reward will only be effective when students are actively working to achieve the goal of learning as I set, this is important for them in order to grow their own personal motivation". In addition, in relation with sources learning that students should access, he said, "they need some sources of learning mathematics, not only from school books, designed by teacher, but also designed by themselves, the benefit is they will be more active, and more appreciated."

In the cases of how students should learn mathematics, Irul's views also consistent with the type of problem-solving beliefs as well. When asked to complete the statement about the best way to learn strategies to solve mathematical problems, he said, "In order students are able to solve problems, students should strive to solve problems by their own way based on their knowledge and experience". In addition, Irul also advise students to learn all the strategies available because he thought, "this [learning problem solving strategies] are important for them, because although it could not be used on a particular problem, however, it will be likely used in other types similiar problem or even unfamiliar". In looking at the use of calculators, Irul suggested, "I will give them opportunity to use a calculator for problems demanding reasoning, not for question that only assess how to calculate. "

Irul's knowledge about mathematical problem solving

Regarding Irul's knowledge, he stated that a mathematics question is called a problem or not depends on whether a student is able to solve and whether the question indicates something challenging and interesting. He also knows that something that students do not have any knowledge about it totally is not a problem for students. According to him, "how could the problem of the equation of a straight line like presented in this question becomes problem for elementary school students? They do not have enough experience about it..." "He also expressed his good understanding on the types of problems. When interviewed about the idea to give examples of open problems, he proposed, "determine as many as possible possibilities of size of a rectangle that has an area of 40 cm^2."

Irul's pedagogical knowledge also seemed sufficient to explain how to implement problem-solving instruction within classroom teaching. He said, "I usually ask students to read the question and asked students to understand the intent of the problem, and then provide direction of students' discussion about what is being asked and what is known, then in groups or pairs, I ask them to discuss to find the idea of strategies. After that, I ask them to work on appropriate strategies. Well, if there are students who need some helps, I will come to guide their strategies. At the end of the lesson, some groups would present their solution to clarify whether there were mistakes as well as paying particular attention to the answers from other friends."

Irul's teaching practice

The following are the summary of Irul's teaching practice on guiding his students solved different six problems considering whether it indicates consultative or directive teaching.

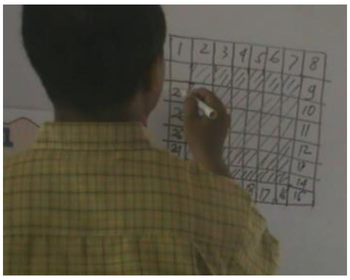
Table 4. A summary of Irul's teaching practice regarding problem solving stages

Problem solving stages	**Problems**					
	Food Boxes	**Design of Swimming Pool**	**Guessing friend's age**	**Book pages**	**Daily rice supply**	**Travelling**
Understanding problem	Consultative	Consultative	Directive	Consultative	Directive	Directive
Devising plan for strategies	Directive	Directive	Directive	Directive	Directive	Directive
Carrying out the plan	Consultative	Consultative	Consultative	Consultative	Consultative	Directive
Looking back	Directive	Directive	Consultative	Consultative	Consultative	Directive

Table 4 shows that the most significant feature describing Irul's practice can be seen from the second stage of his guidance, which is directive teaching on devising plan for strategies for all problems used in his practice. For example, he offered to calculate the number of boxes in every layer using area of rectangle formula on problem 'food boxes', aimed students to see the square number pattern for the given tiles on problem 'design of swimming pool', suggested students to think of possibilities of pages number so that the sum of two first page numbers and two last page numbers must be 98, and told students to use the knowledge of dividing integer by fraction on problem 'daily rice supply'. The talk below exemplifies this finding.

Irul:	How can you analyze the pattern of the design?
Student:	(no respond)
Irul:	Let's make a table representing the number of tiles for grey-colored tile, and white-colored tile [drawing a table]. Now I put that the number of grey tiles in pattern 1 is 1, while the number of white tiles is 8. In pattern 2, there are 4 grey tiles and 12 white tiles. What's next?
Students:	16 white tiles...20 tiles...
Irul:	How about the grey tiles? Can you see the pattern?
Students:	(no respond)
Irul:	This is called square number pattern. So, it start from 1, 4, 9 and so on. Continue this pattern.

Another description of his typical teaching practice seems in his guidance when students clarify their solution in 'looking back' activities. Within all most problems, Irul seemed did not engage students to actively clarify their answers. Rather, he dominantly showed the correct answers of the problem and just asking which group got the answer, asking those who got incorrect ones to revise. The following illustrate this finding on problem 'design of swimming pool'.


demonstration on
solutions

Irul: [while drawing pattern of tiles on whiteboard] Let's count the number of tiles as we predicted
Students: one, two, thre,..., twenty five, twenty six,...
Irul: Now, we can conclude that our prediction is correct

The practice above indicates Irul's directive teaching when checking whether the number of grey tiles and white tiles they predicted fits with the actual number of each of tiles represented in a model of tiles arrangement, instead of letting his students to present their own way to check the reasonablesness of the solution they found.

Despite Irul mostly performed directive teaching for several activities, Irul also hold consultative teaching for several other activities. For example, Irul tried to check the mathematical procedures his students performed by either visiting every group to ensure the correctness of the procedure or asking students to write their calculation on whiteboard. The following depicts this finding on problem 'daily rice supply.'

Irul: Let me see your work...how to obtain this result [pointing
Student: We calculated the number of boxes in the first layer, which is 45 from 9x5
Irul: Don't you see that there are some missing boxes in this part?
Student: Oh ya..I see there are two missing boxes here
Irul: So, what should you do next?
Student: I must remove two boxes, so that the remaining are 43 boxes.

We also found that among the four stages, the process of guding students worked on mathematical procedures in carrying out the plan seemed most aligned with the consultative teaching other than the other three stages.

Discussion and Conclusion

Our finding indicates that the problem solving view hold by Irul seemed consistent with his sufficient knowledge about problem solving. Nevertheless, we found an inconsisteny between his beliefs and knowledge toward his teaching practice. Irul agreed that students should be actively enganged in finding strategies discussing the ideas emerging from their thinking and determine to clarify their misunderstandings. Regarding knowledge, he also professed that in problem solving instruction, a teacher ask students to present their solution to clarify their answers from other friends' opinion. However, when come to his teaching practice, he seemed to peerfom differently with what he espoused on his beliefs and knowledge in most stages of Polya's four processes, particularly on guiding students to devise plan and look back. In most cases, he did not let students to think of the appropriate strategies. Rather, he offered such strategies to the class although he then asked to do with them in groups. He also seemed to confirm the solution found by the students by telling the correctness of such solution instead of asking them to verify other possible procedures and answers, or even extend the problem into a more complex problem.

Concerning on this finding, researchers discussed about this kind of incosistency. Empson and Junk (2004) asserted that even if teachers professed certain belief which are aligned with mathematical teaching reform, such as problem solving view, it does not mean that they will surely perform in ways consistent with those beliefs. Anderson, White, & Sullivan (2005) added that the inconsistency between beliefs and practice may be results of many influences, some of which are social context and teachers' level of thought. Meanwhile, Raymond (1997) added that such inconsistency can be resulted from teachers' social norms and the immediate classroom they encounter. In this study, we found immediate classroom particularly about the mathematics at hand, students' abilities, and time constraint as the main causes of this inconsistency. As evidence, after the lessons ended, we interviewed Irul to tell his experience about how often he offered problem solving tasks like he just used in his class. He told, "I often give problems to my students, however, the problems are not as challenging and complex as the problems I just gave to them". He added, "because several problems seemed very difficult to them, I aimed the big idea on how to solve. In my experience, they will likely find difficulties when asked to determine what formula or what procedure if the problems are given in words". Thus, it indicates his worry about his student's lack ability and the compelexity of problems at hand as well as his teaching experiences.

In further explaining that inconsistencies, Raymond (1997), however, argued that it arise not only because of the immediate classroom and teaching experience but also because of the cumulative effect of other influences on teaching practice, such as time constraints, social teaching norms, past school experience, and teacher education program. However, we did not study these factors intentionally.

To conclude, we would highlight that while the teacher's beliefs, which is closely related to problem solving view, are consistent with his sufficient knowledge of problem solving, there is an inconsistency between such beliefs and knowledge with his teaching practice. The gap appeared primarily on the directive teaching which are aligned to instrumental view he held in most of Polya's process during his teaching practice, primarily when guiding students devised plan for determining appropriate strategies, which is not consistent with beliefs and knowledge.

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DEVELOPING LEARNING MATERIALS IN THE ADDITION AND SUBTRACTION OF FRACTIONS WITH REALISTIC MATHEMATICS EDUCATION FOR FOURTH GRADE

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Abstract

This article aimed to describe the development process of learning materials that consist of lesson plans, student sheet activity, and fraction card media. The study focuses on teaching addition and subtraction of fractions through realistic mathematics education. Exactly, the development process has two goals, for generating a good learning material and describing the effectiveness of learning. The development process used Four-D Model which consist of four steps. The steps are define, design, develop, and disseminate. On the first step (define), the teacher established the terms of learning by determining the fundamental problem, analyzing the students, and also specifying instructional objectives. Furthermore, in the second step (design), they made the prototype of learning materials by constructing test standard, choosing the proper media and selecting the format. On the third step, the learning materials validated by the expert and then revised. If the learning materials were all ready to use, it could be practice in the classroom. From the practice, the teacher got data to measure the quality of learning materials. The final step is disseminating. It is the implementation of learning materials on a broad scale. From this step, the teacher got data to measure the effectiveness of learning. This study generates a good learning material. It also supports the student to learn about addition and subtraction of fractions at grade four.

Keywords: *development research, fractions, learning materials, realistic mathematics education*

Introduction

Fraction is one of the difficult topics in primary school. The difficulties experienced by the teacher at SDN Rejosari. In fact, on the teaching of addition and subtraction of fractions for the uncommon denominator, the teacher usually asks students to equate the denominator of both fractions directly. They didn't motivate the students to find the concept by themselves. This condition was caused by the poor quality of learning materials. The lesson was less meaningful for the student.

Based on the conditions, the researcher was challenged to develop learning materials that could help the teacher on presenting the topic of addition and subtraction of fractions. The learning process was design based on realistic mathematic approach with fraction card media. It was expected to help the student to learn significantly through the process of reinvention (rediscovery).

Literature Review

Teaching math in a diverse classroom is very challenging. It requires not only skills but also the mastery of mathematical concepts. In fact, students' competence in math only measured by a test. Even though, mathematics is able to give effect to the student to solve problems in daily life.

Frei (2008, p. 153) states that if the teacher only emphasizes the skills and training of math concepts, they have hurt students. They should help the student to apply the formula and procedures in real life. Actually, when students learn concepts related to real events, they will be understood. Application of abstract concepts in real life is an effective learning tool.

The effective learning activity was influenced by the teachers' ability in planning and implementing the lesson. The teacher is the facilitator of the learning activity. Therefore, the teacher should be able to create a situation that allows students to learn. The teacher has two tasks, as an instructor and class manager. As an instructor, the teacher is responsible for creating optimal learning activities, while as manager, they have responsibility for the effective learning.

Addition and subtraction of fractions

In primary school, the fractions concept has been taught in third grade. At fourth grade, the student begins to learn about addition and subtraction of fraction. Heruman (2007, p. 55) found that the prerequisites of learning addition and subtraction of fractions are the concept of fractions value, equivalent fractions, as well as the addition and subtraction of integers. The student's mastery of equivalent fractions emphasized in the addition and subtraction of fractions with the uncommon denominator. To build the students' understanding, Reys et al. (2004, p. 278) suggest that learning about addition and subtraction of fractions should start with problems involving the separation and merger. In addition, the model can also be used in the form of images, not just a symbolic sentence like 2/3 + 1/4. Problems and models in the form of pictures can help students understand that the addition and subtraction of fractions can be completed as addition and subtraction using integers. In addition, the student can develop the idea of a reasonable answer. Besides, they will understand why the same denominator required when adding fractions with uncommon denominators. Furthermore, the things that the student know about addition and subtraction of integers can be used to explain the concept of addition and subtraction of fractions. If the student has the capability to calculate fractional parts (for example 3/7 + 2/7 is 5/7) and compare two fractions using an image model, they will find their own way to solve the problem.

Realistic Mathematic Approach

The realistic mathematic approach is taken from the Realistic Mathematics Education (RME) concept by Freudenthal. Accordingly, this theory emphasizes the richness of thematic context, integration of mathematics with other issues and reality, individual difference in the learning process, and the importance of working together in the heterogeneous group (Treffers, 1991, p. 19). Realistic mathematic education integrates ideas about math, how student learn mathematics, and how mathematics should be taught.

The realistic mathematic approach, according to Gravemeijer (1994, p. 90), has three principles, the first is guide reinvention and progressive mathematizing, the second is the didactical phenomenology, and the third is self-developed models. According to the principle of reinvention, the student has the opportunity to experience a process similar to the process in which mathematic is found. The principle of the invention can be inspired through informal solution procedures. In this case, mathematic creates an opportunity for reinvention. In general, the contextual issues needed for any problem solving. The principle of rediscovery can be inspired by the informal procedures, while the progressive mathematical process using mathematical concepts. De Lange (1996) describes the discovery process should be promoted through the exploration of a wide range of real world problems so that mathematic should be close to the students and relevant to daily life.

The principle of didactic phenomenology emphasizes the importance of educational contextual issues. The didactic phenomenon is finding a contextual situation and approach the problem in a particular situation. Next, the process is generalizing in order to achieve a contextual problem-solving procedure that can be used as a basis for vertical mathematics. In other words, the contextual problems are expected to guide the student in the process of developing their math knowledge. The last principle is self-developed models. It has a function to bridge the informal and formal knowledge of mathematics. The students get informal knowledge from the real life before they learn the formal mathematical knowledge.

Gravemeijer (1994, p. 114) describes five characteristics of realistic mathematical approach as follows: the use of context, the use of models, the student contribution, interactivity, and intertwining. In realistic mathematical approach, the learning starts by using contextual issues or problems in daily life. Through the use of context, a student has involved actively in exploring the problem. The exploration is direct to develop a variety of problem-solving strategies. Furthermore, the problem can be express in the form of mathematical models. Modeling was done to simplify problem solving or understanding of the concept. Development of a model is use as a bridge between informal and formal situations. However, a great contribution in the learning process is expected to come from the student. Contributions may be a variety of answers or opinions. With production and construction, the student is encouraged to reflect on the important parts of the learning process.

The learning process becomes meaningful through interaction between the student and student to the teacher. The student interaction occurs through class discussion or discussions in small groups which allows the student to exchange ideas and arguments about the results of their work. Moreover, the interaction between student and teacher could be an explanation, justification, approval, or reflection. It will help the student gets a formal mathematical concept of informal by themselves.

Actually, the concepts in mathematics have relevance because the concept in mathematics is not introduced separately. In the discussion of a topic, particularly include several interrelated concepts. These linkages should be explored in order to introduce and build the understanding of some concepts at the same time.

To start the learning process through realistic mathematic approach, the teacher provides contextual issues in accordance with the subject matter. Then, the student is required to understand the contextual issues. If there are things that have not understood by the student, the teacher explains or give directions. Furthermore, the student individually solves problems on worksheets use their own way. During the student solve problems, the teacher observes and control the activities of the students. Then the teacher gives the student chance to compare and discuss the answer to the problem solved by the group of their friends followed by a class discussion. After reaching agreement on the best strategies through classroom discussions, the teacher guide students to make conclusions in order to obtain a formulation of concepts, principles, or procedures.

Learning Materials

Learning materials are the learning resource that is used in the learning process. According to Ibrahim (2003, p. 3), learning materials needed by the teacher to manage the learning process includes syllabus, lesson plan, student sheet activity, evaluation instrument or test, instructional media, and students' book. Learning materials developed in this research include the lesson plan, student sheet activity, test, and instructional media.

Lesson plans

The lesson plan is learning activity plan for one or more meetings. The lesson plan was developed based on the syllabus to guide the students' learning activities to achieve basic competency. Actually, there are thirteen components of a lesson plan based on the regulation of ministerial education and culture number 65, 2013 About Standard Process, among others: (1) school identity; (2) subject's identity; (3) the class; (4) subject's matter; (5) time allocation; (6) learning objectives; (7) basic competencies and indicators; (8) teaching materials (9) methods of learning; (10) instructional media; (11) learning resources; (12) learning steps; (13) learning assessment.

Student Sheet Activity

Student sheet activity is tasks that must be done by the student (Majid, 2008, p. 176). The sheets typically in the form of instructions and steps to complete a task. Ordinarily, student sheet activity contains a set of basic activities that must be performed by the students to enhance the understanding of the indicators. In the development of these materials, student sheet activity contains contextual problem-solving activities that assisted fraction card media to find the solution or other possible strategies.

Test

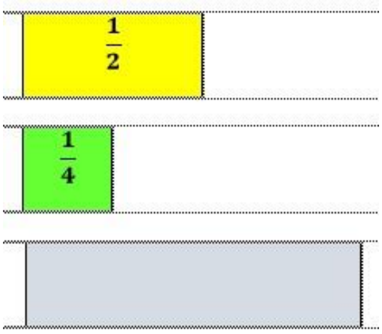
The test is done to measure the standard that have been achieved by the students (Purwanto, 2009, p. 67). Therefore, the test must meet two requirements, namely the validity and reliability. Purwanto (2009, p. 114) describes that a test is considered valid if it measures what it is supposed to measure. There are two types of validity, content validity and construct validity. The validity of the content is carried on testing the validity of its contents to ascertain whether the test item measures precisely the situation to be measured. Construct validity is testing the validity of which is done by looking at the suitability of the construction items are written with the criterion. While reliability refers to the consistency of the scores achieved by the same people when they were retested with the same test on different occasions or under different test conditions.

Instructional Media

Educational practitioners recognize that the instructional media is helpful for the classroom activity, especially for the improvement of student achievement. However, only view teachers who take advantage of, even use the lecture method is still quite popular among teachers in the learning process. The instructional media is everything which can deliver the message of learning resources in a planned manner to create a conducive learning environment in which the recipient can make the learning process effectively and efficiently (Munadi, 2012, p. 8). The definition is in line with the definition given by the Association of Education and Communication Technology / AECT that instructional media is all forms of media used to distribute the messages/information. Instructional media have several uses. In general, the use of the media directed to achieve the learning objectives. Sadiman, et al. (2014, p. 17) describes four uses of media include: (1) clarify the presentation of the message; (2) to overcome the limitations of space, time, and the sense of power; (3) to overcome the passivity of learners; and (4) to overcome the teachers' difficulties in face different characteristics of students

Fraction card media

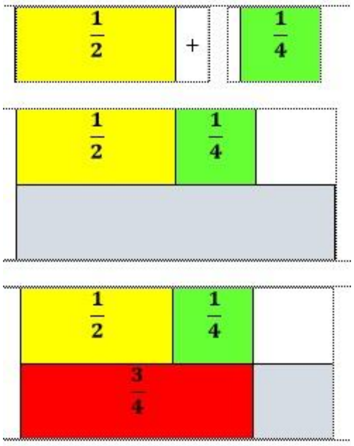
Fraction card is card media made of colored paper. A set of fraction card media consists of three cards, two cards symbolize particular fractions and one card symbolize the unit.



Fraction card media

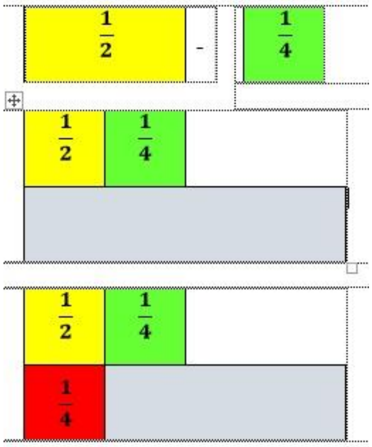
The Use of Fraction Card Media

To determine the number of both fractions, done by connecting both extends well beyond the fraction card. After connecting both fractions cards, the next step is to compare the two parts of the connection card with one part-unit cards. After that, the students gave shade on the unit card that shows the sum of two fractions. The use of fraction card to add two fractions are shown in the following figure.



Addition of fractions using fraction card media

The picture above shows the addition of 1/4 and 1/2. The red part is the sum of the two fractions. Students make a shading part of the card which is the sum of fractions. In addition, students are asked to write a form of the addition of fractions using a formal notation. The difference of fractions is determined by connecting both extend into the fraction card or put a card used for subtracting fractions over the other fraction card. Part of the card that looked represents the difference between the two fractions. Then the students leave the shades on the card that indicates the result of subtraction of fractions. Demonstration of subtraction of fraction can be seen in the image below.



Subtraction of fractions using fraction card media

The picture above shows the subtraction of 1/2 to 1/4. The red part is the result of subtraction. Students shade part on the unit card as the result of subtraction on fractions. Then the student asked to write a form of subtraction of fraction using a formal notation.

The Development of Learning Materials

The development model used in this study is the Four-D models of Thiagarajan, Semmel, and Semmel (1974), consists of four steps, that are define, design, develop, and disseminate. The fourth steps described as follows.

Define

The purpose of defining step is to establish the terms of learning. At this early step, the analysis is done to determine the learning objectives and limitations of the material. Accordingly, the defining step consists of five steps of analysis, namely: the front-end analysis, learner analysis, task analysis, concept analysis, and specifying instructional objectives. This step is used to determine the fundamental problem faced by the teachers.

Front-end analysis required consideration of various alternative development learning devices. While learner analysis is performed to examine the students. However, the identification of students characteristics in accordance with the design and development of learning. The task analysis aims to identify the skills required and analyze them into a subskill's framework. The concept analysis is done through the identification of the main concepts, organize hierarchically and detailing concepts that are relevant based on its properties. Specifying instructional objectives conducted to determine or formulating the learning goals.

Design

The design step aims to create the prototype learning materials. The design step consists of four steps: preparation of standard reference test, media selection, format selection, and initial design. The preparation of standard reference test (criterion-construction tests) is a link between define and design stage. It uses to convert the standard reference test objectives into specific outline learning materials. In addition, media selection step is carried out to determine the right media with the presentation of the subject matter. While format selection tailored to the factors outlined in the learning objectives. The format was chosen for the content design, strategy selection, and learning resources. The last, the initial design of the learning materials for presenting learning to use the right media and the proper sequence.

Develop

The purpose of the develop step is to produce learning materials that are revised based on input from the experts. This step consists of the expert appraisal and the developmental testing. Validation of experts intended to obtain suggestions for improvements. Based on these suggestions, the learning materials repaired to generate a good result. In the developmental testing, the materials are tested to get a response, reactions, suggestions, and comments from students, teachers, and observers. The test result was revised again based on feedback from various parties to produce good materials.

Disseminate

The disseminate step is the use of the materials that have been developed on a broad scale. This step aims to test the effectiveness of the development results.

Learning effectiveness

The effectiveness of learning characterized by the student's achievement towards learning objectives (Kemp, Morrison and Ross, 1994, p. 288). The effectiveness can be measured by learning the test results, project assessment, and observation of the behavior of students during the learning process. In line with these opinions, Pribadi (2011, p. 19) states that effective learning is the learning that is able to bring the students achieve the learning objectives or competencies expected. The effectiveness of learning can also be reviewed through four indicators. Slavin (2005, p. 277) describe the effectiveness of learning to use *QAIT* Model (Quality, Appropriateness, Incentive, Time). Each element of the model can be described as follows. (1) The quality of instruction; Presentation of information or skills can help students to learn the material. The quality of curriculum and learning presentation strongly influence the quality of instruction. (2) Appropriate levels of instruction; the suitability of the learning capacity refers to the extent to which teachers ensure that students are ready to learn new lessons (have the necessary skills and knowledge to learn). In other words, the level of learning is appropriate when it is not too difficult or too easy for students. (3) Incentive; Incentive is a level where teachers ensure that students are motivated to work on learning tasks and learn the material presented. (4) Time; Time is an element that shows a level where students are given enough time to learn the material.

Methodology

This type of research is development research. It aims to develop a good quality of learning materials. The learning materials developed consist of the lesson plan, student’s sheet activity, test, and fractions card media. The research use Four-D models from Thiagarajan, Semmel, and Semmel (1974, p. 5). This development model includes four steps: define, design, develop and disseminate.

The research was conducted in the second semester of the school year 2015/2016 by using experiment class and implementation class. The experiment class is grade four of SDN Rejosari which consist of 19 students. While the implementation class is grade four students of SDN Kroyokulon with the number of students in the implementation class is 17.

The researcher used some instruments to collect the data, among others validation sheet, student’s observation sheet, teacher's observation sheet, student questionnaire responses to the learning materials, and achievement test. While the data analyzed by qualitative and quantitative description.

Results

The learning materials of addition and subtraction of fractions with realistic mathematical approach met the criteria of good learning materials.

Quality of learning materials

Criteria	Description	Category
Validations results	The expert appraisal generally are at a score of 4	Valid
Observation results of teachers' ability	The teachers' ability on the score of more or greater than 3	Good
Students' activities	The students' activities meets the criteria of ideal time with a tolerance of 10%	Effective
Students' test results	79% of the students' test results reached the standard	Completed
Test	The test validity more than 0,40 The test reliability more than 0,40	Valid, reliable
Students' response to the learning materials	More than 75% students thought happily and attracted during the learning process. The students also thought the learning materials were new and clear for them.	Positive

Learning with realistic mathematics approach effectively to teach addition and subtraction of fractions at fourth grade with the following criteria.

Effectiveness of learning materials

Criteria	Description	Category
Students' test results	82 % of student learning outcomes reached the standard	Completed
Observation results of teachers' ability	The teachers' ability being on the score of more or greater than 3	Good
Students' activities	The students' activities meets the criteria of ideal time with a tolerance of 10%	Effective
Students' response to the learning materials	More than 75% students thought happily and attracted during the learning process. The students also thought the learning materials were new and clear for them.	Positive

Conclusion and Suggestion

The process of developing learning materials through realistic mathematics education follows a systematic model of Four-D with four steps: define, design, develop, and disseminate. The development of learning materials met good criteria with validity, practicality, and effectiveness. The validity shown by the results of the expert appraisal, the learning materials, in general, are at a score of 4 in the category of valid. The practicality demonstrated by the teacher's ability to manage to learn are at a score of 3. The effectiveness, demonstrated by the activity of the students who met the criteria for an effective and the classical mastery reached 79%.

The learning of realistic mathematics education with fractions card media was effective to teach addition and subtraction of fractions at grade IV Elementary School. The Indicators of the effectiveness of the study are the classical mastery percentage reached 82%, the teacher's ability in manage the learning process are at a score of 3, the student activity met the effective criteria and the positive response of the students towards learning. Based on the research that has been conducted, researchers gave suggestions to another teacher, that the development of learning materials with realistic mathematical approach can be used as an alternative material in learning addition and subtraction of fractions at grade four. Moreover, other researchers can make this study as a follow-up to enhance the existing weaknesses in order to obtain more accurate research results.

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Perspective of Learning Mathematics by ELPSA Framework in Developing the Character Values of Nation

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Abstract

Learning is conducted in order to establish competencies including knowledge, skills and character building. In learning mathematics, the students need to understand the direct object to build knowledge and skills and also the indirect object to form character or attitude. Therefore, learning design that allows the relationship between direct and indirect object of mathematics is highly needed. ELPSA framework with the sequence of *Experience*, *Language*, *Picture*, *Symbol*, and *Application* can facilitate the development of a correlation between the two objects. Thus, theory of ELPSA framework and learning theories about the direct object and indirect object of mathematics as well as the concept of attitude (character) formed in mathematics were elaborated. The result of the study showed that ELPSA framework can simultaneously facilitate the establishment of correlation between direct and indirect object of mathematics, so the students might be able to understand facts, concepts, principals and skills of mathematics as well as the formation of attitude (character) as a result of learning mathematics.

Keywords: *Character, Direct object, ELPSA, and Indirect object.*

Introduction

National education aims to develop students' potential to become a man of faith and be devoted to The God Almighty, noble, healthy, knowledgeable, skilled, creative, independent, and also become democratic and responsible citizen (Article 3 of Act No. 20, year 2003 on national education system). The development of students' potential is conducted by learning process in establishing competencies include aspect attitude (spiritual and social), and students' knowledge and skill.

The Ministry of Education (2015) stated that learning process is related to three domains namely attitude, knowledge and skill. According to John Dewey (as cited in Ridwan A. S., 2008) "*Learning involves, as just said, at least three factors: knowledge, skill and character*" which means learning consists of three factors which are knowledge, skill and character building. Marthin Luther King (as cited in Rahmi, 2013) said "*Intelligence, that is the true aim of education*", that means intelligence plus character are the true aims in education.

The writer's observation as mathematics *widyaiswara* (teacher trainer) found many designs and implementations of learning mathematics that were developed and applied by teachers only emphasize on establishment of knowledge and skill aspects without character aspect. Rahmi (2013) stated "most of teachers are only focused on improving students' intelligence and less concern to students' character". Agung Prabowo et al. (2010, p. 170) asserted that "the other domains in learning mathematics need to be considered because all this time mathematics is more dominant on cognitive domain". Fadillah (2013) stated "Learning process should make the student become knowledgeable figure and has character value that reflected on his or her everyday life. Similarly, Arifin (2015, p. 13) stated "Learning math is not only focusing on how students comprehend mathematical concepts and principles but also must be accompanied by the character". Meanwhile, Hindarto (as cited in Miftahul et al, 2012, p. 55) mentioned "Character building should be conducted concurrently with academic education"

Those elaborations inspire the writer to review learning theories about mathematics learning process that are related to the domain of attitude, knowledge, and skill developed by Robert Gagne. Robert Gagne (as cited in Suherman Erman et al, 2011) mentioned that "there are two kinds of object in mathematics learned by students namely *direct object* and *indirect object*". Direct object covers fact, concept, principle, and skill: (1) fact is agreement in match in the form of symbol, picture or sign, (2) concept refers to an abstract idea that allows us to categorize objects into examples and are not examples, (3) principle is the most complex object among the other objects, and (4) skill refers to the ability to answer correctly and quickly. In the other hand, indirect object includes logical thinking skill, problem solving skill, analytical thinking skill, positive attitudes towards mathematics, thoroughness, perseverance, discipline, and other things that will be learned implicitly if students learn mathematics. Therefore, learning mathematics does not only present direct object but also indirect object of mathematics. The development of mathematics indirect object for students will give impact on their logical thinking skill and build them to be critical, consistent and independent.

Based on the description, learning design is required to facilitate the relationship between direct and indirect object of mathematics. In this study, the author developed a lesson plan with ELPSA framework (*Experience*, *Language*, *Picture*, *Symbolic*, and *Application*). Lowrie and Patahuddin (2015, pp. 3-4) explained that "ELPSA framework refers to an approach of cyclical learning design. Nevertheless, it is important to remember that ELPSA is not linear process because learning refers to the complex process that cannot be predicted entirely and does not occur in a linear sequence". Furthermore, Lowrie and Patahudin (2015) stated "The five components that proposed are necessary to be considered in designing lesson plan of mathematics, if you expect the students to understand mathematics comprehensively"

The first component of ELPSA is *Experience*. Bodner (cited in Ministry of National Education 2012, p. 7) clearly explained "*Piaget argued that knowledge is constructed as the learner strives to organize his or her experiences in terms of preexisting mental structure or* *schema*". Therefore, the accumulated experience will determine a person's behavior. Pavlov, Skinner and Hull (as cited in Bilson, S, 2008) mentioned that "person's behavior is the result of learning from the accumulated experience". Experience can be introduced through brainstorming, general discussion, using visualization to provoke thoughts and presenting story by teacher or students.

The second component is *Language*. Sutawidjaya (2002, p. 5) revealed that "language is important element learning. It is possible that the students do not understand a mathematical concept not because the concept was too difficult for them, but because the way teacher deliver the material cannot be understood by the students". Therefore, the teacher should present the material using simple language and easy to understand. Language is closely related to everyday social interaction including learning process. One of the factors that determine individual's readiness to receive message is the clarity of the information received (Fabrigar et al., *as* cited in Neila Ramdhani, 2013). The role of language is used to encourage understanding. Language provides mathematical ideas because mathematics is full of symbols. By using simple language, then, the meaning of symbols in mathematics will be easy to be understood by students.

The third component is picture (*pictorial*). The application of picture representation (visualization) presents mathematical ideas. Picture is critical aspect in mathematics. Pictures are often used for bridging students' understanding and preparing the stimuli to complete the math task before introducing the symbols. Bruner (as cited in Russeffendi, 1991, p. 109) expressed that to gain the understanding into the process of learning mathematics, student activities should be directed in three ways; enactive, iconic, and symbolic. In iconic, the students conducted activities related to the mental represented by the objects that it manipulate (explaining with pictures).

The fourth component is the use of symbol (*Symbolic*) with regard to the presentation of mathematical ideas. This component makes mathematics different from other disciplines and sometimes refers to the universal language. Bruner (as cited in Russeffendi, 1991, p. 109) revealed that symbolic way means the students manipulate the symbols or signs from certain objects to get comprehension.

The fifth component refers to *Application*. This stage describes how the knowledge had been obtained (whether facts, concepts, principles, and skills) can be applied in different situations.

ELPSA is a learning framework that let the students to communicate (using language) from every experiences they had, and then express it into the representation of picture and symbol as well as using symbol math problems they experienced (as application). The results of these activities have an impact on the ownership of knowledge and skills. It also can bring positive attitude like honesty, politeness, thoroughness, persistence, and others. These attitudes and behaviors are formed in line with the development of students' knowledge and skills, or an accompanist effect "*nurturant effect*" of teaching and learning process (Ridwan Abdullah Sani, 2008). Thus, ELPSA framework gives space for teachers to design and implement the learning that allows the simultaneous relationship between direct object and indirect object of mathematics. Such relationship has an impact on the understanding of facts, concepts, principles, and skills based on the ownership of characters.

Ministry of National Education (2009, p. 2) explained that character is a personality formed by values, morals, and norms underlying perspectives, thinking, attitude and how individual acts that distinguishes an individual with others. Nation's character is manifested from person's character who became the citizen of the nation. Soedarsono (2006, p. 9) stated that "professionalism is not determined by a person's education level, but by competence and character he or she has, the effort that is based on and guided by the values of courage, passion and true dedication". These ideas indicate that competence shows what can be done by a person, while character shows what the person is. Hence, character is related to personality, value, moral, and manners reflected by individual's appearance.

To create intended values of character, thus schools become the most strategic institution in implementing character building. Bennett and LeCompte (as cited in Slamer Suyanto, 2011, p. 4) asserted that improving character value is not only conducted by parent or family, but also at school as the most strategic place. The implementation of character building started from lesson plan that allows learning process to gain knowledge, skills and attitude simultaneously toward the students. Thereby, the result of learning is comprehension of direct and indirect object create the students' who have positive values: religious, honest, tolerance, discipline, hard worker, creative, independent, democratic, curiosity, national spirit, nationalist, appreciating the achievements, friendly, communicative, peaceful, love reading, environmental care, social care, responsible, as contained in Ministry of National Education (2009, pp. 5-7).

In this review, the writer observed characters that are related to indirect object of mathematics developed by Gagne include the ability to investigate (example: work hard, careful, curiosity), problem-solving skill (for instance: patience, curiosity, responsibility), personal discipline (example: honest, discipline), appreciation on mathematical structure (for instance: orderliness, consistent).

Result and Discussion

RESULT OF STUDY

Every teacher in every educational level is obligated to design lesson plan for the class where he/she teaches (Ministry of National Education, 2015, pp. 152-153). Developing lesson plan by teacher must be done before the learning was conducted. Besides, the lesson plan improved contains basic competence consists of spiritual attitude, social attitude, knowledge and indirect object. In other words, lesson plan contains direct and indirect object.

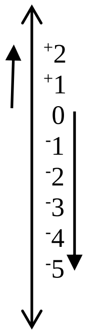
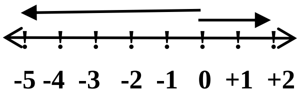
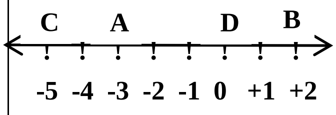
Several results of studies found that teacher who taught mathematics effectively was determined by the ability of designing lesson plan (Sutawijaya et al., 2011). Thus, it is clear that to achieve effectiveness of learning math impacting on a more comprehensive of students' understanding, a lesson plan which facilitates direct object and indirect object simultaneously is absolutely necessary.

Based on the description above, here the writer has outlined the workflow of ELPSA framework that can be developed by the teachers about the correlation between direct and indirect object of mathematics from Gagne's learning theory.

Table 1. The example of the relationship between ELPSA framework with direct and indirect objects on mathematic in Robert Gagne's learning theory.

Topic on mathematics	Component of ELPSA	Direct object of mathematics (cognitive and psychomotor)	Indirect object of mathematics (affective, attitude or character)	Note teaching (Including explanations and questions)
The concept of positive integers and negative integers	**Experience (E)** Teacher starting the lesson by asking question "Could you mention everyday vocabulary that related to opposite words". "For those who can mention the examples of things around us that had opposite condition, please raise your hand". If the students find it difficult (based on the situation),teacher helps them	**Fact**	Curiosity, team work and accuracy.	Discuss in pair and investigate the examples of things around us that have two opposite conditions. Teacher explains! To be able solve these problems, provoke your curiosity, desire to investigate, team work and accuracy.

	by showing pictures that indicate: (1) The height of pulley and the depth of bucket from a well (2) The height of sail and the depth of anchor of a ship (3) Increase or decrease the temperature somewhere in a thermometer			
	Language (L) Teacher explains! Measuring the height and depth is an opposite activity that is going up and down (from the ground surface as a starting point or zero point). Next, the teacher explained "up" with positive (+) sign and "down" with negative (-) sign. Likewise "forward" with positive (+) sign and "backward" with negative (-) sign.	**Fact**	Obedience, curiosity, teamwork, responsibility, and courtesy	Discuss in pair, and then present the result of discussion. a. Suppose the height of pulley is two meters (2m) and the depth of water surface is five meters (5m) from the ground in a well; (1)Which integers are suitable with the height of 2 meters and the depth of 5 meters? (2)Which integer is appropriate with the ground surface? b. Andi takes two steps forward and Budi takes three steps backward from where Ali stands. Which integers are suitable with these statements? Teacher explains! Remember! This agreement should be obeyed together and you must work

				together to solve the problem above.
	a. Picture (P) To assist the students in understanding two problems above, teacher showed a picture of the following line number.  Figure a  Figure b	**Concept**	Curiosity, teamwork, accuracy, tolerance, discipline and responsible	*Observe the picture and discuss it in a group consists of 3-4 members in order to understand the position of numbers on a line number. Write down the symbol of integers indicated by the arrow on picture (a) and picture (b). *Teacher explains! Keep in mind the rule that you have agreed. Teamwork, accuracy, and tolerance are totally required. *Present the result in front of your classmates.
	Symbolic (S) Write down the symbol of integers indicated by line segment AB, DC, AD on the number line bellow:  	Principle	Teamwork, accuracy, and consistency	Discuss and present the result of the task you are doing. Write down the symbol of integers indicated by the picture beside. You should do it together, careful and consistent in solving these problems.
	Aplication (A)If the height of pulley from the ground 2 meters and the distance of surface water of well from the ground 5 meters, how long the rope needed to connect?	Principle and skill	Accuracy, exactness, speed and responsibility	Discuss in pairs and solve the problem properly. Teacher explains! Accuracy and exactness are required in

				answering the question and should accountable.

The explanation above shows that ELPSA bridges the reinforcement of direct object and indirect object of mathematics simultaneously, which is strengthening facts, concepts, principles, and skills. In addition, ELPSA also gives a students opportunity to appreciate each other in teamwork, accuracy, perseverance, persistence, hard work, discipline, tolerance, diligent, and consistent.

DISCUSSION

The main point of what have been explained is the relationship between direct object and indirect object of mathematics toward ELPSA framework as a learning design that is believed to be able to build the values of nation's character. Consequently, the discussion is emphasized on the relationship of; *Experience* component (E), *Language* component (L), *Pictorial* component (E), *Symbolic* component (S), and *Application* component (A) with direct object and indirect object of mathematics.

Table-1 illustrates the introduction of positive and negative integer’s concept. Teacher began the learning by doing brainstorming about the concept of positive and negative integers. "Could you please mention everyday terms that related to opposite words". Who can point out the objects around us that have two opposite conditions? If the students seem confused with the question, the teacher shows the students some pictures they are accustomed that indicate the concept of positive and negative integers. For example, the height of pulley and the depth of bucket from a well, the height of sail and the depth of anchor from the berth ship, the rise and fall of temperature on the thermometer as shown on the pictures bellow.

Picture-1

Picture-2

Picture-3

These descriptions represent the example of **Experience (E)** in ELPSA framework. Lowrie and Patahuddin (2015) stated that the experience can be given by brainstorming, general discussion, using visualization to stimulate ideas, and also presenting story by teacher or students".

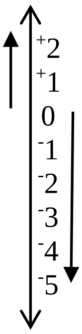
Measuring the height and depth on picture-1 and picture-2 or the rise and fall of temperature on picture-3 are the activities belong to opposite direction. The height of pulley to the top (from the ground level) presented with positive sign and the depth of water surface to the bottom (from the ground level) presented with negative sign on picture-1. The height of mast to the top (from the sea level) presented with positive sign and measuring the depth of anchor to the bottom (from the sea level) presented with negative sign on picture-2. Or the temperature above zero with positive sign and the temperatures below zero with negative sign on picture-3. These are examples of **Language (L)** in ELPSA framework. Lowrie and Patahuddin (2015) said that language is used to encourage understanding. Language also provides mathematical ideas as mathematics contains a huge amount of symbols. By using simple language then the meaning of mathematical symbols will be easy to understand by the students. Similarly, Sutawidjaya (2002, p. 5) explained that “language is an important element in every lesson. It is possible that the students do not understand a mathematical concept not because the concept was too difficult for them, but it is due to the way the teacher deliver the material cannot be understood by the students".

The accumulation of experiences and simple language which can be obviously known by the students will form character and attitude as a result of study. Pavlov, Skinner and Hull, (as cited in Bilson, S, 2008) claimed that a person's behavior is the result of learning from the accumulated experience". In the same line, Fabrigar et al., (as cited in Neila Ramdhani, 2013) stated that one of the factors that determine individual's readiness to receive message is the clarity of the information received.

Suppose that the height of pulley is 2 meters (2m) and the depth of bucket is 5 meters (5m) on picture-1, then the integer corresponding to the height is denoted by +2 and denoted by -5 for the depth of water surface. Number +2 and -5 in Gagne's learning theory is called fact or direct object of mathematics. Gagne stated that fact is agreements in mathematics in the form of symbols, pictures, and signs. Teacher needs to explain that in order to understand +2 and -5 from the the problem, the students should raise their curiosity, willingness to investigate, work hard, teamwork, and accuracy. Teacher's explanation indicates character building toward the students. In line with Agung Prabowo et al. (2010, p. 170) that the other domain in learning mathematics are also need to note because during this time learning mathematics is more dominant on cognitive domain. To make the students' understanding problem (a) and (b) mathematically, then the teacher showed the following picture:

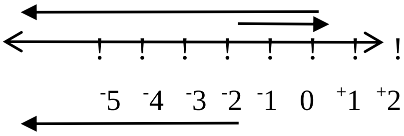
Picture-4

Visualization from the height of pulley and the depth of bucket is depicted by following number line.



Picture -5

Or with the following number line:



Picture 6

Picture-4 and picture-5 are the example of **Picture/Pictorial (P)** within ELPSA framework. In learning theory Gagne represents picture which is called **concept**. Gagne stated that **Concept** is an abstract idea that allows us to categorize object into example and not example. Bruner (as cited in Russeffendi, 1991, p. 109) mentioned that to obtain students' understanding directly, they should be involved in manipulating objects or pictures. It means, to understand a concept, the students are assisted by manipulating objects, materials or pictures. Teacher explains "... remember this is the agreement that need to be obeyed together and you should contribute in teamwork, conscientious, tolerance, and responsible”. Teacher's elaboration indicates character building for students.

Teacher asks the students to write the symbol of the number indicated by the line segment AB, DC and AD on the picture of line number and explains that the number symbol corresponding respectively are +4, -4 and +2. Here is the example of **Symbol/Symbolic (S)** in ELPSA framework. According Gagne's learning theory, the length of line segment AB=+4, the line segment of DC=-4 and line segment AD=+2 refers to **principle**. Gagne stated that principle is the most complex object. Principle is a series of concepts with the correlation between those concepts. To figure out the direct object of mathematics is not sufficient by only manipulating pictures but the most important is the students comprehend symbols in math. Bruner (as cited in Russeffendi, 1991, p. 109) asserted that to gain the understanding, the activity that should be conducted by students is manipulating symbols or signs from certain objects. At this activity, the students are already being able to use notation without depending on real object.

If students already had understood the facts, concepts, and principles appropriately, furthermore, their understanding needs to be improved in order to be able to use it. For instance, the question: if the height of pulley from the ground level 2m and distance of water surface in a well from ground level 5m, what is the approximately length of rope needed to connect the pulley and the water surface?. This is the example of **Application (A)** which belongs to ELPSA framework. In Gagne's learning theory, it is called **skill**. Gagne said that skill is the ability to answer correctly and fast.

This is the process of learning mathematics that forms the knowledge and skills as well as the attitude or character. In the same line, Fadillah (2013) mentioned "Learning process should lead the students to become knowledgeable figure and has character values that reflected on his or her everyday life. Similarly, Arifin (2015, p. 13) stated "Learning math is not only focusing on how students comprehend mathematical concepts and principles but also must be accompanied by the character". These explanations showed that ELPSA facilitates the relationship between direct and indirect object of mathematics simultaneously since lesson plan of ELPSA framework beside allows students to understand the facts, concepts, principles and skills, it also gives students the opportunity to appreciate each other in a teamwork, accuracy, perseverance, hard work, discipline, tolerance, and peaceful.

Similarly, Lowrie and Patahuddin (2015) claimed that ELPSA framework belongs to an approach of cyclical learning design. Then, they further explained that the five components that proposed are necessary to be considered in designing lesson plan of mathematics, if you expect the students to understand mathematics comprehensively". Likewise Gagne (as cited in Suherman Erman et al, 2001, p. 35) mentioned in learning math there are two objects the students obtain, namely direct and indirect object. Indirect object covers the ability of investigating and solving the problems, independent learning, positive attitude toward mathematics, and knowing how to learn properly. Therefore ELPSA bridging the relationship direct object and indirect object and also facilitate the students to understand facts, concepts, principles, and skills as well as build their character in investigating (Example: patient, curiosity, and responsible), personal discipline (Example: honest and discipline), appreciate the structure of mathematics (example: orderliness and consistency). Thus building students' character means building nation's character because the future of nation is absolutely determined by nowadays-students character.

Conclusion

Based on the explanation and discussion, the writer concluded in the following lines:

1. Learning design of mathematics with ELPSA framework facilitate the relation and equality of direct and indirect objects of mathematics.
2. Learning design of mathematics with ELPSA framework can be used to build the character values of the nation because it allows the teachers to improve learning process that assist the students to explore and comprehend the facts, concepts, principles, and skills (direct object of mathematics), besides to give the teacher the chances to explain values of character (indirect object of mathematics) which are formed when the students accomplishing the tasks that deal with knowledge and skill of mathematics. Consequently, build nation's character should begin on building students' character (indirect character) when the students explore and accomplish their mathematics assignment.

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Developing Student’s Book “Basic Algebra” based on Guided Discovery

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Abstract

This study aims at developing mathematics instructional material for 7th grades of junior high school. This material is about basic Algebra based on the basic competencies of determining variables, and characterized by validity, practicality and effectiveness of guided discovery whose procedure include; Understanding problem, Arranging/processing/organizing data, Making conjecture, Stating conjecture, and Testing the understanding of the truth of conjecture. This research is a development research adopting the *four D model* consisting of *define, design, develop, and disseminate.* The product testing was conducted in SMPN 2 Gondanglegi. The compostite score of validation by experts and practisioners showed that the student’s book is valid with content vality of 2.52. The practicality and effectiveness were assessed by two observers using some instruments including lesson plan, feasibility observation sheet, student’s questionnaire, and student’s test in class 7C and 7A. The results showed feasibility for both each class was 2.34 and 2.6 respectively, suggesting the material is practical. Finally, the result of questionnaire and test of students’ mastery indicate hat the student’s book is quite effective. Considering those results, it is expected (1) this book can be applied in teaching and learning activities, (2) it is sugested to provide the answer key and teacher’s guide book, and some parts need to be reworded; (3) it needs to be tested on a wider population.

***Keywords**: development, student’s book, basic Algebra, guided discovery*

Introduction

Variable is an important part of the early algebra concept that must be mastered correctly. Research related variables have been carried out by several experts including (Malisani & Spagnolo, 2009 ; Asquith, Stephens, Knuth, & Alibali, 2007 ; Xiaobao Li, 2006; Soedjadi, 2000). The results showed that a variable material was a crucial material, transition from arithmetics to algebra, so that it took the correct understanding about the variables in a various situations and problems (Malisani & Spagnolo, 2009). Teachers rarely identified understanding the concept of error variables and equals sign, whereas this misunderstanding will hamper the ability of the students to solve the problem (Asquith, Stephens, Knuth, & Alibali, 2007). The students` difficulty was found on how to operate and understand the variable (Xiaobao Li, 2006). Variable in Mathematical objects can be viewed as the transition from Arithmetic to the Algebra. The use of the symbols of variable in junior high school levels will be more and more. Therefore the emphasis of *symbol sense* or understanding of or sensitivity to symbols need a serious attention. This material is very important because it would be used in various fields of work and Science (Soedjadi, 2000).

Research of transition period arithmetic to the algebra has been done by some researchers (Patton & Santos, 2012; Malisani &Spagnolo, 2009; Amit & Neria, 2008; Anthony & Hunter, 2008; Tabach & Arcavi & Hershkowitz, 2008; Livneh & Linchevski, 2007; Middleton & Oksuz, 2007; Breiteig &Grevholm , 2006; Lannin & Barker, 2006; Ferrari & Luigi, 2006; Gallardo & Hernandes, 2005;Kieren, 2004; Healy & Hoyles, 2000; Knuth, 2000). The results showed that writing numbers, then a new visual of Algebra will facilitate the connection of the concept of arithmetic and algebra (Patton & Santos, 2012). Analyzing whether the allegation of "unknown" related to the interpretation of the variables (Malisani & Spagnolo, 2009) The ability of the generalizing is very essential in solving problems in algebra (Amit & Neria, 2008). The process of transition from arithmetic to reasoning algebra is a difficult matter, so that teacher could give the activity related to the pattern (Anthony & Hunter, 2008). Making generalization of arithmetic transition to algebra using computer assistance would be beneficial toward the effectiveness of time (Tabach & Arcavi & Hershkowitz, 2008). A direct intervention in the context of a specific numerical led a better algebraic understanding (Livneh & Linchevski, 2007). The development of middle school students reasoning in the context of Arithmetic and Algebra has some inherent problems (Middleton, 2007). Students could finish, explain and justify why those algebra problem could be solved (Breiteig & Grevholm, 2006). The strategy of generalizing students affected by (a) the input, (b) the structure of duties, the previous strategy (c), (d) the visualization of the condition, and (e) the interaction between teachers and students, and Between students (Lannin & Barker, 2006). The process of symbolizing from Arithmetic to the Algebra was crucial (Ferrari & Luigi, 2006). The students` difficulties in performing operations on negative numbers of Algebra (Hernandes, 2005). Algebraic thinking already beginning since the primary education and it was integrated with the next grade (Kieren, 2004). Many students successfully showed the proof in solving their problems without using Algebra (Healy & Hoyles, 2000). Students successfully solved the problem by connecting between Algebra and graph drawing (Knuth, 2000). Other studies have associated algebra done by experts diantarannya (Koca, 2010; Banerjee, 2008; Berger, 2008; Novotná & Hoch, 2008; Ruthven, Deaney & Hennessy, 2008; Katz, 2007; Radford & Puig, 2007; Francisco & Hähkiöniemi, 2006; Hallagan, 2006). To sum up, they thought that the skill in symbolizying was very important.

The introduction of the new curriculum of 2013, learning requires scientific approach that is characterized by observing ,questioning, associating, conjecturing, experimenting, communicating. In the scientific approach it needs to apply discovery/inquiry learning (Annex B, no. 65 Permendikbud 2013:3)

In the attachments mentioned about principles of learning told that from students are told to the students find out. Learning with a learner find out called the inquiry or discovery. Research related to the principle of discovery learning has been done by some of the experts (Delcourt & McKinnon, 2011; Sikko & Pepin, 2011; Effendi, 2012; Hunter, 2010; Pais,2009; Abdullah & Shariff, 2008; Santos, 2008; In Woo. et al, 2007; Makar, 2007; Jaworski, 2006; Leikin & Rota,2006; Diezmann, 2004; Goos, 2004; Muhsetyo, 2004). In conclusion, inquiry or discovery learning can enhance the learning quality.

The main problem in this research is how the form of the basic Algebra book is characterized by the guided discovery.That is why the researcher needs to make basic Algebra book characterized by the guided discovery which is valid, practical and effective. Based on that problem researcher entitled his research Developing Student's Book "Basic Algebra" based on Guided Discovery

Learning materials have been defined by experts (Dwicahyono & Daryanto,2014; Lestari,2013; Prastowo,2013; Majid,2012;Darmadi, 2009;Widodo & Jasmadi, 2008; Mbulu,2004;Dick & Carey, 2001). Learning materials are a set of systematically arranged in written or not. Hence, it can enable students to learn (Dwicahyono & Daryanto, 2014). Learning materials mean any kinds of materials arranged systematically that allows students to learn and designed according to curriculum (Lestari, 2013). Learning material is any material (whether information, tools, as well as text) that are arranged systematically, which are used in the process of learning with the purpose of the planning and review of the implementation of learning (Prastowo, 2013). Learning materials (instructional materials) generally consist of knowledge, skills, and attitudes that students should be studied in order to achieve the competency (Darmadi, 2009). Learning materials are any materials used to helping teachers/instructor in carrying out activities of teaching and learning (Madjid, 2009). Learning materials are a set of means or learning tools containing material learning, methods, limitations, how to evaluate systematically designed and interesting in order to achieve the expected goal, namely competence or competence with all its complexity

(Widodo & Jasmadi, 2008).

Learning material in this research is in the form of textbooks used by students that contains the activity of learning so that the achievable learning objectives. The activity of the learning organized by the characteristics and the knowledge already owned by the students so that will help students in learning both at school and at home. According to Hobri (2010:31-32) student` books is a part of the source that allows student and teacher to learn. The definition of the student` books was also described by (Trianto, 2013; Prastowo, 2013). Trianto(2013) defined the student's book was a learning activity booklet for students contains of subject matter, the investigation into the discovery of the concept of activities of science in daily life. Whereas Prastowo (2013) defined the student` book was the written material as the author`s idea.

Bell (1978: 241) stated that *discovery learning is learning which occurs as result of learner manipulating, structuring and transforming information so that he or she finds new information. In discovery learning, learner may make a conjecture, formulate an hypothesis, or find a mathematical "truth" by using inductive or deductive processes, observation and extrapolation.The essential of element in discovering new information is that discoverer must take an active part in formulating and attaining the new information.*

Parta (2009:23) stated that according to Akker, J.V. D, Branch,R. M., Gustafan, K., Nieveen, n., Plom, q. (1999:126-127) there are three criteria to measure the quality of the product i.e.validity, practicality and efectiveness in developing the student's book.

METHOD OF DEVELOPMENT

Development model used in this research was Thiagarajan model, Semmel Semmel (1974) and known as 4 D (Four D Model). The four stages contained of define, design, develop and desseminate. The Define phase was aimed at defining learning needs development in analysis purpose and limitations of the material. Design phase aims to design student` book in order to obtain a prototype of the device learning. The develop phase is for generating a prototype 1 device that has been revised based on the input of the experts. The

Desseminate is the application of the product ich has been developed on a broader scale. The procedure is based on the development model of four D model developed by Thiagarajan, Semmel Semmel and (1974). On the Define stage, the early-late analysis, students` analysis, tasks analysis, an analysis of the concept, and learning objective. On the Design stage produces the initial draft of the student books to be validated by experts and practitioners. Design stage also includes preparation of test-based criteria,the selection of media, format selection, and the earlydraft. In he Develop stage, the draft is validated by experts and practitioners. Test subjects in this research grade VII A and VII C SMPN 2 Gondanglegi Malang in the lessons year 2013/2014. Data trial results are obtained from the Product Development is a form of qualitative and quantitative data. Qualitative data was in the form of comments and suggestions of improvements based on the results of the assessment of the validator, and conversion from quantitative data into a letter or category to make decision.

THE RESULT OF DISCUSSION

The results of the calculation of the score of the three validator obtained an average score of the whole aspects was 2.60 consist of 2.52 for content aspect, 2.83for language and 2.44 appearance. Because of there was construct aspect in content aspects with a score of 2.52 based on criteria, so it was concluded that the sudent’s book was valid because the content aspect was in the range $2 \leq Va \leq 3$. The content aspect contained of the concept of truth [2.67], conformity of the order material [2.67], a problem that presented was interesting for students to learn [2.67], learning activities was appropriate with the learning objectives[2.67], the application the first step in the guided discovery that is given of the problem [2.33], allowed students compose or process or analyse data [2.33], there was a statement that requests students to compose a conjecture [2.33], the right order of the conjecture verbalize activities [2], various exercises led students build understanding independently [2.67], systematic,continue activities in order to reflect the application of step the guided discovery [2.67]. From the explanation above, the student` book met the content validity and construct validity so that student` book meet the required standards i.e. characterized by the guided discovery, also it can be tested on the next step.

THE RESULTS OF THE VALIDATION OF INSTRUMENTS

The results of the calculation of the score the third assessment of the validator toward the lesson plan retrieved average score = 2.48, Observation sheet toward the use of student book retrieved = 2.67.From the test mastery of the material obtained = 2.5. From the student questionnaire response. it was obtained = 2.58. In conclusion, all instruments were valid so that can be used in measuring the practicality and effectiveness of the student's book.

DATA ANALYSIS AND PRESENTATION OF TRIAL RESULTS

THE RESULTS OF THE TEST OF PRACTICALITY

At the first trial in the class 7 C, the calculation result score assessment of two observers toward the application of the student` book from first to the fifth meeting obtained the average (IO=2.34). For the second trial in the 7A class obtained the average (IO=2.6). From the first and the second trials, can be concluded that the student` book was valid.

THE RESULTS OF TEST EFFECTIVENESS

At the first trial, from 25 students, all students got score more than 2.66 and they gave positive response and average score 1.3. Then, at the second trial, from 23 students, all students got score more than 2.66. To sum up, the student` book was effective.Overall, the student` book had been valid, practical, and effective.

THE REVIEW OF REVISED PRODUCT

Upon validation of the practitioner and experts retrieved that the student` book was valid. The guided discovery has several advantages *(a) The increase in intellectual potency, (b) the shift from extrinsic to intrinsic rewards, (c) learning the heuristics of discovering, and (d) the aid to memory processing* (Bruner, 2006:58). To study a material on the student` book, students were required actively involved in finding with his own understanding and overcoming possible

the difficulty by asking the teacher. The involvement of students finding a concept according to Hudojo (2005:95) affect to the understanding of the concept became better, long term memory and able to use it into the other context. On the students ' book that studying a concept, students were not given in the form of concept so but students involved the to construct the concept. This characteristic was suitable with Bruner (1973: 406) *"Discovery in learning has precisely the effect upon the learner of leading him to be constructionist ".*

Cobb (2007) learning mathematics is a process where students actively

construct the Mathematics knowledge. Nurhadi (2004:43) stated that teachers in this method should always design the activities refer to inquiry. Subanji (2013) argued that it was important in the discovery learning, a student must be active in formulating and achieving new information. The guided discovery was in accordance contructivist learning which was supported by Kuhlthau (2007)

Guided Inquiry has a solid theoretical foundation is grounded in the constructivist approach to learning. It is based on the work of major educational theorists and researchers, including Dewey, Bruner, Vygotsky, Kelly, and Piaget.

Research study with the method of the invention has been made by some researchers (and Afriati Saragih, 2012; Yunari, 2012; Effendi, 2012; Pais, 2009; Abdullah and Shariff, 2008; Jaworski, 2006; Syafiruddin, 2005; Goos, 2004). The results showed that the the guided discovery can help build and improve understanding (Saragih and Afriati, 2012;Pais, 2009; Jaworski, 2006; Syafiruddin, 2005). The guided discovery also improve the ability of reasoning (Abdullah and Shariff, 2008). Guided Discovery method can also enhance the ability of representation and problem solving of students mathematically (Effendi, 2012). Discovery methods can also increase participation,response, the ideas and opinions of students in learning when teachers are able to give scafolding appropriately. It is suggested in the study need to be created and community culture learning in the classroom through discovery (Goos, 2004).

Moreover it can also improve learning outcomes (Yunari, 2012). Understanding of students toward the Algebra material were very good this was demonstrated mastery of the material in the first trial of 25 students with minimal material mastery criteria 2.66 retrieved results each student gaina value of no less than 2.66. So did a second test on all the students as many as 23 students mastery of the material over 2.66. This aligns research findings (Saragih and Afriati,2012; Yunari,2012; Pais, 2009; Jaworski, 2006; Syafiruddin, 2005) which stated that the guided discovery method can help build and improve students understanding.

The practicality of this book was supported by the existence of the clear instructions in the each activity so that made students to learn independently. The characteristics of student` book according to Lestari (2013:2) that is self contained, self instructional, stand alone, adaptive, and user friendly. From the results of the student response on the first product test score average response from 25 students is 1.3. On the first product test indicated that the students gave positive response.So also at the second product test score average response of 23 students was 1.42. It showed that the students gave positive response. These findings was suitable with Effendi (2012) student has found a positive attitude towards mathematics and learning with the guided discovery.

Ths student` book gave students opportunity to learn independently. The importance of independent study was stated by some scholars and researchers (Hariadi, 2012; Yamin 2012, Chaeruman, 2007; Aisha & Hiltrimartin, 2004; Hasibuan & Moedjiono, 2002; Syam, 1999)

Student's book which had already been validated by experts and practitioners, as well as tested based on that description above has several advantages, as follows:

a. Provide opportunities to the learners can study independently.

b. Include the stages of guided discovery explicitly.

c. Increase the student` confident because the material was given notice of the students` ability.

d. Given notes that must be observed by students about the important things in the writing of mathematical symbols.

e. Help improving social ability, reading and communication.

f. Engage students to be active in learning.

SUGGESTION

At the time of developing the instrument validation of the Lesson Plan questions that lead to the guided discovery was less. Hence, for the next researcher should develop the instrument which contains more questions about the application of Guided Discovery.

The product of this research was still applied in SMPN 2 Gondanglegi. So it was needed to desseminate to the other schools or other regions.

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The Etnomathematics of Calculating An Auspicious Day Process In The Javanese Society as Mathematics Learning

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Abstract

The purpose of this research is to describe the process of calculating an auspicious day in javanese society. This research is focused on the process of calculation an auspicious day for javanese wedding, specifically in Purworejo. This research is included to assessment research of ethnomatematics, because the research reveals daily activity conducted by ethnic at certain cultures. This research is kind of descriptive research with qualitative approach. Data collection methods applied in this research is interview. The subject of this study consisted of one informant who has been knowing the process of calculating an auspicious day in javanese society. In this research, data analysis is performed using descriptive analysis. The analyzed data in this study is the result of the interview. The result of this research shows that the process of calculating an auspicious day for javanese wedding, specifically in purworejo has been doing match activities and can be used as learning mathematics.

Keywords: *an auspicious day, ethnomathematic, mathematic education.*

Introduction

Tandiling (2013) said that etnomathematics is a mathematics applied by a certain culture society, labor/farmer society, kids of certain social class, professional classes, etcetera. In research point of view, etnomathematics is well known as cultural antropology (*cultural antropology of mathematics*) of mathematics and mathematics education. Mathematics is as basic study needed to analyze either basic of calculating science or computation applied in society to enrich the mathematics development.

In daily life, most of the people do not realize that they have been applying mathematics. They think that mathematics is just a subject that is studied in school. Whereas mathematics is often used in every aspects of life, such as measuring, calculating, numbering, and in some of transaction activities. According to D’Ambrosio (Puspadewi and Putra, 2014), mathematics studied in school is known as academic mathematics, meanwhile etnomathematics is mathematics applied by some cultural group which has been identified, such as ethnic society, group of labor, kids from certain age group, professional class and so on. Therefore it can be stated that etnomathematics is mathematics appeared as the result of the impact of the activity in the environment influenced by culture. Having known about etnomathematics, the people can understand that mathematics is a study which is not only able to be studied in a classroom.

Etnomathematics is mathematics both grown and developed in a certain culture. Etnomathematics is perceived as a lense to see and understand that mathematics is a product of culture. The culture in this research refers to language of the society, place, tradition, how to both organize and interprate, conceptualization, and naming the physical and social world (Ascher, 1991). The research of etnomathematics in mathematics has been included into all sectors, such as arsitecture, weaving, sewing, siblings, ornaments, spiritual and religious practice which is often aligned with the pattern occured in nature. According to Puspadewi and Putra (2014) the research about etnomathematics has ever been analyzed, some of them are puzzle game of Hausa culture *Wasakwakwalwa* in North Nigeria, carpenter method of South Africa in deciding center box lid shaped rectangle, and so forth.

Method

The research is included into etnomathematic research, because it reveals the daily activity of one of ethnic group in a certain culture, and kind of descriptive research with qualitative approach (Sugiyono, 2012). The collecting data method applied is interview. The subject of the research is coming out from one intervieweers knowing about the process of calculating an auspicious day in javanese society. The data analysis which is applied in the research is descriptive analysis.

Here are the steps of the research:

1. Introduction, at this step, the writer both decides the area and chooses etnomathematics activity of javanese society. In this research, the javanese society is specified into the javanese society of Purworejo Regency.
2. Making a guidance of interview, the guidance made is only consisted of the outlines of the questions that the writer wants to know about.
3. Implementation, this step is consisted of collecting data taken from interview, about calculating an auspicious day for javanese wedding.
4. Data analysis, analyzing the result of interview, about calculating an auspicious day for javanese wedding.
5. Conclusion, at this step, the writer concludes the data analysis which has been analyzed in the previous step.

Result and Discussion

The Pattern of Calculating Auspicious Day for Javanese Wedding

Weton is a memorial day of birth commemorated in every 35 days. In javanese culture, weton does impact the daily life. One of the weton function is to calculate in searching an auxpicious day for wedding, building house, moving out from the house, or looking for the right time for circumcision. The total of weton can be known from birth day and pasaran usually written by parents.

In the modern life, calculating an auspicious day has been starting to be left behind, but there are still several groups of people believing and using that pattern. In the Javanese society, there is just some of the people in the village that can decide the auspicious day to throw an occassion.

In the calculating system of javanese, there is a basic concept called *cocog*, which means match, such as the matching of either padlock and key, or the man and the woman he will marry. There are several things which has to be paid attention and used in deciding auspicious day, like *netu* the day, and *pasaran* of the birthday date in javanese month of the future bride and the groom.

Deciding an auspicious day, most of the people calculate it based on 7 days (Monday – Sunday), and 5 *pasaran*. Each day has its own pattern which reflects the value of the day and *pasaran.* Here is the value list of the day and the *pasaran.*

Table 1.
Value of each day and *pasaran* of Javanese

No	Day	Value	No	Pasaran	Value
1	Mon	4	**1**	Legi	5
2	Tue	3	**2**	Pahing	9
3	Wed	7	**3**	Pon	7
4	Thu	8	**4**	Wage	4
5	Fri	6	**5**	Kliwon	8
6	Sat	9			
7	Sun	5			

It is only the birth day and the *pasaran* of the bride which is used as reference in deciding an auspicious day for the wedding. For the example, the birth day of the bride is Wednesday of Wage. There are two ways for determaining the auspicious wedding day, the first one is using her birth day, the second one is using her *pasaran.*

1. Using the birth day

   Steps:

   a. Making a day sequence from Wednesday until Tuesday.

Table 2. Sequence Day

Day	**Wed**	**Thu**	**Fri**	**Sat**	**Sun**	**Mon**	**Tue**
Sequence	1	2	3	4	5	6	7

Take her birth day or day that has even sequence (wednesday, thrusday, saturday, and monday) for the example the writer takes thursday which its sequence number is 2.

   b. Matching the selected birth day with *pasaran.*

The rule of deciding *pasaran*, is looking for a mate who is if the *netu* of *pasaran* add up the *netu* of day, then it is divided by four, the residue is 1 or 2. Because the residue of 1 is a symbol of teacher (the person who becomes a role mode) and the one of 2 is a symbol of Wisnu (puppetry character who decends the gods). Meanwhile the symbol of 3 is a symbol of bromo (hot fire) and the 4 is symbol of senility (forgelfulness or the person who has no calculating aspect).

Example: Thursday has *netu* number 8, it can be matched with *netu* number 5 (legi), or *netu* number 9 (pahing). So, the auspicious day for wedding is Thursday of Legi or Thursday of Pahing.

1. Using *Pasaran*

   Steps:

   a. Making *pasaran* sequence from Wage until Wage, in order to decide the *pasaran*.

Table 3. *Pasaran* Sequence

Pasaran	**Wage**	**Kliwon**	**Legi**	**Pahing**	**Pon**
Sequence	1	2	3	4	5

Take her birth *pasaran* or *pasaran* which has even sequence (wage, kliwon, and pahing) for the example, the writers take kliwon *pasaran* because the sequence is 2.

b. Finding match day.

The rule of deciding auspicious day is looking for the day having *neptu* of that day which is if it is added up by *neptu* of *pasaran*, then it is divided by four, the residue is 1 and 2. Example: Kliwon has *neptu* number 8, so that, it can be matched with 3 (Tuesday), 6 (Friday), 9 (Saturday), and 5 (Sunday). Then, if the bride and groom want to throw wedding party in *pasaran* of Kliwon, so the match day will be tuesday of kliwon, friday of kliwon, and sunday of kliwon.

Etnomathematics in deciding auspicious day for wedding.

Based on the statements explained above, it can be concluded that there are several activities about doing mathematical activity. Here are some examples of the mathematical activity which is related to the process of calculating an auspicious day in Javanese culture:

1. Value of day and *pasaran.*

   Each day and each *pasaran* in javanese culture have a certain value, for example, thursday has value of 8, saturday has value of 9. Based on this fact of value, it is stated that javanese society has been doing Mathematics.

2. Deciding an auspicious day for javanese wedding.

   In deciding an auspicious day for javanese wedding, the society can take either birth day of the bride or the day which is even sequence. Based on this fact, the javanese society has already been knowing the pattern of even and odd number. Therefore, the javanese society has already been doing Mathematics in their daily life.

3. The rule of deciding an auspicious day.

   Based of the result of the interview, the rule of deciding an auspicious day can be calculated by the formula, as follows:

$$x = \frac{a+b}{4}, residue\ 1\ or\ 2$$

Which are $x = devided\ number$

$a = netu\ day$

$b = netu\ pasaran$

Potential of Etnomathematics which can be developed in learning activity

Based on the result of the research, it can be stated that there are several potentials which can be developed in learning mathematics. One of them is related to the rule of calculating an auspicious day for wedding. The rule can be developed by the teacher, as a sample question in game format purposed for improving the creative mathematical thinking pattern, for example: *"Bambang and Yuli is javanese future bride and groom using calculating an auspicious day process for their wedding. They come to see Mbah Marijo (one of the elders in that village), in order to ask the auspicious day for their wedding. Bambang was born on Saturday of Wage and Yuli was born on Sunday of Pon. Help mbah marijo to answer some questions from Bambang and Yuli, as follows:*

a. Is it possible to get married on Sunday?

b. If they want to get married on Desember 2016, what date will be match for throwing their wedding?"

Beside, it can be improving critical thinking pattern, it is also expected that connecting the culture to mathematics can be improving the students' motivation in learning Mathematics.

Conclusion

Based on the result of the research, it is concluded that calculating an auspicious day for wedding is included into doing mathematical activity based on the rule of deciding an auspicious day. The formula of deciding an auspicious day for wedding is as follows:

$$x = \frac{a+b}{4}, residue\ 1\ or\ 2$$

Which are $x = devided\ number$

$a = netu\ day$

$b = netu\ pasaran$

Furthermore, it has a potential developed in learning Mathematics in the classroom, for example in material of number or residue theory. The teacher can be developing the theory as sample question in game format purposed for improving the creative mathematical thinking pattern. Beside, it can be improving critical thinking pattern, it is also expected that connecting the culture to mathematics can be improving the students' motivation in learning Mathematics.

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Teaching Elementary Mathematics using Power Point Based Screencast O-Matic Videos

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Abstract

This research was conducted with the use of models of e-learning by using power point based screencast o-matic videos to improve teaching mathematics achievement in elementary school students as a means to support the teaching and learning process and not just implement the teaching materials, but also create an atmosphere of interesting and fun learning. The purpose of this study are: (1) To describe teaching elementary mathematic using power point based screencast o-matic videos in elementary school students (2) Knowing the learning achievement after using power point based screencast o-matic videos. This research data obtained from direct observations during the learning and observation of the form of questionnaires and interviews with the subjects. The result that use power point based screencast o-matic videos can increase interest in learning, as well as the easy absorption of the material so that it can improve student learning outcomes teaching elementary mathematics.

Keywords: *teaching mathematic, screencast-o-matic, interest in learning*

Introduction

E-learning as a media of learning in education are on a very important role and great functionality to the world of education that has been charged with many shortcomings and weaknesses of education such as the limitations of space and time in the learning process emphasizes efficiency in learning in order to get full teaching despite not having to meet also can be accessed anywhere, anytime, in accordance with the tasks given by the teacher normally scheduled time limit specified.

Educational development towards e-learning is a necessity that the standard of quality of education can be improved, because e-learning is just one use of Internet technology in the delivery of learning and broad reach, which is based on three criteria, namely: (1) e-learning is the network with the ability to renew, store, distribute, and share teaching materials or information, (2) delivery through to the end users through a computer using a standard internet technology, (3) focus on a view that is most knowledge about learning behind the paradigm of traditional learning (Rosenberg 2001 ; 28), thus the importance of information technology can be optimized for education. The paradigm shift learning systems began to appear in the process of knowledge transfer. The learning process that is now likely to be more emphasis on the process of teaching (teaching), based on the content (content base), an abstract and only for a certain group (in the process of teaching passive).

The development of learning technology by utilizing various technologies, especially with users of IT has a huge impact on the principle of learning. Learning implies facilitate or assist initiatives of learning for learners. Then the learning is very important concept here is to help the initiative to learn with and without the presence of the teacher, so the teacher's role is lighter but more complex in pursuing various strategies for the learning for learning. The design of learning at grade level is more oriented to improve the knowledge, skills and sensitivity of teachers.

Combines in-person meetings with the electronic learning can enhance the contribution and interactivity between learners. Through face-to-face, learners can get to know fellow students and teachers. Familiarity is very supportive of their collaborative real work. Proper preparation prior to implement a multimedia-based learning plays an important role for the smooth process of learning. All the preparations such as scheduling up to the technical determination of communication during the learning process is an important step in implementing web-based learning. (Dedeh, 2008)

Screencast-o-matic is software that can be used for user friendly by operating system Windows XP, Windows Vista and Windows 7 (Priowirjanto, et al. 2013). Screencast-o-matic can also record webcam activity. This Screen Recoder is usually used to record and share it on youtube tutorial or blog. Screencast-o-matic is a software which can record all e-learning media into a video tutorial which teachers do as though learning in the classroom that can be used as instructional videos

Results of a low learning is not merely a mistake and responsibilities of students, but also because of the teachers' failure in presenting the material in an interesting, election media precisely, the readiness of teachers and prepare materials, and the ability of teachers to give attention to their students (Dedeh, 2008: 107). For the material taught in primary and secondary education have been selected in order to develop students' abilities and personal form so that students are able to follow the development of science and technology. Thus the mathematical function is to develop the ability of reasoning through investigation, exploration, and experimentation as a means of solving problems through thinking and mathematical models as a means of communication through symbols, tables, graphs, diagrams to explain the idea. (Herianto, 2003).

In learning activities teachers will need to select and to develop learning strategies that involve various methods of matching, according to the particularities of materials, suggestions circumstances and situation of students. For that teachers should pay attention to the teaching of mathematics caracter namely: learned gradually, following the spiral method, deductive thinking patterns and embrace the truth of consistency (Yudinugraha, 2008). For that in the selection and use of teaching methods should involve students, lead students to think critically and creatively, it rests on the optimization of the interaction in the learning process and the optimization of the involvement of all the senses of the students.

The research was done by making models of electronic learning using power point based screencast-o-matic videos to increase the variation model of learning mathematics in elementary students as facilities to support teaching and learning process and not just implement the teaching materials, but also create learning scenarios with engagement to invite students to actively and constructively in the learning process. Based on this background the researcher conducting research with the title " teaching elementary mathematics using power point based screencast-o-matic videos" with the formulation of the problem (1) How does teaching elementary mathematics using power point based screencast-o-matic videos in elementary school, (2) How does the interest of student learning using power point based screencast-o-matic videos in elementary school.

RESEARCH METHODS

Qualitative research has the characteristics, they are: (1) natural background, (2) people as a means (instrument), (3) qualitative methods, (4) inductive data analysis, (5) is more concerned with process than results, and (6) design temporary (Moleong, 2009: 8-13).

This study applied learning with using power point based screencast-o-matic videos in elementary school to increase learning achievement in math in elementary schools. The study was conducted in a regular classroom. Researchers act as a key instrument for research who plan, design, implement, collect data, analyze the data, draw conclusions, and make a report. The procedure of this study is descriptive data in the form of description that describes the procedure for teaching elementary mathematics using power point based screencast-o-matic videos.

The design of study can be refined during the study in accordance with the reality on the ground. By looking at the characteristics of this study, that sets of natural, man as a tool, using qualitative methods, data analysis performed inductively, more concerned with process than outcomes and study design which are temporary, then the approach according to this study based on the characteristics of descriptive qualitative research. Descriptive qualitative analysis is done to present data on observations and comments on the questionnaire.

Research data captured by the Likert scale questionnaire using a simple statistic, the data is then processed and compared with the minimum standards that must be met by each component in the aspects analyzed to the data captured by the questionnaire is still a qualitative data prior to analysis must first be converted into the data quantified.

For the field test data obtained from students suggestions or comments analyzed by performing tabulation. Student response data by student questionnaire responses were calculated using a percentage.

The subjects of this study consist of 30 Elementary School students of 5th grade. The data obtained from the test model development using power point based screencast-o-matic videos in elementary school descriptive as a result of questionnaires and field observations in SDN Jaddih 4 Socah Bangkalan, East Java Indonesia

The data obtained in this trial includes several things, they are: (1) the pleasure of students to learn, (2) the accuracy of the draft learning model with the material being taught, (3) Accuracy of delivery of a material through a model of e-learning power point based screencast-o-matic videos. (4) the effectiveness of media power point based screencast-o-matic videos designed. While the data collection instruments, among others: questionnaires, observation sheets, reports teacher educators, and documentation.

DISCUSSION

1. Feedback and assessment of students in math

To test on these media are conducted in SDN Jaddih 4 Socah by the number of students 30 people. Description of questionnaire data media usage screencast o-matic in 5th grade of primary school as follows:

Table 1. The response of students to the use of media screencast o-matic

No.	Aspect	response		Percentage		Category
		yes	No	Yes	No	
1	Students' motivation to learn	26	4	86%	14%	Very good
2	The accuracy of the draft learning model with the material being taught	22	8	73%	27%	Good
3	Accuracy of delivery of a material through e-learning models power point based screencast-o-matic videos	24	6	80%	20%	Good
4	Media effectiveness power point based screencast-o-matic videos	24	6	80%	20%	Good

The result of this media trial conducted in Elementary School Jaddih 4 Socah is 86% of students are motivated to learn to power point based screencast-o-matic videos, while 14% of the students are still not motivated to learn, this is because students are still not familiar with the media. For the accuracy of the design aspects of the learning model with the material, 73% of students said that the material geometry using media screencast o-matic is very precise. While 27% other students said that it is not appropriate, it is because the teachers have little creativity in designing a video screencast o-matic. In the aspect of the precision of delivery of a material through e-learning models with media screncast-o-matic, 80% of students said that it it is appropriate for the material flow in the video is appropriate. And in the aspect of media effectiveness Screencast-o-matic designed 80% of students said that learning with Screencast o-matic media very effectively so that students feel happy in learning.

Description of pretest and posttest data before and after the learning process is done by learning media screencast-0-matic to math instruction. The average value of pretest and posttest are 68.6% and 75.3%. it increased 6.7% following table recapitulation:

Table 2. Description Value Pretest and Posttest

Criteria	Pre-Test	Post-Test
Amount	2060	2210
Average	68.6	75.3
Enhancement	6.7%	

CONCLUSIONS AND RECOMMENDATIONS

Researcher concluded that: (1) E-learning media by using Screencast-o-matic in teaching elementary math in primary students can improve student achievement. (2) the use of media power point based screencast-o-matic videos can foster student interest.

Based on the above research findings can be presented some suggestions in the use of this medium as follows: (1) This learning media power point based screencast-o-matic videos can be a choice in the selection of instructional media into one of the components that are important in the learning process because it can facilitate teachers in achieving the learning objectives. (2) This media power point based screencast-o-matic videos learning can be used by students as a self-learning media to deepen the material being taught or recalls the material being taught. (3) This learning media extremely easy to use and easily applied in teaching, (4) in the manufacture of this media is expected to have the creativity of a teacher so that students are more motivated to learn.

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School Environment as Elementary School Learning Mathematics Students Laboratory

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Abstract

Environment is a laboratory for children to develop themselves both physically and spiritually. The natural environment and social environment provide experiences to children directly. The environment can develop the child to negative or positive direction so that adults around them have to able to direct that the environment is capable of forming the child in a positive direction. School environment as a learning laboratory of mathematics can be used to develop an understanding of the basic concepts of mathematics and to improve critical thinking skills in primary school students. Based on Jean Piaget's theory of cognitive development, elementary school students are in the concrete operational period, but they also have to understand the basic concepts of abstract mathematics. Abstract mathematical concepts can be studied by students in the school environment. Students, with the guidance of teachers, can associate the basic concepts of mathematics with the experience acquired in the school environment. Lessons derived from the environment and the student experience can be easier for students to understand the material because students construct their own knowledge and experience firsthand. Learning environment around students have some positive impacts: (1) To develop students' ability to understand the basic concepts of mathematics, (2) improve critical thinking, (3) Cost-effective, and (4) Students gained a lot of experience. Students learning environment includes observing shapes of objects around them, experience in the use of time, the learning unit, and basic concepts of arithmetic.

***Keywords*: :** environment, laboratory, mathematics

INTRODUCTION

One duty of the State is the nation's intellectual life as mandated in the 1945 opening paragraph IV. To achieve this it is necessary to reform in various sectors especially in the education sector. Education is key to the development of a nation. A good education will print the next generation of literate knowledge, superior moral character and improve the skills possessed. Education is a process of transformation of cultural values in order to build attitudes and personality in accordance with the philosophy of the nation so as to produce a nation who has the mental attitude and the personal qualities of the nation is more reliable in building this nation.

Law number 20 of 2003 on National Education System, explained that education is a conscious and deliberate effort to create an atmosphere of learning and the learning process so that learners are actively developing the potential for him to have the spiritual power of religion, self-control, personality, intelligence, character , as well as the skills needed him, society, nation and country. In the law implied that a good education can develop all the potential of the learners. The potential improvement can be seen through the better learning achievements. Learners can achieve good learning performance based on two things that influence it from within and from outside the child. Dalyono (2005: 55-60) There are several factors that influence a person's achievement of learning outcomes, namely: factors originating from within the learner and factors from outside himself. Factor in people who learn that the internal covering health, intelligence and aptitude, interest and motivation, and learning, while the factors originating from outside the people who learn that external factors include the family, school, community and environment.

A supportive environment will encourage children to eager to understand new concepts. The smoothness of the learning environment is determined by both environmental conditions such as environmental conditions of the school, the family and community environment. Safe and comfortable atmosphere that is created from a good environment will make it easier permeates child receives learning. Motivation child will also increase and resulted in increased learning achievement of children. So far only focused on the children in school classrooms. They acquire concepts and materials through interaction in the classroom. And if you listened to the laboratory environment is an infinity for children to learn and develop the potential in him. Environment will give directly to the experience of the child so that the child is not only bound to the existing knowledge in the classroom. School environment as a learning laboratory mathematics can be used to explore and develop the children's understanding of mathematical concepts. Mathematics is not just learning a concrete concept, but also there is an abstract concept. It is necessary for the media to learn more so that critical berfikis capability can be enhanced. Through the learning environment in schools is expected to increase the learning mathematics reasoning power of children to resolve problems related to mathematics. Lessons derived from the environment will allow the child to understand the concept given because your child will build his own knowledge and experience firsthand. The use of the natural environment for learning requires careful planning so that competence can be achieved. Based on the description above, this paper will discuss the learning environment and the characteristics of elementary school students in the teaching of mathematical concepts.

DISCUSSION

Learning Environment

Suryabrata (2006: 233) states that "the environment is anything different outside individuals where the overall behavior of the individual interacts with his environment, whether consciously or unconsciously, directly or indirectly". Baharuddin (2007: 68) in environmental education can be interpreted as all the factors that are beyond the child and that have meaning for development and continually influence on him. Learning is a process of changing the behavior of individuals who are relatively permanent as a result of experience, habits or exercise reinforced. Prayitno (2009: 203) study is an attempt to master something new. Environment plays an active role in the learning process. Studying the interaction between the individual with the environment. Provide environmental stimulation and individuals respond to these stimuli with marked changes in its behavior. Learning environment around students have some positive impacts: (1) To develop students' ability to understand the basic concepts of mathematics, (2) improve critical thinking, (3) Cost-effective, and (4) Students gained a lot of experience. Hamalik (2001:28) states that "Learning is a process of individual behavior change through interaction with the environment. While the core of the learning experience and experience is gained through interaction with the environment, both physical and social environment". From the above explanation can be concluded learning environment is anything to be around people and affect all kinds of good teaching and learning in the family environment, school environment and community circles. Conducive learning environment will improve learning achievement of each individual. Hutabarat (1995: 203) learning Environment divide into: 1) the physical learning environment is everything that is contained in our place of learning, such as lighting, office chair, and the room is a place to learn. 2) The social environment that requires the presence of a friend of a course of study and others that encourage or inhibit student learning as well as the atmosphere there. The social environment consists of: family environment, school environment, community. Widyaningtyas (2013: 137) Family environment is all conditions and external influences on the lives and development of family members, among others: the way parents educate, relationships between family members, the house, the family's economic situation, understanding parents, cultural background. The school environment is an environment in which teaching and learning take place the students familiarized with the values of the school rules and values of the learning activities of various fields of study, among other things: the methods of teaching, the curriculum, the relationship of teachers with students, school discipline, learning tool, time school, lessons above standard size, the state of the building, method of learning, homework. While communities are supportive environment between the school and family environment that includes student activities in society, mass media, where people hang out and form kehidupaan.

Factors that support the management of learning environments include: 1) The good study include spatial location, a place of learning, the application of sufficient light, good air, the spatial arrangement of the classroom, 2) Media study provided a good learner and equipment will minimize disruption to learn, 3) Discipline learned also need to be considered in improving learning outcomes as in the actions taken in each of the activities of children that will shape the personality and learning styles of children. 4) Hygiene classroom and school environment will make the child feel comfortable in learning.

General Characteristics of Students Progress Elementary School Age

According Desminta (2009: 35) children of school age have characteristics different premises kids younger age. He played, happy to move, enjoy working in groups, and like to feel or do something directly. Therefore, teachers should develop develop lessons that contain games, cultivate students move or move, work or study in groups and provides the opportunity to be directly involved in learning, both in the classroom and outside the classroom.

School-age children is the stage where a child has reached maturity and are ready cognitively, psychologically and physically to do the learning in the school environment. This is in line with the opinion of L. Zulkifli (2009: 52) states that after the child reaches the age of six or seven years, physical and spiritual development of the perfect start. Child out of the family environment and entering the school environment, which is a great environment influence on the development of physical and spiritual. At that age children are expected to acquire basic knowledge deemed essential to the success of the adjustment in adult life; and learn various skills of particular importance, both curricular and extracurricular skills (Hurlock, 1980:146).

According Havighurst (in Desminta, 2009: 35), the task of development of primary school age children include:

1. Mastering the necessary physical skills in games and physical activity
2. Develop a healthy life
3. Learn to get along and work in groups
4. Learn to run a social role

5. Learn to read, write, and count to be able to participate in society
6. Obtain a number of concepts necessary for effective thinking
7. Develop a conscience
8. Achieving personal independence

While efforts to achieve these developmental tasks according Desminta (2009: 36) teachers are required to provide assistance in the form of:

1. Create an environment of peers who teach physical skills
2. Conduct learning that gives students the chance to hang out and work with peers so that developing social personality
3. Develop a learning activity that gives concrete or direct experience in developing the concept both in the classroom and outside the classroom
4. Conduct a study to develop values so that students are able to determine the choice of a stable and into the handle itself.

Cognitive Development School Age Children

Cognitive development is the development associated with thinking or what we often refer to as intelligence. Intelligence is associated with the ability of a child's brain in acquiring and processing an information to then draw conclusions. People who are very famous for his research on cognitive development is Jean Piaget. Piaget (2010: 145) which was translated into Indonesian by Nur Mohamad split into four stages of cognitive development stages, namely:

1. Stage sensorimotor (birth - 2 years)

   In the sensorimotor stage children have the ability formation of the concept of "object permanence" and gradual progress (stage by stage) on the behavior of reflection to behaviors that lead to the goal.
2. preoperational phase (2-7 years)

   In the preoperational stage of development the child has the ability to use symbols to express the objects in the world. Thought still ego-centric
3. Phase concrete operations (7-11 years)

   At the stage of concrete operations child had an improvement in the ability to think logically. New capabilities including the use of operations that can be reversed. Thought no longer centered but not centered, and solving problem not so constrained by the egocentric
4. stage of formal operations (11 - adult)

> At the stage of formal operations child has a pure abstract thought and symbolic possible. The problems can be solved through the use of systematic experimentation.

Piaget (2010: 152) defines the operational concrete phase as directly related to the stage of objects and groups of objects (classes), with relations between objects and object counting.

Soemanto (2006: 97) refer to this as the cognitive development and mental development is a development that is heavily influenced by heredity. Although the difference between the mental abilities of children of preschool preoperational to concrete operational children of primary school is very large, kids tangibly operation still think like an adult.

In the concrete operational stage the child can not understand something that is theoretical and abstract. Kids can only understand the concept, seeing relationships, and solve the problem with something that is real, there is a real and concrete objects that can be observed by the child, or something that is familiar with their life.

Unlike the preoperational, concrete operational child has been progressing in mastering the concept of reversibility. Reversibility is the ability to perform mental operations and then reverse that thought to go back to the starting point. Concrete operational child is able to master this concept so no problem anymore with conservation. Conservation is the concept that certain properties of an object (as eg weight) remained equally unaffected by changes in other properties (such as length).

According Desminta (2013: 130) in line with the increase in children's ability to explore the environment, because it increases the amount of coordination and motor control coupled with the increased ability to inquire with using words that can understand other people, then the cognitive world of children growing rapidly and creative.

Concrete operational child is already able to understand that if there was a glass of orange juice from a glass that is slim and tall in a short glass and fat are the same volume, although the water level in the glasses is different. Concrete operational child is able to logic that the juice that is on the first glass is just as much on the second glass, and when the second glass of juice on ditngakan again on the first glass of juice, the higher will be the same as before. This capability not possessed by children at earlier developmental stages.

Children's concrete operations have also been able to give meaning inferred from the stimulation in the context of the relevant information. Preschool children generally respond to everything that appears by their views, while school-age children are able to draw conclusions based on data from previous observations. According Muhibbinsyah (2014: 71) set of steps to think the child will become the basis for the establishment of intelligence. Intelligence is a process, phase, or specific operational steps that underlie all human thought and knowledge, in addition to an understanding of the formation process.

Ability controlled other school-age children is their ability to recognize an object by its characteristics appropriate, and can arrange them in a certain order. School-age children can identify with the right relationships in seriasi. Suppose Tomi higher than Bob, and Bob higher than Rudi. School-age children can draw the appropriate conclusions that Tomi higher than with Rudi, but it is not the case with preoperational child, preoperational child will have difficulty in drawing the appropriate conclusions from such information.

Children can think about what will happen, as long as the object it looks. For example, children can guess what will happen when the rubber on the catapult is pulled and released. Children can understand the time and space that is good enough to draw a map of the house meraka to school and build an understanding of the events in the past. Hurlock (1980: 162) allude to this by saying that the school-age children can connect new meaning to the old concept is based on what was learned after school. In addition, children receive a new meaning of the mass media, especially movies, radio and television.

It proves that children can draw a hypothesis of a scene by connecting with concepts they have learned in school, even though limited to a simple concept and a very closely related to their daily lives.

The ability of end reached school age children a skill in which an individual can think simultaneously about a whole class of objects and the relationship between the subordinate classes. School-age children can make comparisons within a class like comparing the number of boys and girls. He also understands that boys and girls are members of the children. Concrete operational child is able to make a true comparison between the number of subordinate (boys and girls) with the ordinate (children), and this ability is not owned subsidiary preoperational. Opersaional preschool can make a comparison on the ordinate are similar but are not able to do a comparison between the ordinate with a subordinate.

Mathematics

Susanto (2013: 186) Learning mathematics is an effort to improve the mastery of mathematical concepts in the wake of teachers in teaching and learning in the classroom. Kehiduppan everyday reality there apart from mathematics (Susanto, 2013: 189).

The process of learning mathematics in elementary school have a goal for students to understand arithmetic operations well, able to solve problems in mathematics sergeant communicate ideas (Susanto, 2013: 189-190). Mathematics is part of an exact science, derived from the Latin term meaning Mathematics Relating to learning related to science. Based on the origin of mathematics is the knowledge gained from studying (Haryono:2014:6). Mathematics is an abstract idea which is very important in everyday life.

Environment students can help students to learn to understand math, some basic concepts that can be studied students in the neighborhood include the operations of addition, multiplication, pembegian, reduction can take advantage of rock or a stick that is around students, students can learn about the forms of flat wake.

Mathematics learning environment based on RPP remains institute that has been created by teachers, includes introduction, core activities and weekend activities. Preliminary activities do teachers to convey the purpose of the activity. Teachers prepare teaching aids eg high measuring devices (meter) related material is high, wide. The learning objectives can be used as a learning landing will be achieved. Lessons are conducted in the school environment remains under the supervision of a teacher of mathematics.

Conclusion

There are several factors that influence a person's achievement of learning outcomes, namely: factors originating from within the learner and factors from outside himself. Factor in people who learn that the internal covering health, intelligence and aptitude, interest and motivation, and learning, while the factors originating from outside the people who learn that external factors include the family, school, community and environment.

A supportive environment will encourage children to eager to understand new concepts. The smoothness of the learning environment is determined by environmental conditions such good school environment. The school is the second home for students in which students spend more time at school than at home. Owned facilities such as school gardens, libraries classroom, cafeteria, playground, etc. can be used by students as a means of learning mathematics. School students can help students to learn to understand mathematics. Some of the basic concepts that can be studied students in the school's neighborhood, among others, the operations of addition, multiplication, division, subtraction, and wake up flat by using objects that are around students.

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use *and*

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Developing Geometry Instruction Based on A Worked Example Approach

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Abstract

A set of worksheet of solid geometry for eight graders was developed based on a worked example approach to facilitate the acquisition of problem solving ability. This mixed method research using an embedded design was aimed to describe how to apply the principle of the worked example approach on developing worksheet and to describe its quality based on the validity of the content, the practicality of uses and the effectiveness of the impact. The development of the worksheet followed the ADDIE steps. The worksheet was consulted to worked example experts and revised several times before it was implemented in the trial classroom. A number of 31 eight graders from a junior high school in Tempel, Yogyakarta participated in the implementation of the worksheet. Quality on the implementation of the worked example approach into the instruction was developed further by revising and refining after trying-out. The result showed that the principle of the worked example approach can be applied in the worksheet that is by managing intrinsic cognitive load in accord with student's level of prior knowledge and complexity of the material, reducing extraneous cognitive load and maximizing germane cognitive load by presenting worked examples and isomorphic problem solving in a schematic manner. Based on supporting quantitative data, it was found that the validity (correctness of the content) was averaged 4.47 (content is very good) by the instructional experts and 4.31 (content is very good) by the media experts. The practicality was averaged 3.78 (very easy to follow) by the teacher and 3.14 (easy to follow) by the students. In the problem solving ability test, about 68% students in the trial class scored more than the minimum passing grade that was 70. This means the worksheet was effective to facilitate the problem solving ability.

***Keywords**: worksheet, worked example, problem solving, mathematics, cognitive load theory.*

Introduction

Innovative efforts to improve problem-solving abilities should always be made. According Retnowati (2016), resolving problems effectively and efficiently requires conceptual / factual / declarative knowledge and procedural knowledge. Students who do not have sufficient prior knowledge to solve the problem will experience obstacles or difficulties for advancing their mathematical thinking such as to determe the solution of a given mathematics problem.

Sweller (1994) explained that students who do have limited prior knowledge will tend to use heuristic strategies (e.g., trial and error) that are considered facilitating students to learn problem-solving inefficiently. This might be caused that students just focus more on the final answer without necessarily understand the underlying mathematical knowledge solution of the problem. Thus, Sweller stated that heuristic strategies cause burden on cognitive process for students and could not lead students to form new knowledge. Sweller named the theory of instructional design that minimizes the load of thinking as Cognitive Load Theory (see Sweller, Ayres and Kalyuga, 2011). This theory has derived the principles of learning focusing on the acquisition of problem solving through various experiments in mathematics and the others. This theory explains that the effectiveness of learning might be determined by two aspects, intrinsic cognitive load (due to the natural complexity of the material) and extraneous cognitive load (which deals with the presentation of the learning material) (Sweller, et. Al., 2011; Retnowati, Ayres and Sweller, 2010). Instructional design should be presented to students with manageable intrinsic and extraneous cognitive load. According to this theory, if these cognitive load sources can be managed, students can optimize their cognitive ability to construct new knowledge.

Cognitive Load Theory suggests when students do not have sufficient prior knowledge to solve problems and learn the underlying knowledge, then students should be given an explicit instruction (Sweller, 1994). An example of explicit instruction is the worked example approach (Kirschner, Sweller & Clark, 2006). Atkinson (2000) summaries that a worked example consists of steps to solve the problem like the steps used by experts that are easy to follow and learn. As described by Hillen (2012: 90) that by studying worked examples, students can acquire problem-solving strategies. The worked example provides guidance that assist students to understand the problem and how the solution is represented. This approach is particularly effective for novice learners. The worked example provides novices knowledge base to understand what and how to do it. This knowledge is useful for enhancing their learning and problem solving (van Gog and Kester, 2012).

The effectiveness of worked examples in geometry learning has been shown by Retnowati, Ayres and Sweller (2010). The study compared the worked example and the problem solving approach when individual or group work settings are occupied. The worked example was composed by pairs of a worked example and similar problems. Students were asked to study the example and then complete the similar problem without seeing the example. In the problem solving approach, students were asked to learn the geometry by solving the given problems. Students in the Retnowati, et al.'s experiment were categorised as novices. The results showed that students in the worked example approach learned problem solving better than those in the problem solving approach. The author found that worked examples might reduce extraneous cognitive load and hence assist students to learn while doing problem solving. On the other hand, students in the problem solving approach failed to learn as much as their counterpart because they have limited capacity to understand the underlying concept and procedure while solving problems. Research indicated the effectiveness of worked example approach for facilitating novice learners to study compared to the problem solving approach in many domains (for more review, Atkinson, et al., 2000).

Worked example should be designed in accord with the principles of cognitive load theory, that it should present a low extraneous cognitive load (Sweller, et al., 2011). Based on the review of Retnowati (2012, 2016), it can be summarized that in order to create an effective worked example, at least these five principles should be followed: 1) creating pairs of similar worked example and problem solving, 2) using variation of problem contexts, 3) arranging level of complexity in order, 4) avoiding split attention effect and 5) avoiding redundancy effect. These five principles are proposed to minimize extraneous cognitive load. Nevertheless, instructional designers may improve the effectiveness of the worked example by managing intrinsic cognitive load and creating medium to stimulate germane cognitive load (Retnowati, 2012, 2016).

As discussed above, worked examples have been shown to be useful to present learning material to novice students. However, this approach is rarely found to be implemented at schools or trainings. Indeed, such efforts are needed to mediate research into practice. Therefore, it is necessary to develop a learning material based on the worked example approach. Furthermore, a learning material that is often required at mathematics classroom is namely a worksheet. A worksheet guides students to study a particular topic or to acquire a specific competency. A worksheet that is developed based on the worked example approach should have worked example instruction as the main activity to be followed by students. According to Suyitno (1997: 40), worksheet can help students to understand concepts and procedures if it can facilitate systematic learning activities.

Turning to the topic of geometry, according Safrina, Ikhsan and Ahmad (2014: 11) geometry is often difficult to learn. Geometry contains both abstract and concrete aspects. Similarly, Kariadinata (2010) argued that the geometry is difficult to learn because it requires students to understand abstract concepts through visualizations. Such manipulations or pictures might assist students to understand this, however, when the focus is problem solving ability, then more creative geometry problems that are suitable for the cognitive level of the students might be desired. This research is proposed to develop worksheet based on the worked example approach for facilitating problem solving ability and to improve the quality by validation, implementation and evaluation.

Method

This research may be categorised as a research and development activity where the developed product is student worksheet of geometry. The paradigm of this research is mixed method with embedded design (Creswell and Clark, 2010) where this research is largely a qualitative research while quantitative data used as supporting data of research results. The product was developed by following ADDIE procedure consisting of analysis, design, development, implementation and evaluation (Sugiyono, 2015: 39). A number of 31 students in a Junior High School in Tempel district participated in this research, during the implementation of the worksheet in May 2016.

The data are qualitative and quantitative data. The qualitative data obtained from each step of designing the worksheet by taking field-note to record all aspects of constructing the worksheet, including when discussing the design with the experts or teachers until it was decided that the principles of worked example designing have been implemented. For the analysis of qualitative data, data was acquired continuously, especially during development and learning by worksheet. Qualitative data analysis was used to describe how the development was done from the first step that was the analysis curriculum and students, then the designing and development, the implementation at school and eventually the evaluation of the product. Meanwhile, the quantitative data used to support the qualitative data was obtained from assessment forms that were prepared for looking at the validity of the content of the worksheet, and also the practical aspect of using the worksheet. These used five Likert's scale. A problem solving test (essay) was also used after the try-out (implementation in the classroom) to see the effectiveness of the worksheet. The final product was revised based on all suggestions of the qualitative and quantitative data. Some classifications (Widoyoko, 2009) were determined by the authors to decide the validity, practicality and effectiveness level, as can be seen in the Table 1 below.

Table 1. Qualification of Validity

Average Score	Classification
$\bar{X} > \bar{X}_i + 0,6 \times \text{sb}_i$	Valid
$\bar{X} \leq \bar{X}_i + 0,6 \times \text{sb}_i$	Invalid

Table 2. Qualification of Practicality

Average Score	Classification
$\bar{X} > \bar{X}_i + 0,5 \times \text{sb}_i$	Practical
$\bar{X} \leq \bar{X}_i + 0,5 \times \text{sb}_i$	Impractical

Table 3. Qualification of Efectiveness

Average Score	Classification
$\bar{X} \geq 51\%$	Effective
$\bar{X} < 51\%$	Ineffective

Note:

$\bar{X}$ = Average score

$\bar{X}_i$ = Ideal (theoretic) average score

$= \frac{1}{2}(\text{ideal max score- ideal min score})$

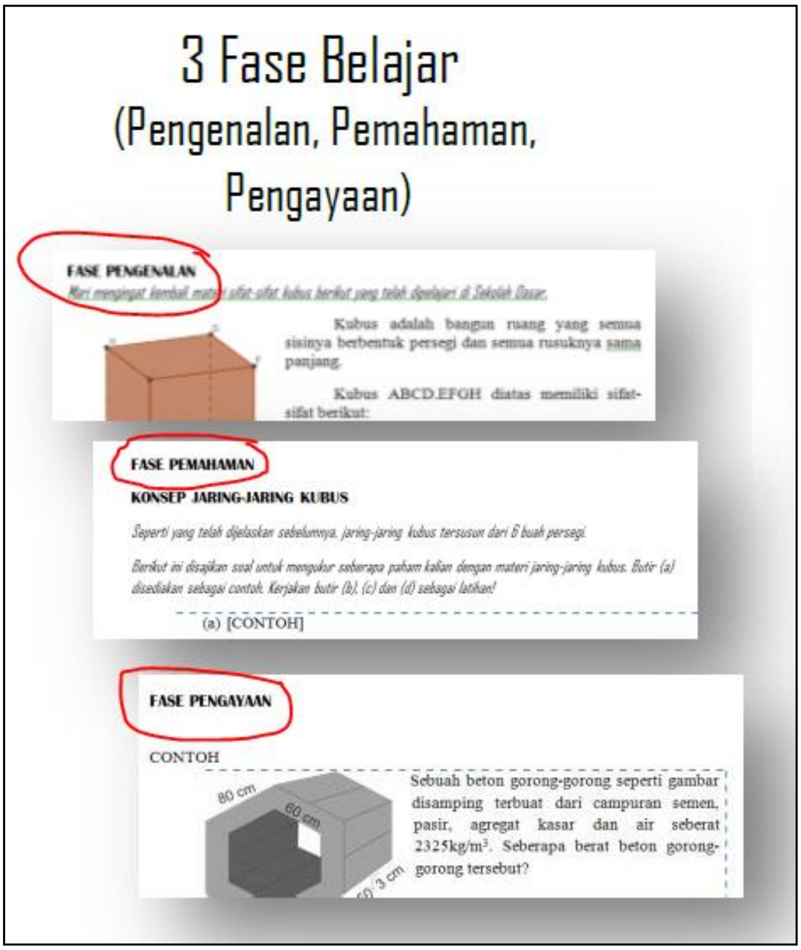
$\text{sb}_i = \frac{1}{6}(\text{ideal max score - ideal min score})$

Result and Discussion

The current research has developed a qualified set of worksheets for learning year eight geometry topics based on the worked example approach. The quality of the product was maintained during the development by applying the ADDIE procedures, as well as collecting the data of the validity, practicality and effectiveness both concurrently in qualitative and quantitative approach as described in the method section above. The main learning topic is three-dimensional figure, specifically cube, prisms and pyramid; which includes nets, surface area and volume.

The overall procedure of developing the worksheet based on the worked example approach followed the ADDIE that can be described below. In the first step, analyze, the reserchers collected detailed information on the proposed subject (Year 8 students) who would utilize the worksheet. Since the subject uses a national curriculum, it may be assumed some prior knowledge they have possessed. For the proposed users of the worksheet, they should have learned about three dimensional figures in primary school. Detail information of how much they have learned was explored through the curriculum and also by asking relevant teachers and students. It was concluded that at year eight, students should be able to focus their learning of three dimensional figures on more challenging problem solving. Therefore, the worksheet contains higher level of problem solving about nets, surface area and volume of cube, rectangular prism and pyramid.

In the design step, researchers applied the principles of worked examples as discussed above. It was designed that in every topic, learning was facilitated from less to more complex. These learning phases are named the *introduction phase*, *understanding phase* and *enrichment phase*. In the introductory phase, students can activate their prior knowledge and have induction for the new problem solving. In the following phase, students learn new (novice) problem solving by worked example approach. In the last phase, students enhance their learning by more challenging problem solving. Key answers are provided in the worksheet but students are instructed to clarify their results after making some attempts during learning. Figure 1 below shows a page in the worksheet describing the three phases.



3 Fase Belajar
(Pengenalan, Pemahaman, Pengayaan)

FASE PENGENALAN

Mari mengingat kembali materi sifat-sifat kubus berikut yang telah dipelajari di Sekolah Dasar.

Kubus adalah bangun ruang yang semua sisinya berbentuk persegi dan semua rusuknya sama panjang.

Kubus ABCD.EFGH diatas memiliki sifat-sifat berikut:

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KONSEP JARING-JARING KUBUS

Seperti yang telah dijelaskan sebelumnya, jaring-jaring kubus tersusun dari 6 buah persegi.

Berikut ini disajikan soal untuk mengukur seberapa paham kalian dengan materi jaring-jaring kubus. Butir (a) disediakan sebagai contoh. Kerjakan butir (b), (c) dan (d) sebagai latihan!

(a) [CONTOH]

FASE PENGAYAAN

CONTOH

Sebuah beton gorong-gorong seperti gambar disamping terbuat dari campuran semen, pasir, agregat kasar dan air seberat 2325kg/m^3. Seberapa berat beton gorong-gorong tersebut?

Figure 1. Worksheet Phase

The worked examples were designed using a strategy of pairing. As can be seen in Figure 2, the position of pairs of example [contoh] and problem solving [soal] is varied, depending on the length of the to-be-learned problems. It was noted that the most important is that the design of the pairs was isomorphic and the level of difficulty increases from low to high. By this design, the extraneous cognitive load caused by the presentation of the problem may be lower and hence improve germane cognitive load.

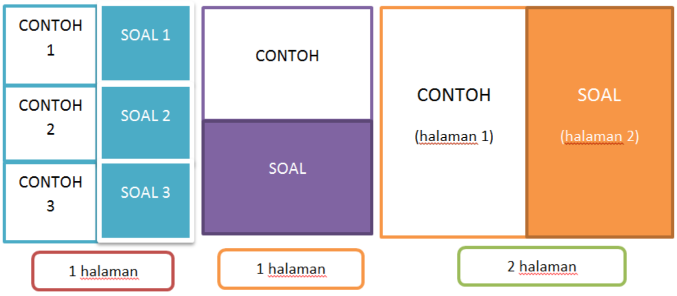
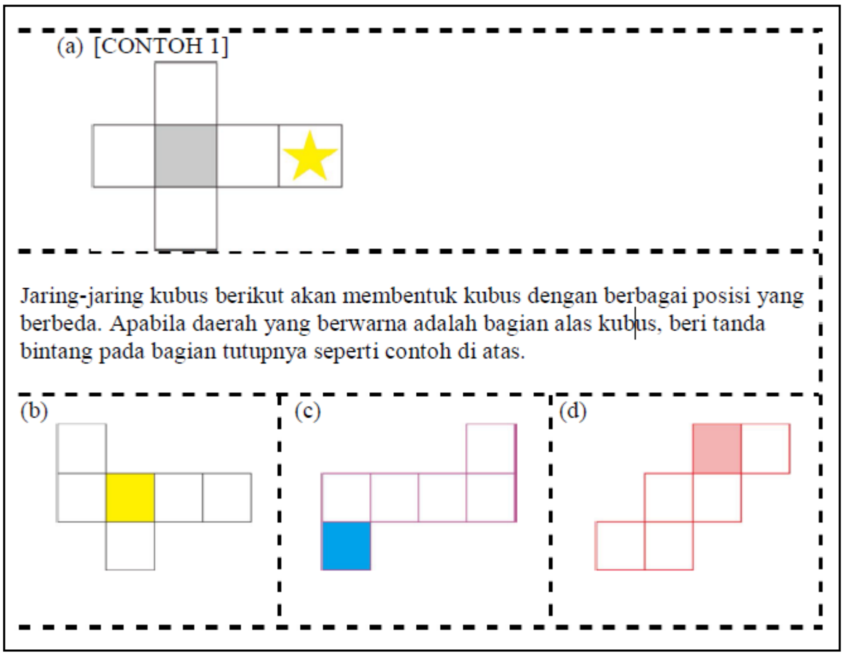


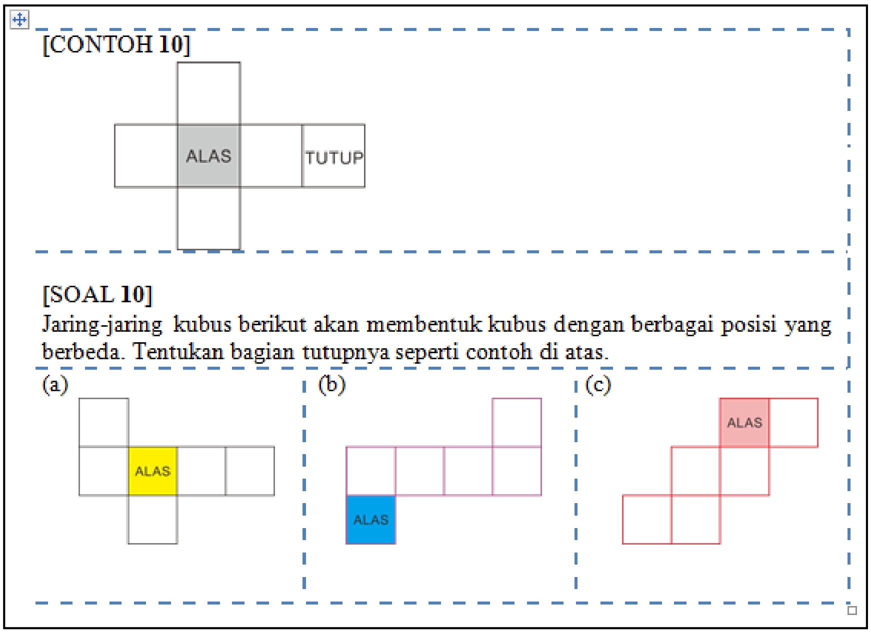
Figure 2. The format of writing a couple of examples and problems

During the development step, researchers consulted the on-progress results to the experts in order to obtain data of validity, practical and effectiveness. Focused discussions were conducted many times (part-by-part of the worksheet) to see whether the development of the worksheet has been in accord with the principle of the worked example approach. Through the discussion, it was found that applying the principle of avoiding split attention and redundancy effects was the most critical. For an example, the previous design of net problem solving was as can be seen in Figure 3 below which was agreed that such design may cause a split attention. The split attention lies on the star, shaded area and the written instruction explaining these symbols. This design was then revised as can be seen in Figure 4 and assumed that this has minimal extraneous cognitive load. In this design, the star and written instruction were removed and replaced by a shaded area and the written explanation in the area. By this way, students would be easily grab the information and learn from the worked example and isomorphic problems more efficient.



Jaring-jaring kubus berikut akan membentuk kubus dengan berbagai posisi yang berbeda. Apabila daerah yang berwarna adalah bagian alas kubus, beri tanda bintang pada bagian tutupnya seperti contoh di atas.

Figure 3. Example of *Split-attention*



[SOAL 10]

Jaring-jaring kubus berikut akan membentuk kubus dengan berbagai posisi yang berbeda. Tentukan bagian tutupnya seperti contoh di atas.

Figure 4. Example of well integrated worked example.

Discussions to reach the quality aspect of the worksheet were done continuously to explore whether the worksheet has been developed following the worked example principles. This also included the level of complexity of every material, the context choices, variation of the problem solving, lay-out and figures. After some confidence was gained by revising the developed worksheet according to the discussion results, the implementation step followed. There were two batches of implementation. In the first one, several students were involved to give information whether the instruction in the worksheet is easy to follow. During this implementation step, the researcher also collected some quantitative data using assessment sheets. These instruments were used to assess the validity and practicality aspects. Questions in the validity assessment were about whether the problem context, level of complexity, sequence of the topics and variability of the problem solving have been relevant to the learning outcome. As well as whether the principle of worked example approach has been applied accordingly and hence possibly lower cognitive load during learning using the worksheet. The results showed that the geometry experts scored 4.47 of the maximum score of 5 (criteria contents valid), and the media experts scored 4,31 of the maximum score 5 (criterion media valid).

As described by Plomp and Nieveen (2013), a product is practical when it provide convenience and usefulness. Attractiveness was added in the practical component. Questions in the practicality assessment were about the appearance, readability, print-out of figures, color, lay-out, distractions and strategically aim to achieve learning outcome. The results showed that an average score 3.14 of the maximum score of 4 (practical) from the students while an average score higher that is at 3.78 out of a maximum score of 4 (practical) from the teacher. A product was considered effective if it can facilitate the learning process to give a good result in order towards the goal (Plomp and Nieveen, 2013). Specifically, effectiveness of the worksheet to facilitate problem solving skills would be reach if students are able to acquire conceptual and procedural knowledge from the worked example instruction. The effectiveness of worked example can be achieved if the worked example do not impose high extraneous cognitive load (Retnowati, 2012). Through discussions during the design and development steps, researchers attempted to minimize cognitive load may be caused by the material in the worksheet. To support this data, a problem solving test was given to thirty one students in the second implementation step. Before completing the test, the students were asked to study the worksheet accordingly. The result showed that 68% of students could score above seventy (of the maximum score of 100).

In the evaluation step (the last step of the ADDIE), the researcher made some little adjustment to the worksheet to improve the quality. Through consultation and discussion with the expert, the final worksheet was produced. This is maybe an early conclusion however it could be said that the developed worksheet has been able to assist students managing their cognitive load during learning, as can be supported by both qualitative and quantitative data.

Conclusion

This research yields a qualified worksheet based on the worked example approach following the ADDIE procedure. Qualitative research was conducted to explore the design and development by discussions part-by-part with experts, teacher and also the subject user, the student. The learning material was geometry for year eight in Indonesia and focused on the acquisition of problem solving skills. The worked example approach was applied by: 1) creating pairs of similar worked example and problem solving, 2) using variation of problem contexts, 3) arranging level of complexity in order, 4) avoiding split attention effect and 5) avoiding redundancy effect. Quantitative data was collecting for assessing the quality of the overall worksheet. The data support the discussion and revision results in the development of the worksheet. This worksheet may be an exemplary educational product that was developed based on cognitive load theory, and hence suggested to be implemented at educational setting. Furthermore, similar research is very useful to assist the implementation of instructional theory into practice.

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What can We Learn from ELPSA, SA and PSA? Experience of SEAMEO QITEP in Mathematics

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Abstract

The Problems of Education in Indonesia as mentioned by the former Minister of Education and Culture of The Republic of Indonesia, Professor Anies, was: ‘How to help Indonesian students to be independent learners and have good characters.’ Therefore the question can be aroused was: ‘What kinds of knowledge, skills and attitudes are needed by our students to survive in the 21st Century and beyond?’ Shadiq (2016a) stated that to change and improve the quality of teaching and learning process from a “typical” or “traditional” mathematics classroom to the new one and more innovative is not easy. Another question can be aroused was: ‘How to Change the Real Teaching Practice?’ Mathematics teachers need to experience in ways that they will be expected to teach it. Masami Isoda (2011) proposed PSA (Problem Solving Approach) which consists of four steps: (1) Problem Posing, (2) Independent Solving, (3) Comparison and Discussion and (4) Summary and Integration. From Indonesia, we can learn the Scientific Approach which consists of five steps: (1) observing, (2) questioning, (3) collecting data, (4) reasoning and (5) Communicating. While Lowrie and Patahuddin (2015) proposed ELPSA (Experiences, Language, Pictures, Symbols, Application) as a lesson design framework for mathematics teaching and learning process. Based on those three framework, the teaching and learning process of mathematics can be designed, started with problem which was proposed by Lowrie and Patahuddin (2015), at least 11 alternatives can be identified to solve the problem.

Keywords: *ELPSA, SA, PSA, independent learners.*

Introduction

The Problems of Education in Indonesia as mentioned by the former Minister of Education and Culture of the Republic of Indonesia (Kemdikbud, 2014), Professor Anies, was: ‘How to help Indonesian students to be independent learners and have good characters.’ Therefore the question can be aroused was: ‘What kinds of knowledge, skills and attitudes are needed by our students to be independent learners and have good characters; so that they can survive in the 21st Century and beyond?’ Shadiq (2016b) proposed this question as a basis in solving those problems: “How to help our students to learn mathematics: (1) meaningfully, (2) joyfully, (3) help learners to learn to think and (4) to help them to be an independent learner?” Shadiq (2016a) also stated that to change and improve the quality of teaching and learning process from a “typical” or “traditional” mathematics classroom to the new one and more innovative is not easy. Therefore another question can be aroused was: “How to change the real teaching practice?” It is clear that mathematics teachers need to experience in ways that they will be expected to teach it, but how?

How to help Learner to be Independent?

In Japan, Isoda & Katagiri (2012:31) stated that the general aims of education in Japan is stated as follows.

> " … *To develop qualifications and competencies in each individual school child, including the ability to find issues by oneself, to learn by oneself, to think by oneself, to make decisions independently and to act. So that each child or student can solve problems more skilfully, regardless of how society might change in the future*."

In anticipating the change in the future, learn from Japanese educator, Indonesian mathematics teachers should enhance the Indonesian students to find issues by oneself, to learn by oneself, to think by oneself, to make decisions independently and to act. From the statement above, it can be concluded that the focus of the Japanese Mathematics Education was problem solving. In line with that, the focus of the Singaporean Mathematics Education (Shadiq, 2016b) was also on problem solving as shown on Figure 1 below.

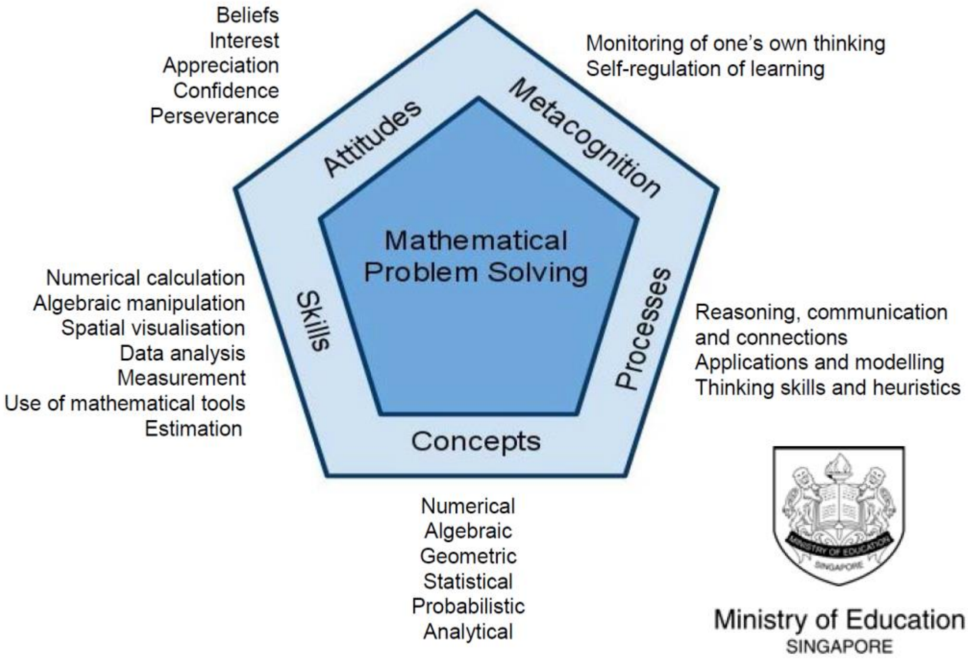

Figure 1. The focus of the Singaporean Mathematics Education was also on problem solving.

In addition, SEAMEO RECSAM (2015:7) proposed the 'SEA-BES Common Core Regional Learning Standards in Mathematics Framework for the 21st Century' which consists of:

1. Content / Strands, such as: Numbers & Operations, Quantity & Measurement, Shapes, Figures and Solids, Pattern & Data Representations, Extension of Number and Operations, Measurement & Relations, Plane Figures & Space Solids, Data Representations & Graphs, Numbers & Algebra, Space & Geometry, Relationship & Functions, and Statistics & Probability
2. Mathematical Processes, such as: Mathematical Thinking and Mathematical Activities.

3. Values, Attitudes and Habits for Human Character, such as: Mathematical Values, Mathematical Attitude and Habits of mind for Citizen to live.

Those document can be found and downloaded from: https://fadjarp3g.wordpress.com /2016/07/27/ standards-for- mathematics- and-science-teacher-and -standards-on-basic-education-for-mathematics-and-science/

Mathematics could be seen as the language that describes patterns (De Lange: 2004:8, NCTM: 2000). Based on that statement, during the teaching and learning of mathematics in their class, students can learn to think, to solve problem, to reason, and to communicate. Therefore, Marquis de Condorcet as quoted by Fitzgerald and James (2007: ix) stated: “Mathematics … is the best training for our abilities, as it develops both the power and the precision of our thinking.” In addition, the National Research Council from USA (NRC, 1989:1), reminds us 24 years ago that: “Communication has created a world economy in which working smarter is more important than merely working harder. … We need workers who can absorb new ideas, to adapt to change, to cope with ambiguity, to perceive patterns, and to solve unconventional problems.” These two statements show the importance and relevancy of mathematics enhance the ability of our students thinking.

To ensure that the teaching and learning of mathematics in Japan focus on problem solving, Isoda (2012) proposed PSA (‘Problem Solving Approach’) which consists of four steps: (1) Problem Posing, (2) Independent Solving, (3) Comparison and Discussion and (4) Summary and Integration. From Indonesia (Shadiq, 2015), we can learn the ‘Scientific Approach’, proposed by Ministry of Education and Culture of the Republic of Indonesia, which consists of five steps: (1) observing, (2) questioning, (3) collecting data, (4) reasoning and (5) Communicating.

What Can We Learn from SEAMEO QITEP in Mathematics

Shadiq (2016b) proposed four important questions as follow as a basis: “How to help our students to learn mathematics: (1) meaningfully, (2) joyfully, (3) help learners to learn to think and (4) how to help them to be an independent learner?” The fact is, many mathematics educators focus on skills and offer mostly procedural practice. That problem can be found in Indonesia and some SEAMEO member countries. They still use the paradigm of transferring knowledge from teachers’ brain to students’ brain. Another type of mathematics program leans more toward exploration of mathematical concepts through conceptual investigation. The teacher focuses attention on the pupil’s learning. However, to change and to improve the quality of teaching and learning process from a “typical” mathematics classroom to the new one and more innovative is not easy.

The Southeast Asian Ministers of Education Organization (SEAMEO) Regional Centre for the Quality Improvement of Teachers and Education Personnel in Mathematics (SEAQiM) is an organization which runs under the flag of SEAMEO and the government of Indonesia. SEAMEO itself is a regional intergovernmental organization established in 1965 among the governments of Southeast Asian countries to promote regional cooperation in education, science and culture in the region. SEAMEO QITEP in Mathematics was established in 2009 with the purpose to develop the capacity of mathematics teachers and other education personnel including school supervisors, and headmasters across the Southeast Asian region. SEAMEO Regional Centre for QITEP in Mathematics is located in Yogyakarta, a city which is famous for Javanese fine art, culture and education.

SEAMEO QITEP in Mathematics since 2010 to 2013 actively participated in the APEC-Tsukuba conference on Lesson Study. Therefore, Japanese ‘Problem Solving Approach’ (PSA) for students and the Japanese ‘Lesson Study’ approach for mathematics teachers were usually implemented to enhance the competence of students and mathematics teachers in the region. Since 2012 SEAMEO QITEP in Mathematics (Shadiq, 2015) conducted studies related to Disaster Risk Reduction (DRR), in 2012 the study related to earthquake and tsunami, in 2013 related to flood, in 2014 related to volcanic eruption, and in 2014 related to landslide. Every year, SEAMEO QITEP in Mathematics conducted a workshop to develop proposal and instrument of the study. The workshops were attended by practitioner mathematics teachers and specialists from SEAMEO QITEP in Mathematics as teams.

One of the significant result was regarding to the importance of first step of PSA in supporting the SA. The Problem-Solving Approach (PSA), especially the first step, from Japan can support the Scientific Approach from Indonesia. The PSA and SA can be compared (Shadiq, 2015) in this following table.

No	The PSA (Japanese)	No	The SA (Indonesian)
1.	Problem posing		
2.	Estimating the ways of solutions (planning and predicting the solution),	1.	Observing
		2.	Questioning
		3.	Experimenting
3.	Independent solving	4.	Reasoning
4.	Comparison and discussion	5.	Communicating
5.	Summary and integration		

Based on the table above, it can be concluded that the teaching learning process should be started with contextual problem which is in line with the first step of PSA to ensure that the SA can be observed during the teaching and learning of mathematics.

Another significant result was regarding to the difficulty in changing and improving the quality of teaching and learning process from a "typical" or "traditional" mathematics classroom to the new one and more innovative. This result was consistent to the conclusions of the research conducted by Shadiq (2010:56-57) that most teachers of mathematics in their schools use or implement the traditional ways during the learning and teaching process of mathematics. They still use the paradigm of transferring knowledge from teachers' brain to students' brain. On the other hand, Goos, Stillman and Vale (2007:4) stated: "Whether we are aware of it or not, all of us have our own beliefs about what mathematics is and why it is important." Furthermore, Goos and Vale (2007:4) quoted Barkatsas and Malone (2005:71) which stated, "'Mathematics teachers' beliefs have an impact on their classroom practice, on the ways they perceive teaching, learning, and assessment, and on the ways they perceive students' potential, abilities, dispositions, and capabilities."

In line with that, Shadiq (2016a) also stated that to change and improve the quality of teaching and learning process from a "typical" or "traditional" mathematics classroom to the new one and more innovative is not easy. Lesson learnt from the research finding was: How to Change the Real Teaching Practice? In addition, mathematics teachers need to experience in ways that they will be expected to teach it. While Lowrie and Patahuddin (2015) proposed ELPSA (Experiences, Language, Pictures, Symbols, Application) as a lesson design framework for mathematics teaching and learning process.

What Can We Learn from ELPSA

Based on those frameworks, the teaching and learning process of mathematics can be designed, started with problem from Lowrie and Patahuddin (2015). The Original problem as follow: "Can you find the Number of Matchsticks on Fig 30?" While the modified problem is as follow: "How many ways to find the Number of Matchsticks on Fig 30?"

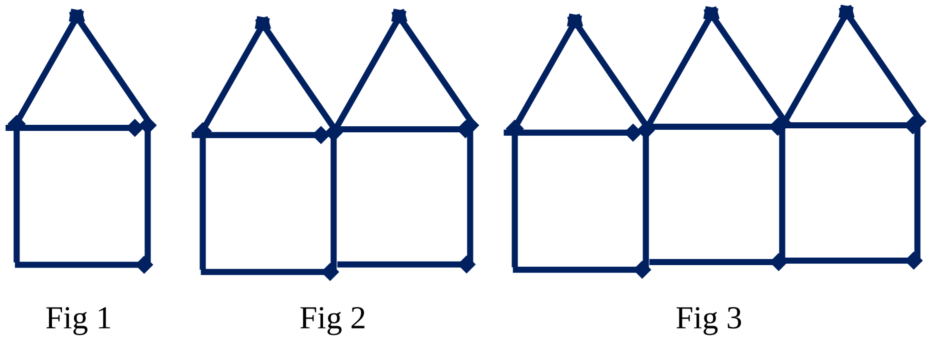

Figure 2. The Problem: "Can you find the Number of Matchsticks on Fig 30?"

At least 11 alternatives can be identified in solving the problem as follow.

Alternative 1

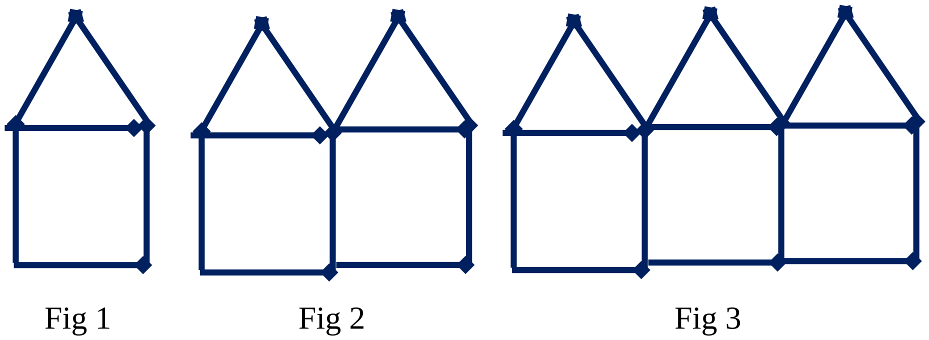

Figure 3. The 1st Alternative in Finding the Number of Matchsticks on Fig 30

By Counting and calculating, students can discover this table or other ways.

Fig …	1	2	3	4	5	6	7	---	30
Number of Matchsticks	6	11	16	21	26	31	36	---	?

So, based on that table or by counting, students can decide that on Fig 30, the number of matchsticks is 151.

Alternative 2

The table above can be tabulated in this another way.

Fig …	2	4	6	8	10	---	30
Number of Matchsticks	11	21	31	41	?	---	?

By looking for the pattern on the table above, students can study the relationship of ‘Number of Fig … ‘ and ‘the Number of Matchsticks’ needed, especially of the Number of Fig … ‘ for the even numbers. They can learn that for ‘Number of Fig … ‘ for the even numbers, for example, on Fig 10, then the number of matches is 51. Remember that 5 is a half of 10. So, based on that table or, students can decide that on Fig 30, the number of matchsticks is 151.

Alternative 3

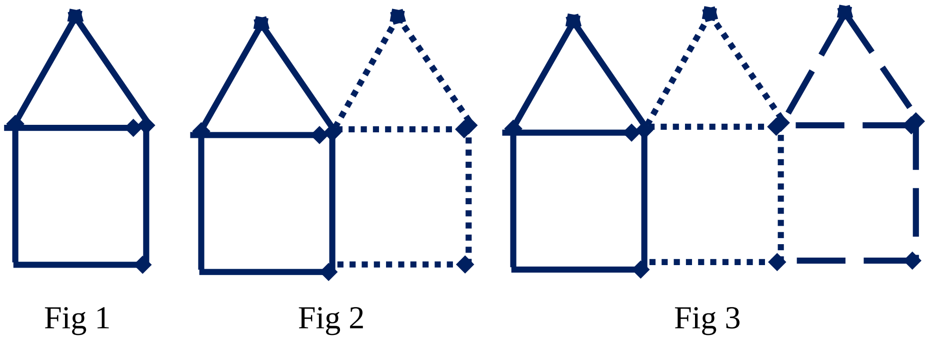

Figure 4. The 3rd Alternative in Finding the Number of Matchsticks on Fig 30

By using ELPS, see the diagram above, students can be facilitated to look for pattern and to draw diagram suitable for the condition that the number of matchsticks on the first house is 6, while the number of matchsticks on each of the next house is 5. In other symbol, students can learn that: F1 → 6, F2 → 6+5, F3 → 6+2×5, F4 → 6+3×5, ... So, on Figure 10, the number of matchsticks is 6+9×5 = 51, while, on Fig 30, the number of matchsticks is 6+29×5 = 151.

Alternative 4

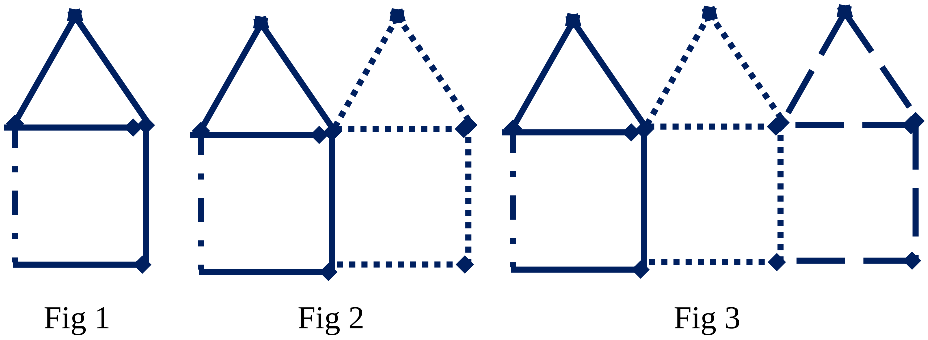

Figure 5. The 4th Alternative in Finding the Number of Matchsticks on Fig 30

By using E themselves that the Number of Matchstick on the first figure is 1+5, on the second figure is 1+2×5, and on the third figure is 1+3×5, and so on. In other symbol, students can learn that: F1 → 1+5, F2 → 1+2×5, F3 → 1+3×5, F4 → 1+4×5, ... So, on Figure 10, the number of matchsticks is 1+10×5 = 51, while, on Fig 30, the number of matchsticks is 1+30×5 = 151.

Alternative 5

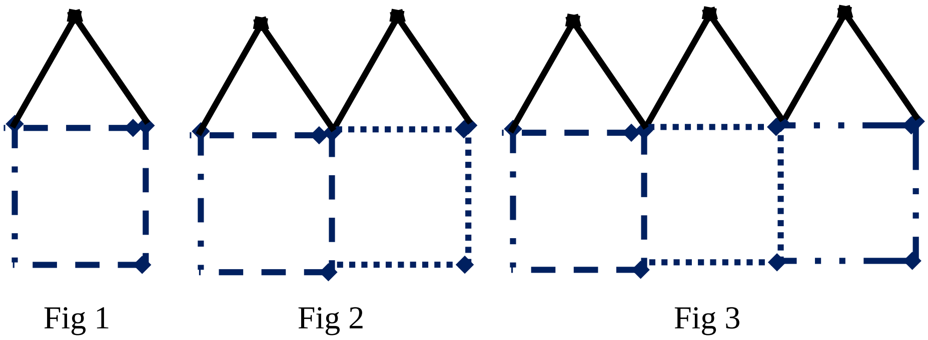

Figure 6. The 5th Alternative in Finding the Number of Matchsticks on Fig 30

By using ELPS and the diagram above, students can discover that the Number of Matchstick on each figure consist of three parts: (1) one matchstick on the most left-hand side of each figure, (2) two matchsticks on each roof of the house of each figure, (3) three matchsticks on the ceiling, right-hand side and ground floor on the house of each figure. In other symbol, students can learn that: F1 → 1+2+3, F2 → 1+2×2+2×3, F3 → 1+3×2+3×3, ... So, on Figure 10, the number of matchsticks is 1+10×2+10×3 = 51, while, on Fig 30, the number of matchsticks is 1+30×2+30×3 = 151.

Alternative 6

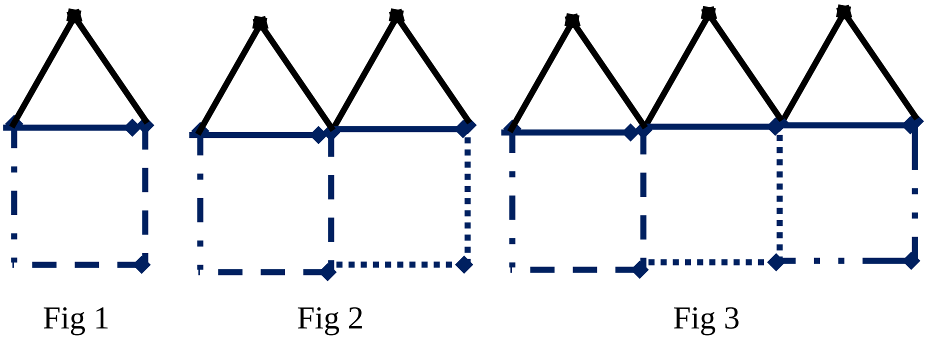

Figure 7. The 6th Alternative in Finding the Number of Matchsticks on Fig 30

By using ELPS and the diagram above, students can discover that the Number of Matchstick on each figure consist of three parts: (1) one matchstick on the most left-hand side of each figure, (2) three matchsticks on roof and the ceiling on the house of each ach figure, (3) two matchsticks on the right-hand side and ground floor on the house of each figure. In other symbol, students can learn that: F1 → 1+2+3, F2 → 1+2×2+2×3, F3 → 1+3×2+3×3, ... So, on Figure 10, the number of matchsticks is 1+10×2+10×3 = 51, while, on Fig 30, the number of matchsticks is 1+30×2+30×3 = 151.

Alternative 7

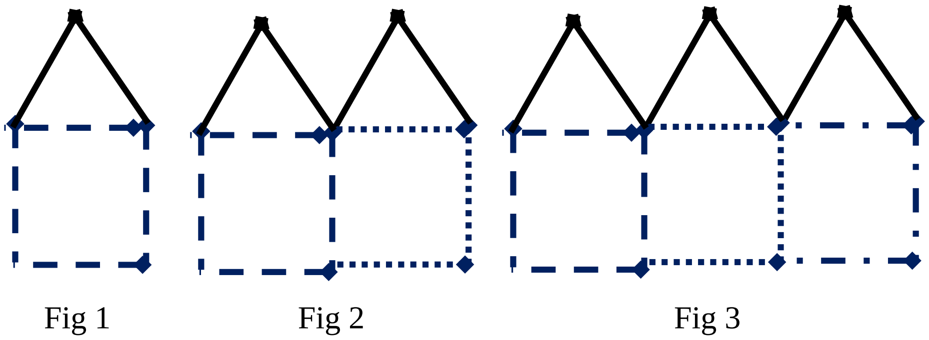

Figure 8. The 7th Alternative in Finding the Number of Matchsticks on Fig 30

By using ELPS and the diagram above, students can discover that the Number of Matchstick on each figure consist of: (1) four matchsticks on the first house and three matchsticks on the next house of each figure, and (2) two matchsticks on each house of each figure. In other symbol, students can learn that: F1 → 4+2, F2 → 4+3+2×2, F3 → 4+2×3+3×2, ... So, on Figure 10, the number of matchsticks is 4+9×3+10×2 = 51, while, on Fig 30, the number of matchsticks is 4+29×3+30×2 = 151.

Alternative 8

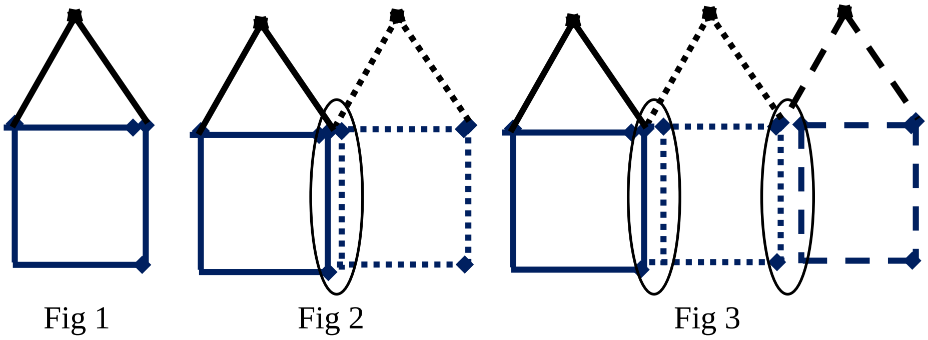

Figure 9. The 8th Alternative in Finding the Number of Matchsticks on Fig 30

By using ELPS and the diagram above, students can discover that the Number of Matchstick on each house consist of six matchsticks, however the middle walls of the house has been counted twice, so the students have to subtract by the number of the middle walls. In other symbol, students can learn that: F1 → 6, F2 → 2×6 – 1, F3 → 3×6 – 2, , ... So, on Figure 10, the number of matchsticks is 10×6 – 9 = 51, while, on Fig 30, the number of matchsticks is 30×6 – 29 = 151.

Alternative 9

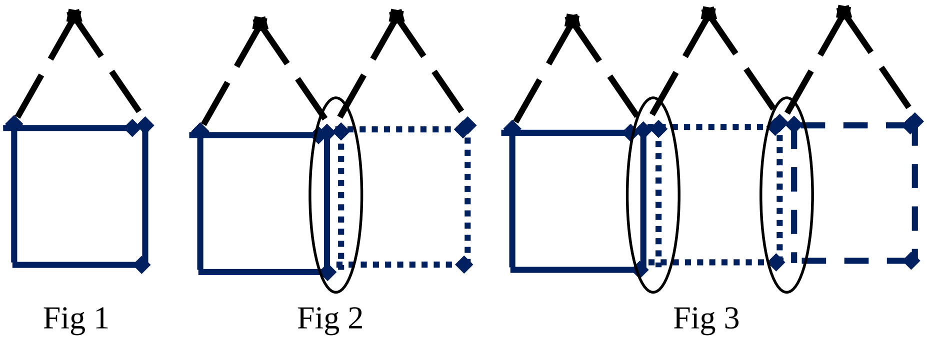

Figure 10. The 9th Alternative in Finding the Number of Matchsticks on Fig 30

By using ELPS and the diagram above, students can discover that each house consist of: (1) two matchsticks on the roof of each house of each figure, (2) the Number of Matchstick on each house is four matchsticks, however the middle walls of the house has been counted twice, so the students have to subtract by the number of the middle walls. In other symbol, students can learn that: F1 → 2+4, F2 → 2×2+2×4 – 1, F3 → 3×2+3×4 – 2, ... So, on Figure 10, the number of matchsticks is 10.2+10.4 – 9 = 51, while, on Fig 30, the number of matchsticks is 30.2+30.4 – 29 = 151.

Alternative 10

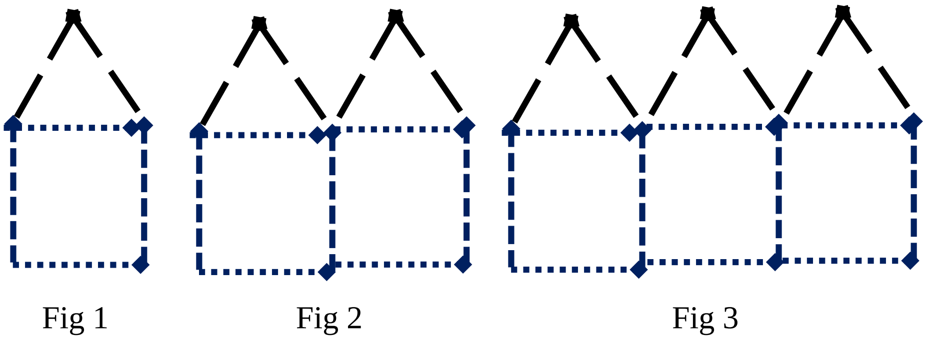

Figure 11. The 10th Alternative in Finding the Number of Matchsticks on Fig 30

By using ELPS and the diagram above, students can discover that each house consist of: (1) two matchsticks on the roof each house, (2) two matchsticks on the two horizontal line, (3) the number of house plus one matchsticks is needed as represented by the vertical line. In other symbol, students can learn that: F1 → 2+2+1+1, F2 → 2×2+2×2+2+1, F3 → 3×2+3×2+3+1,... . So, on Figure 10, the number of matchsticks is 10×2+10×2+10+1 = 51, while, on Fig 30, the number of matchsticks is 30×2+30×2+30+1 = 151.

Alternative 11

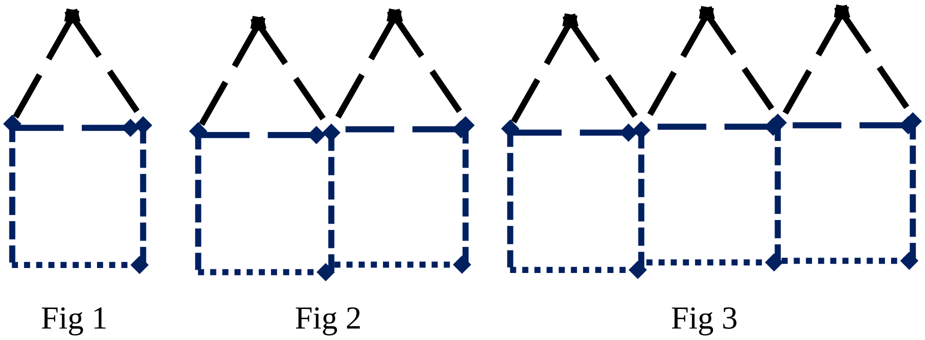

Figure 12. The 11th Alternative in Finding the Number of Matchsticks on Fig 30

By using ELPS and the diagram above, students can discover that each house consist of: (1) three matchsticks on the roof and ceiling of each house, (2) one matchstick on the ground floor of each house, (3) the number of house plus one matchsticks as represented by the vertical lines. In other symbol, students can learn that: F1 → 3+1+1+1, F2 → 2×3+2×1+2+1, F3 → 3×3+3×1+3+1, ... So, on Figure 10, the number of matchsticks is 10×3+10×1+10+1 = 51, while, on Fig 30, the number of matchsticks is 30×3+30×1+30+1= 151.

The Conclusions

The example of a lesson above that started with problem, as proposed by Isoda, from Lowrie and Patahuddin (2015) can be used and applied in the teaching and learning of mathematics that can help learners to learn mathematics: (1) meaningfully, (2) joyfully, (3) help learners to learn to think and (4) how to help them to be an independent learner. In addition, a lesson that started with problem from Lowrie and Patahuddin (2015) can be used and applied to help learners to enhance the attitude and values of learners, which consist of: (1) the beauty of mathematics, (2) the curiosity to learn mathematics, (3) the reasonableness of mathematics results and (4) good or positive appreciation toward the teaching and learning of mathematics, that had been proposed by Isoda (2015a). Also, a lesson that started with problem from Lowrie and Patahuddin (2015) can be used and applied to help learners to enhance the Indonesian Scientific Approach which consists (MoEC, 2013) of: (1) observing, (2) questioning, (3) reasoning (4) collecting data or experimenting and (5) communicating. It can be concluded that the teaching and learning of mathematics should be started with problems (contextual, realistic or mathematics) to ensure that our students can be facilitated to learn to think and to reason.

In solving the problem from Lowrie and Patahuddin (2015), the second step on PSA, proposed by Isoda (2015a), that is ‘independent solving’ can be elaborated and helped by using ELPSA (Experience, Language, Pictorial, Symbolic and Application), proposed by Lowrie (Lowrie & Patahuddin, 2015) or Indonesian Scientific Approach (SA), which consists (MoEC, 2013) of: (1) observing, (2) questioning, (3) reasoning (4) collecting data or experimenting and (5) communicating. It cannot be denied, concerning the importance of the first step of PSA, namely ‘Problem Posing’ in helping learners to learn to think and to be independent learners. Also, the importance of the all aspects of ELPSA (Experience, Language, Pictorial, Symbolic and Application) and all steps of the Indonesian Scientific Approach which consists (MoEC, 2013) of: (1) observing, (2) questioning, (3) reasoning (4) collecting data or experimenting and (5) communicating in helping the second steps of PSA, namely ‘Independent Solving’ can be executed by Indonesian students.

The Recommendations

Based on the explanation above, it can be concluded that the teaching learning process should be started with contextual problem which is in line with the first step of PSA to ensure that the SA and ELPSA can be observed during the teaching and learning of mathematics. Every mathematics teacher and educator was hoped and be motivated to improve his/her competency to produce such high quality of teaching and learning resource materials for teachers (including in designing Lesson Plan that start with activities or contextual/realistic/mathematical problem, hypothetical learning trajectory) as real examples for mathematics teachers.

As mentioned earlier, this research also implemented the lesson study to innovate mathematics teachers by using the step of Plan → Do → See. However, the result of this research was fail to change and improve the quality of teaching and learning process from a “typical” or “traditional” mathematics classroom to the new one and more innovative is not easy. Based on the result, it is recommended that teachers need to experience mathematics in ways that they will be expected to teach it. Mathematics teachers need concrete examples as soon as possible, ideally when they are sitting in pre-service institutions, so that they will be ready to facilitate their students in mathematics classes. Teachers are more likely to implement the new approaches in their own classes if they have experienced it in their own learning experiences.

In Japan, the lesson study activity was supported by university experts. Learn from this, every pre-service and in-service institution has to work with and help elementary and secondary school teachers. Learn from Japan also, every pre-service and in-service institution should focus on helping and facilitating learners. The main focus of pre-service institution should be on how to produce mathematics teachers who can help their students to learn mathematics meaningfully, learn to thank, and to learn mathematics by/for themselves.

The main focus of in-service institution should be on how to maintain and improve the quality of mathematics teachers that can help their students to learn mathematics meaningfully, learn to think, and to learn mathematics by/for themselves to anticipate the change in technology and in society. The lecture of pre-service and in-service institution should be having experience to work with students.

Every mathematics teacher and educator was hoped and be motivated to improve his/her competency to produce such high quality resource materials for teachers (such as mathematics text books, example of lesson plan, and materials from website/blog, periodicals, films, or VCD), such as proposed by Lowrie and Patahuddin (2015).

Further research should be designed to find the Indonesian mathematics teachers' beliefs which have an impact on their classroom practice, on the ways they perceive teaching, learning, and assessment, and on the ways they perceive students' potential, abilities, dispositions, and capabilities. Also important to find the best way to change the beliefs.

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PARALLEL COORDINATES: THE CONCEPT THAT PRE-SERVICE MATHEMATICS TEACHERS NEED TO KNOW

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Abstract

Some currant issues about what mathematics concepts need to be learned by pre-service mathematics teachers are discussed in this paper. It is argued that pre-service mathematics teachers need to learn non-conventional mathematics concepts which are not belong to advanced mathematical concepts. In particular, how to embed a non-conventional mathematics concept into one of the courses is examined. Argumentation of why pre-service mathematics teachers need to learn coordinates parallel is build and developed. An analysis of why this concept is needed to develop advanced mathematical thinking such as mathematical representation, creativity, and abstraction are discussed. Finally, a parallel coordinates as one of non-conventional mathematics concepts is described and examined from the perspective of curriculum development for pre-service mathematics teachers, to show that this concept could be embedded in one of courses in order to lead the mathematical abstraction process of mathematics future teachers.

Keywords: *Parallel Coordinates, mathematical abstraction, non-conventional mathematics concepts, teacher education, analytic geometry*

Introduction

This paper is an exploration of the questions: why concepts of Parallel Coordinates need to be learned by pre-service mathematics teachers and how it can be embedded into curriculum for pre-service mathematics teachers. Pre-service mathematics teachers in this paper refer to students of mathematics education department prepared for teaching mathematics in secondary and high school level.

In order to enhance education quality in Indonesia, it can be started from teachers education program as a legal institution that preparing student teachers to be qualified teachers in the future. Most of us would agree that mathematics teachers need to master both mathematical and pedagogical concepts to be able to teach mathematics. In addition they also need to have good skill for teaching and understanding their students beside they also need to have good understanding of curriculum. Shulman (1986) and his colleagues suggested three categories of knowledge for teachers: (a) subject-matter content knowledge, (b) pedagogical content knowledge, and (c) curricular knowledge. *Content knowledge* can be described as knowledge about the subject in this case refers to mathematics and its structures. *Pedagogical content knowledge* can be articulated as knowledge about how to reformulate and represent mathematical concepts into various ways using common rules and mathematical procedures, completed by knowledge how to analyze students' answers and explanations (Hill, Rowan, & Ball, 2005). Finally *curricular knowledge* encompasses what might be called the 'scope and sequence' of a subject and materials used in teaching (Graeber, 1999). Teacher of mathematics not only need to comprehend mathematical concepts that they will teach but also have to know how to present those concepts in various ways, provide students with explanation of common rules and mathematical procedures, analyze students' answers and explanation

Of course, based on the aims, this writing will focus on domain of *content knowledge* which is related to mathematical concepts for pre-service mathematics teachers without trying to segregate *content knowledge* from *pedagogy knowledge* for pre-service teachers. Ball, Thames, and Phelps (2008) proposed new point of view related to what content need to be learned by pre-service mathematics teachers, they included a new category within subject matter knowledge called as "*horizon knowledge*". *Horizon knowledge* is an awareness of how mathematical topic are related over the span of mathematics included in curriculum and it also includes the vision useful in seeing connection to much later mathematical ideas. One of the impacts from their ideas is that this new category can be used for designing support material for teachers as well as teacher education and professional development.

Unfortunately, they have not defined what kind of mathematical concepts that can be taught for pre-service teachers so that they will have horizon knowledge. Here I argue that one of the mathematical concepts that can be choosen as an alternative is *non-conventional mathematical concept*. This term is adopted from Zaskis (1999) that defined conventional mathematical concept as object of school mathematics. In this paper *non-conventional* mathematical concept means as mathematical concepts that are not belong to objects of school mathematics and also are not included in advanced mathematics such as abstract algebra, real analysis, numerical method, etc.

One of *non-conventional* mathematical concepts is Parallel Coordinates. This topic was invented by Philbert Maurice d'Ocagne, a French mathematician in 1885, then Alfred Inselberg continued to develop mathematical model of parallel coordinates and generalize many related concepts using projective geometry. Finally, he published the first textbook covering mathematical proofs, generalizations, and applications to data analysis for this multidimensional visualization technique (Nurhasanah & Siregar (2015) stated that this topic can be used to develop mathematical abstraction of pre-service mathematics teachers.

Concept of Parallel Coordinates

Parallel Coordinates is as a system for doing and visualizing analytic and synthetic multi-dimensional Geometry (Inselberg & Dimsdale, 1990. It is widely used in visualization technique for multivariate data and high-dimensional geometry (Heinrich & Weiskop, 2013).

Parallel coordinates can be constructed using Cartesian coordinate components by placing axes in parallel with respect to embedding 2D Cartesian coordinate system in the plane. It is freely to choose the orientation of the axes, it can be horizontal or vertical or not even both, but the most common implementation use is horizontal or vertical layouts. For reason of simplicity and consistency, vertical axes will be used in this paper.

A point C with coordinates (c_1, c_2, ...,c_N) in N-dimensional Geometry is represented by the polylines whose N vertices are at (i – 1, c_i) on $\bar{X}_i$-axis for i = 1, 2, ..., N. In consequence, there is a 1-1 correspondence between points in R^N and planar polygonal lines with vertices on $\bar{X}_1$, $\bar{X}_2$, ..., $\bar{X}_n$ is established.

Below is an example of parallel coordinates in 4D:

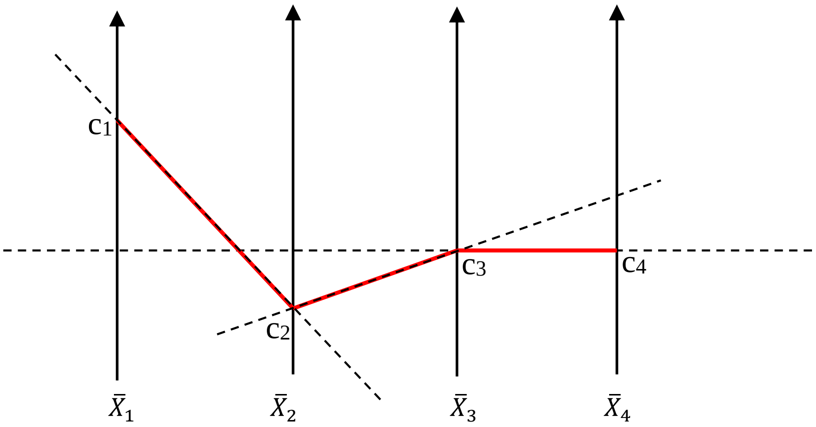

Figure 1. Polylines in parallel coordinate 4D

It can be seen that parallel coordinates can be used to represent objects in *N*-Dimensions in the form of 2 Dimensions. After it developed as a coordinate system, more application are available for tis concepts such as in collision avoidance algorithms for air traffic control, data mining, optimization, computer vision, process control, intrusion detection, and of course visualization in multivariate data, etc.

Why it is important for pre-service mathematics teachers?

Issues of usefulness and applicability of advanced mathematics concept for pre-service mathematics teachers are always up to date to be discussed related to *content curriculum*. Zaskis (1999) even stated that it is surprising that students-teachers mathematics mostly who asking question “When we ever use this (advanced mathematics concepts)?”

It is understandable that somehow students-teachers mathematics seem reluctant to accept this idea because based on my experiences, mathematics students teachers have various background so that they found advanced mathematics subjects such as Real Analysis, Complex Analysis, Abstract Algebra, etc as difficult subjects and for them those subjects could probably the most struggling class that they ever take. Another reason also found because of the lack of information about applications of those subjects.

Despite all of those opinions, teaching mathematics is a challenging job that cannot be just stable but very dynamic and complicated so that pre-service teachers need to be well prepared with all types of knowledge for teaching mathematics.

Ball, et al (2008) tried to propose idea related to Shulman’s domain of mathematical knowledge for teaching. They stated that there are three types of knowledge needed for teaching mathematics in *domain of subject matter knowledge*: (a) Common Content Knowledge (CCK); (b) Horizon Content Knowledge; and (c) Specialized Content Knowledge. CCK is mathematical knowledge known in common with others who know and use mathematics; personally I tend to describe as mathematics for school or elementary mathematics concepts. Horizon Content Knowledge is defined as an awareness of how mathematical topics are related over the span of mathematics included in curriculum and it also includes the vision useful in seeing connections to much latter mathematical ideas.

Based on these explanation it is clear cut that elementary mathematics concept only is far from enough for pre-service mathematics teachers. Many researchers and experts mostly have in common in argumentation about this issue. Usiskin (2003) argued that teachers need to know more advanced mathematics in order to: prepare students who need to have an advanced knowledge of mathematics for their careers; know the various ways in which mathematics they teach is applied later; distinguish those ideas that are fundamental from those that are enrichment; know the different ways in which people who use mathematics approach problem. While Zaskis (1999) believed that non-conventional mathematics concepts combine with relevant mathematical activities that engage students in meaningful problem solving can help them to see and appreciate the internal beauty and power of mathematics, beyond its applicability to `real life’.

Based on explanation in previous paragraphs, to be able to have *horizon content knowledge* and *specialized content knowledge*, advanced mathematical concepts cannot be separated from subject matter domain for mathematics teachers so that they aware with applicability, usability, and connectivity of mathematical concepts. If topics on advanced mathematical concepts seem to be too much for pre-service mathematics teachers, so idea of non-conventional mathematics concepts proposed by Zaskis (1999) can be the perfect answer for this problem. These types of mathematical concepts also powerful in providing mathematical activities such as meaningful problem solving, mathematical abstraction, inspection of definition, developing of explicit and fluent mathematical notation, knowing how knowledge is generated and structured in the discipline, extending procedures, generalizing, and providing experiences in mathematizing (Zaskis, 1999; Ball, et al, 2008; Nurhasanah & Siregar, 2015). Having rich experiences in doing mathematics for pre-service mathematics teachers can help them in understanding students' thinking processes and provide them with alternatives in designing learning instruction.

Parallel coordinates as one of the non-conventional mathematics concepts is potensial as an alternative for enriching pre-service's horizon content knowledge and their specialized content knowledge. It is definitely a new concept for them that can be constructed using their prior knowledge in the topic of Cartesian coordinate which is familiar for them since they were in high school. It is also can open their minds in order to generalize concept of dimension in Geometry. One of the drawbacks of the concept of Cartesian coordinates is that it is still limited for representing object in 3-dimension only and it is become an obstacle for understanding concept of dimension (Skordulis et al, 2008).

There are two basic concepts needed for exploring parallel coordinates: projective plane model and *point-line* duality. Both concepts are not included in curriculum for pre-service mathematics teachers so that analysis of content is a must in order to adjust where those concepts can be embedded. In addition, analysis of content also needed to design an appropriate learning context related to parallel coordinates concept. The next paragraph will explain how this concept can be embedded in subject of Analytic Geometry in curriculum for pre-service mathematics teachers.

How Parallel Coordinates is Embedded into Curriculum for Pre-service Mathematics Teachers?

Looking back into history of the parallel coordinate's concept, D'Ocagne (1885) written a book of this concept as a method in Geometry Transformation before it was systematically developed as a coordinates system by Alfred Inselberg starting in 1977. Later this concept was developed more rapidly in multidimensional geometry, visualization, and multivariate statistics.

Unfortunately, most curriculums for pre-service mathematics teachers do not include such topics as part of content knowledge for their student. An adjustment need to be designed in order to embed this concept into one of the subjects. Giving this topic as a new subject for pre-service mathematics teachers without preliminary study and analysis is unwise so that the most appropriate choice is to pick some suitable concepts which is potential to be generalized and not too far from students' prior knowledge and having relation to existing subject knowledge.

Based on content curriculum for pre-service mathematics teachers, the parallel coordinate can be embedded in Calculus or Analytic Geometry. In both subjects topic of Cartesian coordinate is an important part for learning further concepts such as graph of function, linear function, slope, tangent, lines, planes etc. Refer to its history; in this paper parallel coordinate is embedded in Analytic Geometry.

Based on analysis of related topic in Analytic Geometry, concept of parallel coordinates are limited in 2 Dimension as a starting point, below are topics that designed for pre-service mathematics teachers:

1. Cartesian coordinates system as the basis for constructing parallel coordinate in 2-Dimension.
2. Representation of a point in N-dimension (c_1, c_2, ...,c_N) of Parallel coordinates
3. Representation of a point A (x,y) from Cartesian coordinate 2-Dimension in Parallel Coordinate 2-Dimension.
4. Representation of a Line $\ell \equiv y = mx + b$ in Parallel coordinate 2-Dimension.
5. Definition of point in infinity and concept of duality.
6. Parallel lines and perpendicular lines in Parallel coordinate 2-Dimension.
7. Lines intersection in parallel coordinates 2-Dimension.
8. The role of the tangent lines in Parallel coordinates.
9. Concepts of Region in Parallel coordinate.

Topics were selected based on the relationship with topics in Analytic Geometry. Those selected topics were constituted for designing learning modul and learning activities. Below is an example of the part of modul for student:

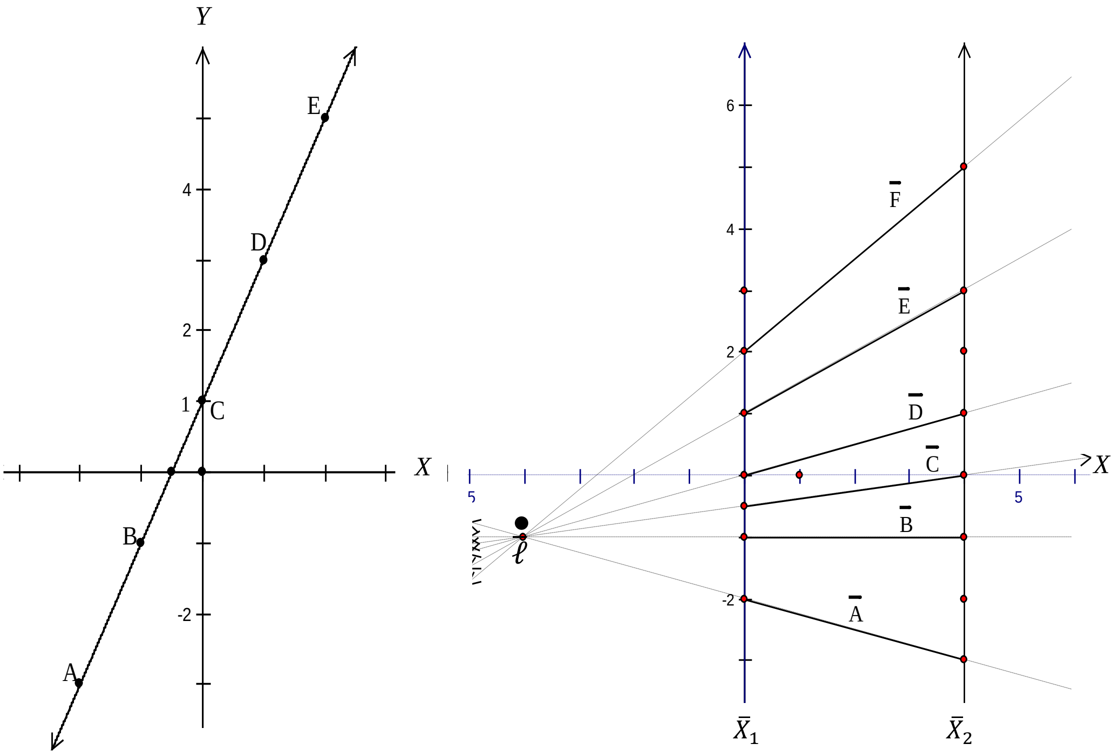

Figure 1. An Example of Lines representation in Cartesian dan Parallel Coordinates

Conclusion

On preface of his book, Inselberg (2009) stated that *parallel coordinates is a general-purpose VISUAL multidimensional coordinate system* which contains theorems on the unique representation of multidimensional objects, and geometrical algorithms for intersections, containment, minimal distances, proximities, etc, so that it can be adopted for visual experimentation and explanation in teaching mathematics and the science at the university, high school and even elementary-school levels. Refer to this statement, this paper contain idea of how to present topic on Parallel Coordinates which are suitable for pre-service mathematics teachers in very early stage.

Argumentation behind this idea is that parallel coordinate as one of non-conventional mathematics concept is potential to be adapted in learning context design for stimulating mathematical experiences such as problem solving activities, mathematical abstraction processes, construction definition, etc for pre-service teachers. Those mathematical experiences are very important to develop horizon *content knowledge* and *specialized content knowledge* of pre-service mathematics teachers.

This paper is only an onset of idea for next long journey of finding and developing curriculum for getting professional mathematics teachers. This step will open many opportunities to develop concept of parallel coordinates, non-conventional mathematics concepts, and mathematics content knowledge for teaching, mathematical experiences needed by pre-service teachers and so on.

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COOPERATIVE LEARNING MODEL OF *GAG* (GEOMETRY AUGMENTED GAMES) FOR 6TH GRADE ELEMENTARY SCHOOL STUDENTS

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Abstract

The aim of this study is improving students' mathematics' achievement and their life skills by implementing the cooperative learning of *GAG* (Geometry Augmented Games) for 6th grade elementary school students of SD Negeri Sumber 1 Berbah Sleman. The best practice is based on the experiences in implementing problem solving approach of cooperative learning. *GAG* is an integration of ICT and environment based games in geometry learning. This learning method is a joy full and challenging learning. Implementing *GAG* in cooperative learning has proved that this method has succeed improve students' mathematics achievements and life skill. The result found that students' mathematics' achievement increased from 50% to 85.71% of students' scores are $\geq$ 72. Students also increased their life skill of working in group (collaboration), problem solving, communicating with others, and using technology for learning.

***Keywords**: Cooperative Learning, Geometry, Augmented Games, ICT, and Environment*

INTRODUCTION

Background

Mathematics is universal science to develop modern technology. It is very important for human living and life. Mathematics is all around us; it underpins much of our daily lives and our futures as individuals and collectively (Ofsted, 2011: 4). It is therefore of fundamental importance to ensure that children have the best possible grounding in mathematics during their primary years. United Stated Common Core Standard for Mathematics stated that what students can learn at any particular grade level depends upon what they have learned before (www.corestandards.org). Number, or arithmetic, is a key component of this (Ofsted, 2011:4).

Starting in elementary school, children should be learning beginning concepts in algebra, geometry, measurement, statistics and logic (U.S. Department of Education Office in Communication and Outreach, 2005: ii). Furthermore they said that students should be learning how to solve problems by applying knowledge of math to new situations. They should be learning to think of themselves as mathematicians—able to reason mathematically and to communicate mathematical ideas by talking and writing. In Indonesia, Kurikulum Tingkat Satuan Pendidikan (KTSP) for elementary school also stated the same. It stated that mathematics must be given start from elementary school to give students ability to think logically, analytically, systematically, critically, and creatively, and having skill to work together in collaboration (www.file.upi.edu). This curriculum said that problem based learning is mathematics' focus include close problem with single solution, open problem with not only one solution, and problem with various solution. To improve problem solving skill we must develop the skill to understand the problem, make mathematics' model, solve the problem, and interpret the solution.

Like many other teacher, the writer also has tried to do her best to help her students learning mathematics. Unfortunately, the 6th grade students of SD Negeri Sumber 1 Berbah Sleman is having mathematics' problem in geometry. About 60% of the students cannot distinguish how to measure and find out the area and volume of geometry. Meanwhile, measuring and finding out the area of 2-dimensional shape and the volume of 3-dimensional shape were already given when they were in previous grade, grade 4th and 5th. When they were asked by the writer the reason of their difficulties the answer is quite surprising, "Because I cannot memorize the formulas", one of the student said. Of course this problem reduce students' achievement in mathematics. There're about 14 or 50% from 28 students who's mathematics' achievement are under passing grade (the passing grade is ≥ 72).

Following the curiosity, the writer investigated the students personally one by one in rehearsal period while they were in relax time. The findings were sadden the writer because not only 1 or 2 students who were fear of mathematics in their previous grades, but there're 14 from 28 students who confessed it. So the investigation went on. The next finding sadden the writer more, because from 14 students who didn't fear of math there're 6 students who stated that they were hesitated and got tired to learn math. The worst finding that sadden more was "The previous teachers taught us by drawing picture on the blackboard, wrote the formulas, had us to do the same on our books, then asked us to memorize them before gave us task and assignment." confessed the students.

Based on those problems, the writer tried the best by doing some trials of joyful and meaningful ways of teaching geometry by applying game to make the students interested in learning it. From those trials, there is one practice that succeed the students in geometry learning of measuring area and finding out the volume. The best practice conducted cooperative learning using games. The games were named as GAG (Geometry Augmented Games). This is a problem solving approach on how to solve mathematics problem by integrating games in mathematics activity. So this article will tell the reader how cooperative learning model of GAG were implemented in the classroom. Even though this best practice that hasn't perfect yet, but the writer hope the reader will get inspiration to do the same in their classroom. Furthermore, the writer really hope that this game can be improve it to be better in the future.

The Strategy to Solve the Problem

As the writer told before this best practice is based on the experiences in implementing problem solving approach of cooperative learning of GAG (Geometry Augmented Games) for 6th grade students of SD Negeri Sumber 1 Berbah, Sleman. According to the meaning, augmented reality games is the integration of game visual and audio content with the user's environment in real time (www.techtarget.com). This games often superimposes a pre created environment on top of a user's actual environment.

Unlike the definition has mention before, the augmented games in Geometry Augmented Games is much simple than that. This game is an integration of ICT and environment in geometry learning through games. What kind of ICT that the writer have used in this learning? The writer only used laptop, internet connection, and hand phones. The learning process is simple that can be applied by other teacher to the other topics lesson or maybe other subject material. Whether the environment, the writer only use any 2 or 3-dimensional shapes that were found at school. In detail the learning process was running in these stages:

1. One day before the learning, the 28 students were divided into 7 groups. Each group consist of 4 students. Every group were asked to bring hand phone that can be connected to the internet for the next day. In this stage, the games rules haven't explained to surprise the students. It is believe that the surprise will make the activities will be more joyful and challenging.
2. In the learning day, after the explanation of the game's rules to the students, the writer brought them to leave the classroom to search for the pictures of 2 and 3-dimensional shapes that the writer have shared via WhatsApp, Messenger, Instagram, and BBM. The pictures of geometry were taken in the school environment, so that the students will measure and calculate the real thing in reality. It can help the students to make their own understanding of using mathematics' problem solving in their daily life.

Figure 1. *Some of 2 and 3-Dimensional Shapes that were Found at School*

3. After found the picture the students must measure the geometry shapes (it can be glasses, boxes, ball, dustbin can, etc.). Then, they must find out the area or volume based on the task that the picture has mention. During their activity, the writer observed them using the observation instrument to know their collaboration run. The writer also made some notes to know how their works are to be done. It is important for teacher to know how this strategy is really work or not. It is also important for teacher to analyze which one need to be improved and developed. Some investigation are also needed to make sure that students really know what they do and love the learning.

Figure 2. ***Students' Activity***

To create the games, the writer prepared the picture one day before after the students leave school. The writer took a picture in every 2 or 3-dimensional shape that were found. Then the writer decided which one is appropriate to be mathematics problem. After that, the writer wrote the task on the picture. Last, the writer create video of the problems using video slides in PhotoGrid application that easily can be downloaded in Play Store freely. The writer choose this application because it is easier to create video using hand phone. The mathematics problems were made with video to make it more interesting to share it in social media. It will be easier for the students because they only needed once download. At that time the writer made sure the internet connection was running well too.

4. After finished, all groups must return to the classroom to discuss the result and how they can get it. Each group may have various ways to found out the formulas of areas and volume. The different ways were discussed then made a conclusion. It is important for teacher to know the process too.

Figure 3. ***Discussion the Result and How***

During the discussion, every group must present their findings.

5. Reflection and appreciation for each group whether their result are good or not good enough to motivate them to do better in the future.
6. Last, evaluation was held individually to measure the final result of the learning.

Based on the strategy above, the most important in this games based learning is about the students' chance to solve mathematic problems by their own solution and their opportunity to work in collaboration. The writer wanted them to love mathematics learning by making it more joyful and challenging. So, problem based learning for mathematics the writer applied by creating it to be fun and challenging games. In short, from those strategy, the learning process can be described in 6 stages, as follows:

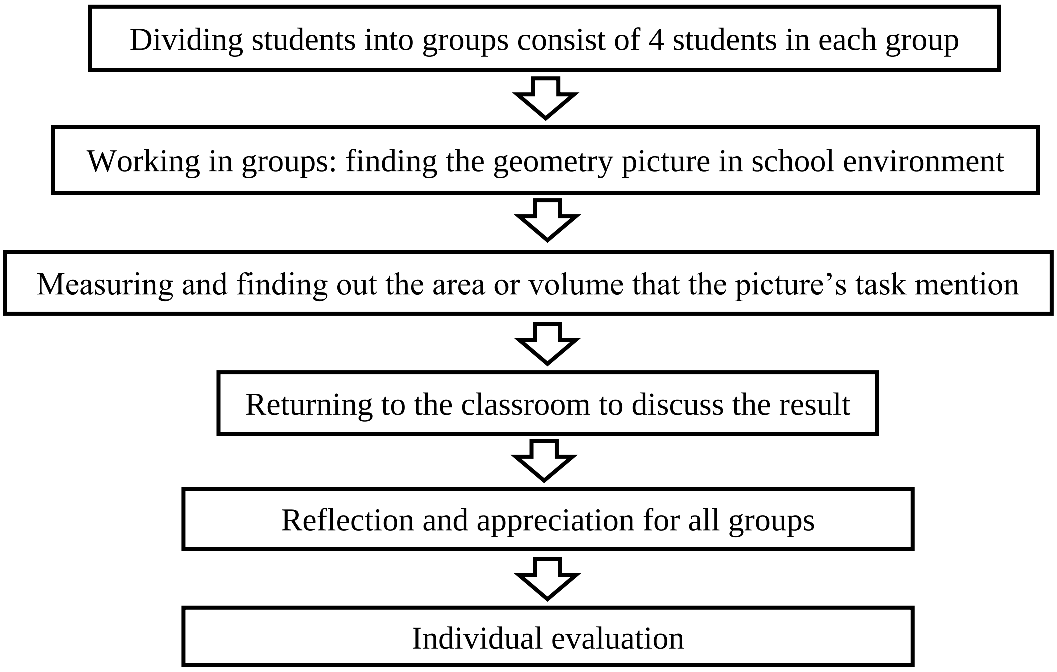

Figure 4. ***6 Stages of Cooperative Learning Model of GAG (Geometry Augmented Games)***

From the 6 stages above, we can see that the students were actively followed the lesson. They were actively involved in the learning process. In those stages there's also reflection and appreciation in the strategy. This condition because the writer had the students to develop their confidence in their mathematics ability. Therefor the writer wanted the students to know that problems can be solved in different ways, wrong answer sometimes can be useful. The students must be brave to try and take a risk, and also being able to do mathematics in their brain (U.S. Department of Education Office in Communication and Outreach, 2005: 3-4). This is important to build their academic and life skills that will be beneficial in their future.

The Aim of the Study

The aim of the study is improving students' mathematics' achievement and life skills by implementing the cooperative learning of *GAG* (Geometry Augmented Games) for 6th grade elementary school students of SD Negeri Sumber 1 Berbah Sleman.

The Advantages of the Study

The advantages of the study gained in learning practices are: 1) the increase of teachers' confident in developing a variety of teaching methods, and then publish it to be used by other teachers in teaching; 2) the use of technology, cooperative learning, games, and environment in learning provide valuable experience for students, as well as add their insights to utilize technology for positive things; 3) increasing repertoire of methods and models of learning using technology.

DISCUSSION

The Argument of the Chosen Strategy

Mathematics is a fundamental human activity – a way of making sense of the world. Children possess a natural curiosity and interest in mathematics, and come to school with an understanding of mathematical concepts and problem-solving strategies that they have discovered through explorations of the world around them (Ginsburg in The Expert Panel, 2004: 1). It is important for educators to provide experiences that continue to foster students' understanding and appreciation of mathematics. By providing mathematics programs in which students explore and make sense of mathematical patterns and relationships, we can help students develop mathematical knowledge that allows them to solve problems and explore new ideas, in and out of the classroom (The Expert Panel, 2004: 1).

In this study, the writer choose cooperative learning because it is believed will help the students to build their collaborative skill. So that it will be useful for their life in the future. Theoretically, cooperative learning is a model of learning in which learners learn and work in small groups in a collaborative whose members consist of 4 to 6 people, with a heterogeneous group structure (Slavin in Etin Solihatin & Raharjo, 2007: 4). According to those theory, the learning success of the group depends on the capabilities and activities of the group, either individually, or in groups.

Cooperative learning will provide opportunity for students to improve their communication skill. Some studies showed that students need many opportunity to talk in a linguistically rich environment. Researchers found that students' learning enhanced when they have many opportunity to elaborate on ideas through talk (Pressley in Karnasih, 2015: 89). Meanwhile, in previous studies of cooperative learning has been proven to improve learning outcomes of students.

Utilizing ICT (Information and Communication Technology) in the learning becomes the element of surprise (surprise element) for learners. As integrated component of teaching and learning, ICT allows learning experiences which are innovative, accelerated, enriched, and deepen the skills acquisition, motivating, and engaging and relating school experience to work practice and an authentic context (Pannen, 2015: 35). The integration of ICT in the teaching and learning of mathematics increases students motivation in learning mathematics, allows individual learning based on an individual's learning style and pace, and it has been excellent in increasing the cognitive and affective skill as compared to the traditional teaching and learning process (Etikawati in Pannen, 2015: 39).

In previous studies have proven that the use of technology in learning more successful in increasing the knowledge input learners. It is like the statement "We know that ICT-based education gives a learner more opportunities to assimilate the concept through auditory and visual memory of the human brain. So in ICT-based education, the number of inputs will be higher than the traditional one "(Aktaruzzaman, Md., Shamin, Md.R.H., and Clemen, C.K., 2011: 116). This means that ICT-based education provides more opportunities for learning to assimilate the concept in audio and visual in the human brain. Thus, the ICT-based learning input amount will be higher than traditional learning. This further adds to the confidence the author to integrate technology in learning.

The reason why the writer using games in this study because students love playing games. That's why engaging games in mathematics learning will invite their interest to learn deeper. Theoretically, mathematics games and puzzles provide a rich context for practicing mathematics concepts. Games provide an excellent environment to explore ideas of the computational thinking (Moursund, D., 2007: 8). It can help the students to think independently joyful. The writer wanted to prove that mathematics is a joyful and challenging learning that will be very useful for them in the future. Well-designed mathematics games are another way for teachers to have students hone their proficiency with numbers (Kamii in The Expert Panel, 2004: 23).

Advocates of game-based learning in higher education cite the ability of digital games to teach and reinforce skills important for future jobs such as collaboration, problem-solving, and communication (McClarty, K.L., et.al., 2012: 4). While in the past educators have been reluctant to use video games or computer games in the classroom, there is an increasing interest across broad and varied parts of the educational establishment to look at the use of digital games as serious learning and assessment tools.

In this study, the writer involved learning environment in order to make learning more meaningful and realistic for students. In the journal article compiled by Worosetyaningsih also proved that the use of the environment and learning media ICT (Information Communication Technology) to improve learning outcomes Integrated Social Science students of class VII B of SMP N 2 Ngemplak the school year 2013/2014 (Worosetyaningsih, T., 2015: 38). Making a relation between mathematics learning and students life is very important. It is not only to make it meaningful but also easier for them.

The Result of the Chosen Strategy

During the lesson, the writer observed the students using observation instrument of collaboration and made some note to know how their effort to solve the problems. The writer also did some interview to know how their feelings about the learning by integrating ICT and environment into games. The students seemed very enthusiast and enjoy the lesson. They stated that learning mathematics using games succeed in decreasing their stress. “It made mathematics more fun.” one of the student said. “It’s a kind of refreshing to reduce stress” the other one said.

During the learning process the writer found the students improve their skill of collaboration as well as their communication skills. They laughed, smiled, and let out an argument during their working. Over all, the cooperative learning of GAG is succeed creating joyful and challenging learning.

To ensure that cooperative learning model of GAG (Geometry Augmented Games) succeed improve the students’ achievements in 2 and 3-Dimensional shapes measurement, the writer gave them individual assessment/evaluation. The results are as follows in this table:

Table 1. The Recapitulation Result of the Mathematics Assesment of 6th Grade Students of SD Negeri Sumber 1 Berbah

No.	Scores	Number of Students Percent					
		Pre GAG		GAG 1st		GAG 2nd	
1.	00-31	1	3.57%	0	0	0	0
2.	32-41	2	7.14%	2	7.14%	0	0
3.	42-51	4	14.29%	3	10.71%	0	0
4.	52-62	2	7.14&	4	14.29%	2	7.14%
5.	62-71	5	17.85%	5	17.86%	2	7.14%
6.	72-81	10	35.71%	8	28.57%	7	25%
7.	82-91	2	7.14%	6	21.43%	10	35.71%
8.	92 -100	2	7.14%	5	17.85%	7	25%
	Jumlah	**28**	**100%**	**28**	**100%**	**28**	**100%**

Based on the table of recapitulation result of the mathematics assessment for measuring the area of 2 dimensional shapes and volume of 3-dimensional shapes, we can see that there is a very significant improvement in students' achievement. Before the implementation of cooperative learning model of GAG (Geometry Augmented Games) was held, the students who have completed the learning were only 14 or 50% of them. After the first implementation, the achievement was improve into 19 students or 67.85% have completed. At the second implementation, the improvement has completed the requirement that there're 24 students or 85.71% have completed.

The Strength of the Strategy

The successful implementation of the chosen strategy succeed to overcome the problems cannot be separated from their supporting factors. In this best practice, the details of these factors are as follows:

1. the great enthusiasm of the students towards learning that was carried out by utilizing the technology, games, and environment;
2. the granting rewards to the success of students, both verbal and non-verbal;
3. the packaging of learning that do make the students feel fun and pleasure in learning, as well as unencumbered as when learning is done conventionally;
4. respect, collaboration, and a good response from the head of the school and other teachers

The Weakness of the Strategy

During the lesson, some problem happened but didn't affect the process. The first weakness is the problem that happened due to the bad internet connection. So that the students didn't receive the games at the same time. This condition caused a little chaos among the member of the groups. The second weakness is because this learning model is depended on the internet connection.

Development Alternative

For the next teachers who want to apply this learning method could develop some alternative strategies to make better result, such as:

1. improve the internet connection so that all groups can get the games at the same time;
2. apply this learning model into other topics that can be applied by games and technology, and engaged by environment;
3. create the offline version so that when the internet connection problem happened, the learning process will still running well.

CONCLUSION AND SUGGESTION

Conclusion

GAG is an integration of ICT and environment based games in geometry learning. This learning method is a joy full and challenging learning. Implementing *GAG* in cooperative learning has proved that this method has succeed improve students' mathematics achievement and also their life skill. Before the implementation of cooperative learning model of GAG (Geometry Augmented Games) was held, the students who have passed the passing grade of the mathematics learning in geometry were only 14 or 50% of them. After the first implementation, the achievement was improve into 19 students or 67.85% have passed. At the second implementation, the improvement has completed the requirement that there're 24 students or 85.71% have completed (score ≥ 72). Students also improved their life skill of working in group (collaboration), problem solving, communicating with others, and using technology for learning.

Recommendation

For the next teachers who want to implement this learning method could develop some alternative strategies to make better result, such as:

1. improve the internet connection so that all groups can get the games at the same time.
2. apply this learning model into other topics that can be applied by games and technology, and engaged by environment

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TEACHING MATERIAL BASED ON LEARNING STYLE WHICH BASED ON STAGES OF MATHEMATICAL REFLECTIVE THINKING ABILITY

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Abstract

This study is aimed to develop teaching material based on learning style whether visual, auditory, and kinesthetic based on mathematical reflective thinking ability (MRTA) stages for Senior High School students. This teaching material development passes through 3 stages, namely: need analysis, product development, and field-test. Need analysis is done by literature review activity and survey. The result of survey shows that Teaching Material is needed to enhance MRTA stages which facilitate students' various learning styles. The activity of development is done by designing computer- based Teaching Material which facilitate students with various learning styles and validate the product to expert. Computer- based design contain 8 indicators of MRTA. Field-test is done through one stage through limited scale test, and data which are collected is analyzed in qualitative and quantitative descriptive and taken through questionnaire, document and field note. The result of study shows that Teaching Material which is developed had facilitated students with visual, auditory and kinesthetic learning style.

Keywords: *learning style, mathematical reflective thinking, mathematical reflective disposition,*

INTRODUCTION

Mathematical reflective thinking ability is thought important, because this ability become foundation for mathematical critical thinking ability and another mathematical abilities. This is in accord with Ennis (1987) and Bruning et al (Jiuan, 2007) who argued that someone who had possessed critical thinking ability must also have reflective thinking ability. This makes reflective thinking ability become foundation to acquire critical thinking ability.

Zehavi and Mann (2006) categorized reflective thinking ability in analyzing problem solving in analytic geometry field which consist of some stages, namely: choosing the technique, monitoring solution process, insight, and conceptualization (relation of concept and meaning). Shermis and Given (Noer, 2010) said that reflective thinking comprise abilities to: identity the conclusion; identity reason and proof; identify assumption and valued conflict; identity descriptive assumptions; evaluate reasoning; identify omitted information. Based on Shermis opinion then reflective thinking ability has characteristic which is similar with critical thinking ability. According to Given, reflective thinking is considering individual's success and failure and asking what have been done, what needs and what does not need improvement.

From opinion of some scholars suggested before then mathematical reflective thinking ability defined as critical thinking ability which empower past knowledge in order to be able to interpret a case based on mathematical concept involved; able to evaluate and examine the correctness of an argument based on concept/nature used; able to draw analogy from two similar cases; able to analyze and clarify question and answer; able to generalize; and able to differentiate between relevant and irrelevant data. If that ability arise then mathematical reflective thinking disposition will arise indirectly.

Critical thinking process will arise after reflective thinking process is done. For example, in process to prove concept and formula in mathematics. Concept and formula proving in mathematics can be regarded as correct in its process if reflective activity is done first. That reflective activity for instance, whether from stage 1 to stage 2, and to the next stage had been correct based on rule, theorem, algebra which is related with it so someone empower his/her knowledge in the past and relate it with process of identify, relate or connect one concept with another concept. After reflective ability is empowered, then students are able to judge and decide the correctness of concept or the correctness of that formula proving.

Mathematical Reflective Thinking Ability (MRTA) which had been developed by Nindiasari (2013) is the ability to interpret a case based on mathematical concept involved, to evaluate the correctness of argument, to draw analogy from two similar cases, to analyze and clarify question and answer, to generalize, to differentiate between relevant and irrelevant data. That mathematical reflective thinking ability developed is intended to Senior High School students but it is possible also to be developed beyond that level.

Based on result study conducted by Nindiasari & Novaliyosi (2014) in each indicator of MRTA has 7 stages, namely: stage 1- observing, stage 2- understanding the problem, stage 3- collecting data, stage 4- making assessment from data collected, stage 5- choosing strategies in solving the problem, stage 6- conceptualization, and stage 7- monitoring the solution. In fact, those seven stages are intended to school with low category and their students are still in stage of collecting data.

According to Nindiasari (2014), MRTA stages comprise 3 stages, namely: stage 1- trying to understand mathematics well, stage 2 – sensitive toward the others' feeling, and stage 3- trying to solve mathematical problem well. Students in some level of school had reached these MRTA stages.

Students learn in their own styles. This learning style is manifestation of individual difference which should be noticed by teachers. Learning styles comprise visual, audio and kinesthetic. If teacher understand student's learning style then student will be motivated to learn. Knowing student or child's learning style is important because it will be able to enhance the effectiveness in learning as said by Ghufron and Rini (2014). Students' various learning styles in class demand teacher to have innovative thinking to apply learning model and media based on learning style. According to Ghufron and Rini in 2014, learning style is how individual learn or the way used by each person to concentrated on process, and obtain difficult and new information through different perception. From this definition, then it is very important how model and media which will be developed can be applied or can be accepted by students who have various learning styles.

The arrangement of this Teaching Material is aimed to enhance MRTA stages based on learning style.

METHOD OF STUDY

This study is development study which passes through 3 stages, namely: need analysis, product development, and small scale field-test. Need analysis is done by studying mathematical reflective thinking disposition and ability stages and learning style. Product development is done by making draft of computer- based teaching learning development in which first making its storyboard and printed teaching material which support that computer-based teaching material. After developing draft, expert-test is done. The result of expert-test conclude that product developed is deserved to be field-tested and there are some suggestions for improvement. After doing improvement, small scale trial is done.

Subject of this study are Senior High School students who had learned Probability material. In this study, the subject are 11th graders of Senior High School with total of 10 students who have visual, audio, and kinesthetic learning styles. Instruments used in this study are questionnaire for experts related with media, MRTA stages, mathematics education and mathematics. Another instruments are learning style scale and questionnaire of opinion about media, observation sheet, and field notes.

RESULT AND DISCUSSION

Before teaching material draft is made, need analysis is done first. Need analysis from this study is there is a need of Teaching Material which facilitate students' various learning styles that is through computer- based teaching material. Because based on result of field survey conducted by Nindiasari & Novaliyosi (2014), from students who are given printed teaching material to enhance MRTA stages not all of them reach MRTA stages particularly in school with low category, from the existing 7 stage (stage 1- observing ; stage 2- understanding the problem; stage 3- collecting data, stage 4- making assessment toward data collected; stage 5 – choosing strategies in solving the problem and insight; stage 6 – conceptualization; stage 7 - monitoring the solution) students just reach in stage of collecting data.   Based on survey also, students in each school in whatever category have different learning style. Therefore, teaching material is needed to facilitate students with various abilities and learning styles to enhance MRTA ability stage.

Teaching Material draft which is developed is teaching material to facilitate students' learning style whether auditory, kinesthetic, and visual namely interactive and computer- based teaching material and its printed teaching material is Story Board. Computer- based teaching material comprise main menu containing: Direction, competence which will be achieved, and explanation of MRTA indicator. In this explanation of indicator there are 8 indicators of MRTA such as showed in Figure 1.

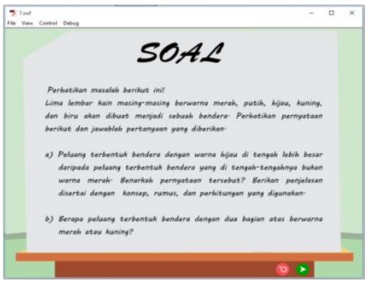
Indikator dari Kemampuan Berpikir Reflektif Matematis (KBRM) terdiri dari 8 indikator. Klik pada salah satu angka untuk mengetahui 8 indikator tersebut.

Figure 1

Indicator Menu

In this menu, students can choose indicator to be seen and understood. After they choosing one of indicator then students can choose again the supporting material to solve MRTA problem. MRTA problem contain questions posing to enhance MRTA stages. MRTA problem below is one of problems from indicator namely: able to identify mathematical concept and formula which is involved in mathematical problem which is not simple.

Besides presented in written form, item and question posing stage is guided by the voice which read it. This is intended to facilitate students with auditory learning style. To facilitate students with visual and kinesthetic learning style, when understanding the material and try to solve MRTA problem, they are accompanied by printed material.



SOAL

Perhatikan masalah berikut ini!

Lima lembar kain masing-masing berwarna merah, putih, hijau, kuning, dan biru akan dibuat menjadi sebuah bendera· Perhatikan pernyataan berikut dan jawablah pertanyaan yang diberikan·

a) Peluang terbentuk bendera dengan warna hijau di tengah lebih besar daripada peluang terbentuk bendera yang di tengah-tengahnya bukan warna merah· Benarkah pernyataan tersebut? Berikan penjelasan disertai dengan konsep, rumus, dan perhitungan yang digunakan·

b) Berapa peluang terbentuk bendera dengan dua bagian atas berwarna merah atau kuning?

Figure 2

Example of MRTA Problem

PROBLEM

Consider this problem below!

Five pieces of clothes each has red, white, green, yellow, and blue colors will be made to become a flag. Notice the following statements and answer the questions given:

a) *The probability of flag created with green color in the middle is bigger than the probability of flag created which in the middle is not red color. Is that statement true? Give explanation accompanied by concept, formula, and calculation used.*
b) *What is probability of flag created with two upper parts have red or yellow color?*

For enhancement of MRTA stages, questions are posed such as: “Read carefully the problem above, then write what you can understand from the problem above? What material that you had been learned before to solve that problem?”. Then in computer- based teaching material, there are choice of answers to be chosen by students. This question posing is intended that students passing through stage 1- observing and stage 2- understanding the problem in MRTA stage such as in Figure 3a.

.

SOAL

Untuk Menjawab soal di atas, perhatikan langkah-langkah di bawah ini!

Baca dengan seksama soal di atas· Kemudian tuliskan apa yang dapat kalian pahami mengenai soal di atas? Materi apa yang sudah kalian pelajari sebelumnya untuk menyelesaikan masalah tersebut?

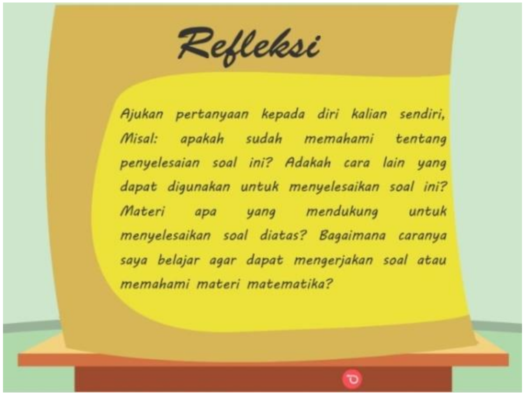
a· Soal diatas dapat diselesaikan dengan konsep peluang, peluang dua kejadian saling lepas, dan peluang komplemen suatu kejadian, sehingga perlu dipahami manakah yang dimaksud dengan kejadian dan ruang sampelnya serta manakah kejadian kompelen dan dua kejadian saling lepas·

b· Soal diatas dapat diselesaika dengan konsep peluang, yang didalamnya perlu bantuan konsep kombinasi·

Figure 3a
Example of Question posing to Enhancement
of MRTA Stages

Related with choice of solution strategy which is contained in MRTA stages, the questions are posed as follow, “Based on problem understanding, data and its assessment, how your strategy or way to solve the problem in the item above?”. Then, based on step of that strategy, students are asked to make solution and write formula and concept which support the solution. Instruction or another question posing is, students are asked to check their work result which is guided by choice. This question posing is expected capable to enhance understanding of material. This understanding of material will give effect to enhancement of students’ learning achievement. This because there is connection between effect of learning style with learning achievement which is in accord with result of study conducted by Bire Ludji, Garadus, and Bire (2014) who concluded that visual, kinesthetic and auditory learning styles simultaneously/together or separately can influence students’ learning achievement.

The last menu students can choose is reflection section, as for its reflection menu can be seen in Figure 4 below.



Refleksi

Ajukan pertanyaan kepada diri kalian sendiri, Misal: apakah sudah memahami tentang penyelesaian soal ini? Adakah cara lain yang dapat digunakan untuk menyelesaikan soal ini? Materi apa yang mendukung untuk menyelesaikan soal diatas? Bagaimana caranya saya belajar agar dapat mengerjakan soal atau memahami materi matematika?

Figure 4. Reflection Menu

REFLECTION

> *Ask question for yourself. For example: Do I have understood about this problem solution? Is there another way which can be used to solve this problem? What is the supporting material to solve the problem above? How I learn to be able to solve the problem or understand mathematics material?*

In this section of reflection, questions are posed to students themselves in order that they always do self evaluation, control their thinking activity, whether they had understood or not of what had been done, if they had not understood yet then what should do for the next time. If students are able to self reflect then they will get used to asses themselves and able to plan the next steps. With question posing to monitor their own thinking, their understanding, monitor thinking patterns, then indirectly students are skillful in using their metacognitive ability. By using metacognitive ability, students had showed their professional in judging their own thinking. This is in accord with Tan (2004) who argued that ideally students should judge their thinking before, after and after solving the problem or decision making process, able to monitor their understanding level, aware of their own environment, judge the wrong thinking patterns, which all of them are metacognitive. Reflection activity as professionalism development tool is in accord with Naghdipour & Emagwali (2003) opinion.

In all indicators of MRTA which are trained in this teaching material, questions are posed which are intended to enhance MRTA stages. After being arranged, expert-test is done toward this draft of computer- based teaching material related with media, enhancement of MRTA stages and also related with mathematics education and Mathematics.

Table 2. Questionnaire Result of Multimedia Expert-Test

No	Aspect	Percentage of Final Score
1	Display	80
2	Coloring	80
3	Background Design	100
4	Animation and Illustration	80
5	Structure of Material Placement	80
	Average	84

The recommendation in general related with media expert is that teaching material is packaged in the form of Compact Disk (CD) and color variations among indicators need to be added.

Table 3. Questionnaire Result of Expert-Test Related With Mathematical Reflective Thinking Ability (MRTA) Stages

No	Aspect	Percentage of Final Score (%)
1	Observing Items	100
2	Understanding The Problem	100
	Collecting Data	100
3	Making Assessment from Data Collected	100
4	Choosing Strategies in Solving Problem and Insight	80
	Conceptualization	80
5	Monitoring Solution	100
6	Reflection Activity	80
7	Facilitate Students to Try to Understand Mathematics Well	100
8	Facilitate Students to Sensitive Toward Other People’s Feeling Related with Mathematics	80
9	Facilitate Students to Try to Solve Mathematics Problem Well	100
	Average	92.72

From Table of questionnaire result related with MRTA stages respectively, MRTA stages comprise aspects of observing the items, understanding the problem, collecting data, making assessment from data collected, choosing strategies in solving problem and insight, conceptualization, monitoring solution, reflection activity, facilitate students to try to understand mathematics well.

Related with mathematics education and mathematics aspect, all of them are categorized very good and percentage of final score is 100%. As for aspects assessed are concept accuracy, fact and data accuracy, example and case accuracy, drawing and illustration accuracy, term accuracy, notation, symbol and icon accuracy, and facilitate students learning who have auditory, visual and kinesthetic learning style. After expert-test is done, then small scale field-test is done.

This small scale field-test is given to 10 students of 11th grade in one of Senior High School in Serang City who had learned Probability lesson. Those ten students have various learning styles (auditory, visual and kinesthetic). The categorization of learning styles is by scaling of learning style whose scoring is processed a posteriori. Scoring by a posteriori in attitude scale according to Yaniawati (2001) is scale by calculating each item based on respondent answer, thus sore of each item can be different. From result of calculation for learning style scale, it is obtained 14 item of statements which are valid with reliability of 0.77 and included in high category.

Based on result of students categorization by giving questionnaire in the form of learning style scale and observation in class then it is obtained the students who are included in category of visual, audio and kinesthetic learning styles by 30%, 20%, and 50% respectively. From this categorization, their behavior and view are observed through questionnaire toward interactive media and printed media for enhancement of reflective mathematical reflective ability stage and mathematical reflective disposition stage.

The result of their view related with computer based interactive media based on learning style category are as follow:

Table 4. Description of Students' View Related With Computer Based Interactive Media Based on Learning Style Category

Learning Style	Description
Visual	1. Initial display of learning CD media is clear and interesting. 2. Color combination is good. 3. Illustration of picture and animation is interesting and support understanding toward material. 4. Examples of item and exercise are appropriate. 5. Learn mathematics is fun. 6. Animation displayed is interesting and support understanding toward material. 7. There is new thing in Probability material.
Auditory	1. Initial display of learning CD media is clear and interesting. 2. Color combination is good. 3. Illustration of picture and animation is interesting and not represent learning material. 4. Examples of item and exercise is appropriate. 5. Learn mathematics is fun. 6. Animation displayed is interesting and support understanding toward material. 7. There is new thing in Probability material. 8. Interesting and represent learning material.
Kinesthetic	1. Initial display of CD media is less clear but interesting. 2. Color combination is good. 3. Illustration of picture and animation is interesting and represent learning material. 4. Example of item and exercise is appropriate. 5. Learn mathematics is fun by using interactive learning CD.

	6. Animation displayed is interesting and support understanding toward material. 7. Animation displayed is interesting but not support understanding toward material. 8. There is new thing in Probability material.

Entirely, based on learning style category, students argued that computer based learning style has clear and interesting display, good color combination, illustration of picture and animation which is interesting and represent learning material, material presentation which is enough, and examples of item and exercise which are appropriate. They also argued that learning with computer- based teaching material is fun and animation presented is interesting and increase understanding of material delivered. They give suggestion that animation and picture need to be improved.

In order to be success, in delivering material or concept understanding, teacher needs to understand their students' learning style. This as explained by Prashing (2007) that teacher need to know students' learning style so they can be facilitated. Visual learning style is learning style which is depend on vision and verbal. Student with this learning style usually likes picture and diagram and likes video. Therefore, the way student learn with this style should be accompanied by notes or handout, there is picture or graphic, multimedia, memorize with association of pictures. Students' interest in learning style toward picture is showed by Table 2 in which they say that initial display of media is interesting, color combination is good, illustration of picture and animation is interesting and represent learning material. Animation displayed is interesting and support understanding toward material, and interest to animation display and its pictures is increased.

Students with auditory learning style have type of learning by listening something and usually enjoy to listen audio cassette, discussion, debate and verbal instruction. One characteristic of students with this learning style is that students learn by listening and being read then discussed. This views is in accord with opinion of students with auditory learning style who said that picture and media are good but not represent learning material. This indicate that students with auditory learning style not show interest in picture and animation, and more tend being read.

Kinesthetic learning style is learning style through physical activity and direct involvement. The characteristic of students with this learning style are there is physical activity during memorizing or learning, read with fingers, speak slowly and body is moved. Students with kinesthetic learning style enjoy to learn mathematics through this interactive media, because they are facilitated by interactive activity which present teaching material for enhancement of MRTA.

It is expected that this learning media with computer- based teaching material can create interesting learning environment. As said by Prashing (2007) that students need learning environment as good as possible. Through this computer- based teaching material media, students' thinking ability particularly MRTA and its enhancement can be better. Because supported by interesting environment through media so students can have a fun while learning, discuss with their friends, and are given opportunity to be flexible in the way of thinking and can self adjust.

In fact, the media resulted makes students who have various learning style whether visual, auditory and kinesthetic shows interest in following mathematics learning, This shows that media which facilitate various learning styles can attract students' interest to learn mathematics. They more easily in understanding the material, which is showed by their view that example of items are appropriate with exercise, animation is interesting and support understanding of material, there is new thing related with Probability material. Besides, based on observation they are able to follow and fill the questions related with enhancement of MRTA stages, such as "are those data enough to solve the problem above?" from this statement there is choice of answers yes or no, if yes there is choice of reason which should chosen by students. Related with the choice of solution strategy contained in MRTA stages, some questions are posed as follow, "Based on understanding of problem, data and its assessment, how your strategy and your way to solve the problem in item above?" Then, based on that step of strategy, students are asked to make solution and write formula or concept which support the solution. Another instruction or question posing is, students are asked to check their work result which is guided by choice. It is expected that this question posing can enhance understanding toward material. This material understanding will be effected on enhancement of students' learning achievement. This is because there is relation of influence in using learning style with learning achievement as in accord with result of study conducted by Bire Ludji, Bire and Garadus (2014) who concluded that visual, kinesthetic and auditory learning styles are simultaneously/together or separately can influence students' learning style.

CONCLUSION

Conclusion which can be drawn from this result of study and discussion above is that Teaching Material which can facilitate students' various learning styles (auditory, kinesthetic and visual) is computer- based interactive teaching material which is accompanied by printed teaching material which contain menu of direction, competence, indicator of MRTA in which in each indicator students can choose the material, items which are guided by question posing to enhance MRTA, and the last section is reflection. It contain interesting animation or picture and the sound which support it. This teaching material had been able to facilitate students with various learning styles (visual, auditory and kinesthetic) which is showed by students who enjoy to follow learning with interactive teaching material, and understand the material delivered. Based on result of study, the recommendation can be suggested as follow. First, for teachers, they need to give teaching material with computer- based media and printed teaching material which facilitate students' various learning styles. It is expected that they create interesting learning environment to make students succeed particularly in enhancement of MRTA stages through various learning media, pose the questions and instruction which force students to enhance MRTA stages. For students, it is expected that they hold discussion with more able students, often ask questions to teacher, search media or learning sources which facilitate learning style. For another researcher, it is hoped that they test the effectiveness related with enhancement of MRTA stages based on category of learning styles (visual, auditory and kinesthetic).

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Misconception of Preliminary's Construction Concept Math Teacher On ASEAN Country at Century 21

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Abstract

Teachers are spearhead of education worldwide that should improve the quality of teaching needs. One important component in the learning of mathematics is how to make understand student a mathematical concept not only through memorization and procedural material, but also a meaning of the concept. Teachers should engage students in understanding mathematical concepts by relating the factual material. One of the discussions was how misconceptions of students in the initial construction of mathematical concepts are scattered in algebra, geometry, and number operations there are a lot of mistakes. The expected result is turn back teachers will repair misconceptions so students can get correct information about concept mathematics. This paper is a preliminary study of the need for early construction error analysis about mathematical concepts. The author get data from the questionnaire written about right and wrong type question then teacher give explanation about their choice. Data is taken from teacher who attending at Course on Utilization and Development of IT-based Mathematics Learning for Senior High School on 21 october- 3 November 2015 consist of 30 teacher's around ASEAN Country. The results of True-False questions that given to each participant using a scale of 0-100 without regard to fault of their reasons, obtained a mean score of 51.03 which is quite extreme. While Results of interviews with participants there are a lot of misconceptions, where participants only understand the concepts only procedurally without understanding factually. In addition, participants have weakness attention to the initial construction of a mathematical concept that alot of them give wrong answers.

***Keywords**: Misconceptions , initial construction , a mathematical concept , procedural material , factual material*

Introduction

Education in the school became one of the significant intellectual development of children so that the received information stored in long-term memory. Therefore, the teacher as 'the spearhead' of education around the world should constantly improve their quality in the way of teaching and guiding student. All Schools not only as a venue for the learning process, without ignoring the sense of learning itself, because many students called study if there is a change of intellectual from capable to not capable, and also from not knowing to knowing. so the teacher's role is indispensable to familiarize students about think mathematical concept correctly.

Some experts argue that among students tend to be more visually strong and weak at the way of analysis a problem, namely Sinnett, (Sinnett & Soto-Faraco, 2007) states that: "Perceiving and processing visual information by sensory and mental processes are referred to as visual perception processes. Humans display a robust tendency to rely more on visual information than other forms of sensory information".

Observation and visual information processing with a process called sensory and mental processes visual tendencies. Humans displays a strong tendency to rely on visual information from the forms of other sensory information. This suggests that teachers should begin with the activity observed first, but in fact the teachers tend to give the concept directly. (Gal Hagar & Lichevski L, 2010) in his writings contain about: "In this paper, we aim to explain, discuss, and exemplify how processes are priorto Accompanying roomates" reasoning, "influence the cognitive process"

In his writings, they aim to explain, discuss, and provide examples of how that process ever to accompany "reasoning," the cognitive processes of students. It showed that after observing students should continue to ask, then gather information to students until student have ability in reasoning. Many researchers state that student difficulties in constructing and solving mathematical problems are often reflected in the form of a mathematical error (Brodie, 2010).

Students often experience their interference thinking about the mathematical concept where students tend to analogize the properties that they have received valid on the entire matter. In harmony with the opinion that reviewed by Brodie so that not a few students are often trapped by his own understanding is wrong. This causes an error against the construction of students' mathematical concepts.

One important component in the learning of mathematics is how to understand a mathematical concept is not otly just limited to memorization and procedural material, but also the meaning of the concept are important. This statement is contrary with statment that is encountered by researchers in the classroom that most teachers put more emphasis on the procedural course, so that the student is easy to forget the concept (Subanji, 2006).

Basically, the students are in the actual zone which is the zone of proximal development, with the assistance of scaffolding technique will get maximum results in a level of thinking as expressed by Anghileri (Angileri, 2006): "Mathematics teaching is informed by the social constructivist paradigm for the teaching-learning process in the which students construct meaning as they Actively Participate in substantial ways Increasingly in the re-enactment of established mathematical practices".

Learning math is formed by a social constructivist to teaching-learning process that students are actively building a significant learning because they feel the substantive involvement in the reenactment of mathematical practice to build their concept. So, teacher with a good level of thinking can facilitate students to discover mathematical concepts significantly. Teachers tend to be impatient in waiting for an answer students so often specific learning that one-way direction communication and there was a teacher only one of learning center. This causes students to become passive and teachers acted arrogantly without seeing the creative side of their opinion (Hudojo, 1988)

Teachers should engage students in understanding mathematical concepts by relating concept of mathematic with the factual material. Most students only remember concept without understanding how is the logical reason from the concept (Krismanto, 2003). Therefore, students often answer questions based on the habits of the general lack of attention to sufficient conditions for a mathematical concept spread in algebra, geometry, and number operations so that contained many misconception that need for discussion about this issue.

Learning that teachers are often concerned with the achievement of the learning materials but override the students in depth understanding of the subject material listed in the syllabus, standard of competence and basic competence. This condition led teachers only focus more on the pursuit of material from the pursuit of students' understanding of the material (Pribadi, 2009).

In addition, it should emphasize students' learning of mathematics to the instructions explicitly and implicitly that correlate constructivist method, and rendering the significant image contained in the book of mathematics, which are related to each other (Hasegawa, 1997). Therefore, the students recalled significantly and do not easily forget the concept given by their teachers.

Data Retrieval

Researchers took Data is taken from teacher who attending at Course on Utilization and Development of IT-based Mathematics Learning for Senior High School on 21 october- 3 November 2015 consist of 30 teacher’s around ASEAN Country. All teacher come from many country who are selected as talent teacher in ASEAN Country.

This research is a qualitative and quantitative description. Steps invention misconceptions teachers to construct beginning math concept carried through has five logical step sequences, namely, (1) Presentation of the phenomenon of questions foundation students often wrong: planning, (2) Preparation of humbug: involve the specific terms of a concept that is rarely discussed by the teacher, (3) to formulate questions: write about the spread in some topics are scattered in algebra, geometry, and number operations, (4) Establish resolution procedures: Create an answer key True-False, (5) Analyze the reasons: an experiment / exploration students how to reason they answered right or wrong answer.

Each teacher is given Table True-False questions about the material that is spread on algebra, geometry, and number operations as follows:

Table 1. Question True –False

NO.	QUESTION	TRUE	FALSE	REASON
1.	If $x^2 = 16$, so a solution is $x = 4$			
2.	If there are two function with formula *f(a)* = *f(b)*, so there is theorem $a = b$			
3.	Form $\left\{\begin{array}{l} y = 3 \\ x = 2 \end{array}\right.$ that equations are system of linear two equation			
4.	Unit for area is cm^2 , is invented by multiplication of cm x cm			
5.	If $f(x) = \sqrt{x}$ and $g(x) = x^2$, so $(gof)(-2) = -2$			
6.	$^{(-2)}\log(-8) = 3$			
7.	A set of solution from: $\left\{\begin{array}{l} x + 2y = 3 \\ 3x + 6y = 9 \end{array}\right.$ Is $\{ \infty \}$			
8.	$2x + 3y = 5xy$			
9.	$\lim\limits_{x \to 0} \dfrac{f(3h + x) - f(3h)}{x} = f'(3h)$			
10.	If two lines are perpendicular, so m_1 x m_2 = -1 (note: *m* is gradient)			

Instruments of True-False questions at the top, are done by theirselves. Then move the answers he got on a letter as agreed by their knowledge and education. Each teacher fill the question on the table so that all members of teacher are in harsh working conditions because is monitored by researcher. Researcher walk around to make sure that teachers do the True-False question independently.

Each member of the teacher take a job fair so that all members of the teacher are in harsh working conditions are monitored by researcher. Researcher go around to make sure that teacher fulfill the task in an orderly manner and independently.

Figure 1. Making an answer on table

Teachers work by to solve problems of True-False that each number consecutively with the. In True-False question can be listened to students enthusiastically and its are open questions that have answers or ways more than one, in which teachers are asked taskbar whether the statement beside it is true or is false and give reasons why they claim is true or false come.

Discussion Result

In the beginning of event, teachers do apersepsi through frequently asked questions about some of the math concepts that they have learned in class X and XI once, and also we convey the scope of activity and item analysis to be achieved at this meeting is to make deepen the concept of early mathematics scattered on algebra, geometry, and number operations.

In addition, in this section the researcher gives motivation to the teachers about the usefulness of studying this material to align the common problems that often arise in our students, the initial construction errors are very basic mathematical concepts. Therefore, students do not do the same wrong thing in the future.

At its core activities phase (1) Presentation of the phenomenon of questions foundation students often they do wrong way: planning, Teachers collaborate with colleagues who teach math class X and XI with a discussion about what is often students make mistakes construction initial drafts scattered on algebra, geometry and number operations. Therefore, didapatlah 10 True-False questions that will be tested to the learners.

In phase (2) Preparation of humbug: it involves special requirements a concept that is rarely discussed by the teacher. Teachers conduct literature studies against multiple sources of textbooks and Student Activity Sheet (LKS). Then each book Math Class X, Class XI and LKS examined the conditions that are often forgotten or misprints to gather information about the procedure can be done to make the matter. Therefore, the matter we can get 10 True-False questions including the detractors.

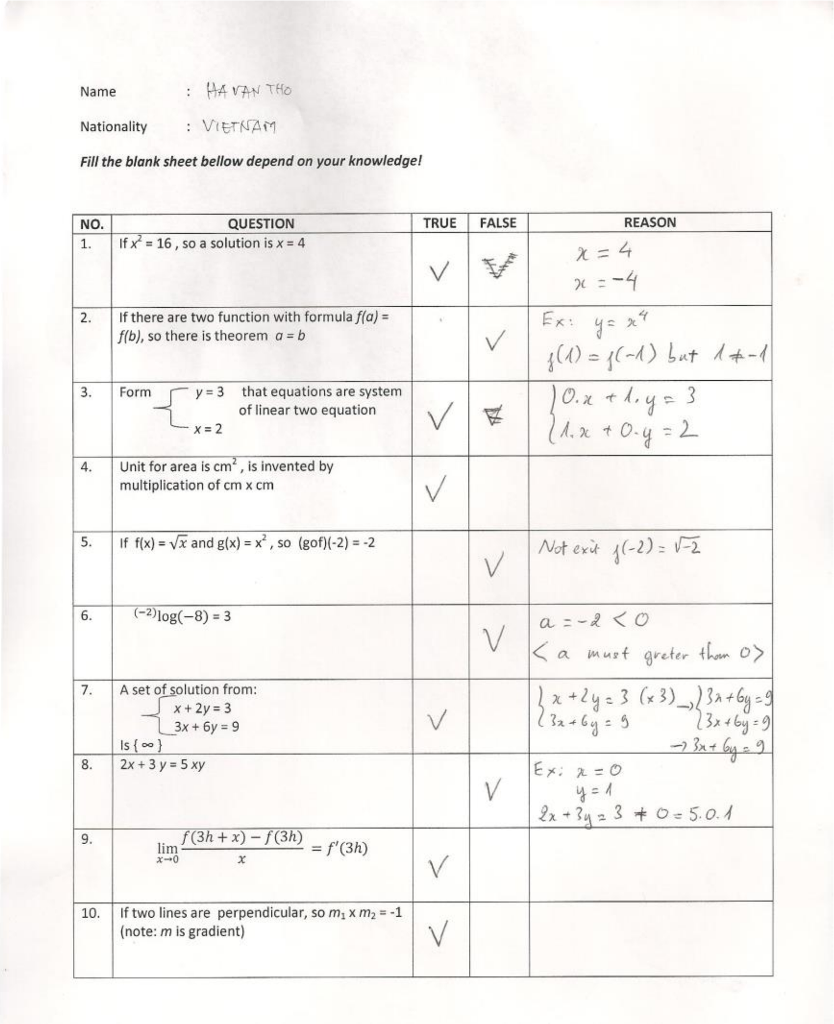
In phase (3) to formulate questions: write about the spread in some topics are scattered in algebra, geometry, and number operations (Look at Table 1) .

In phase (4) Develop procedures for settling: Creating True-False answer keys are as follows:

Table 2. Key Question True –False

NO.	QUESTION	TRUE	FALSE	REASON
1.	If $x^2 = 16$, so a solution is $x = 4$		√	False, Because a solution not only 3 but also -3.
2.	If there are two function with formula *f(a)* = *f(b)*, so there is theorem *a* = *b*		√	False, because there are even fuction for example *f(x)* = x^2 , where f(-2) =f(2) but -2 ≠ 2.
3.	Form $\begin{cases} y = 3 \\ x = 2 \end{cases}$ that equations are system of linear two equation	√		True, because there are two questions that not all coeficient x and y are zero (0).
4.	Unit for area is cm^2 , is invented by multiplication of cm x cm		√	False, because area unit come from 1 unit/ square unit cannot be result of cm x cm, Take attention that area is can be result of area triagle, area perpendicular etc.
5.	If f(x) = $\sqrt{x}$ and g(x) = x^2 , so (gof)(-2) = -2		√	False, because $\sqrt{-2}$ not defined so that cannot be procced any more.
6.	$^{(-2)}\log(-8) = 3$		√	False, because requirement of basis and number which we look for rank must non negative numbers
7.	A set of solution from: $\begin{cases} x + 2y = 3 \\ 3x + 6y = 9 \end{cases}$ Is { ∞ }		√	False, because the answer Infinity and must not enclosed in curly braces, as the information ∞ is not a member of the set.
8.	$2x + 3y = 5xy$		√	False, because x and y expresses a different number, not because x and y something different bodies/ object
9.	$\lim\limits_{x \to 0} \dfrac{f(3h + x) - f(3h)}{x} = f'(3h)$	√		True, because in accordance with the definition of the derivative function, not only f'(x)
10.	If two lines are perpendicular, so m_1 x m_2 = -1 (note: *m* is gradient)		√	False, because there are two lines equations x =0 and y = 0 are mutually perpendicular , but m_1 x m_2 ≠ -1.

The results of one of the group results displayed by learners there are many errors that involve any of initial construction on mathematical concepts with the result that the value range 0-100 is 60. The analysis found that teachers only work as a habit without understanding for the concepts and terms of entry into force of the concept. Picture taken by photographing one of the answers to the following teachers:



Name : HA VAN THO

Nationality : VIETNAM

Fill the blank sheet bellow depend on your knowledge!

NO.	QUESTION	TRUE	FALSE	REASON
1.	If $x^2 = 16$, so a solution is $x = 4$	✓	~~✓~~	$x = 4$ $x = -4$
2.	If there are two function with formula $f(a) = f(b)$, so there is theorem $a = b$		✓	Ex: $y = x^4$ $f(1) = f(-1)$ but $1 \neq -1$
3.	Form $\begin{cases} y = 3 \\ x = 2 \end{cases}$ that equations are system of linear two equation	✓	~~✓~~	$\begin{cases} 0.x + 1.y = 3 \\ 1.x + 0.y = 2 \end{cases}$
4.	Unit for area is cm^2 , is invented by multiplication of cm x cm	✓		
5.	If $f(x) = \sqrt{x}$ and $g(x) = x^2$, so $(gof)(-2) = -2$		✓	Not exit $f(-2) = \sqrt{-2}$
6.	$^{(-2)}\log(-8) = 3$		✓	$a = -2 < 0$ < a must greter than 0>
7.	A set of solution from: $\begin{cases} x + 2y = 3 \\ 3x + 6y = 9 \end{cases}$ Is { ∞ }	✓		$\begin{cases} x + 2y = 3 \ (\times 3) \\ 3x + 6y = 9 \end{cases} \rightarrow \begin{cases} 3x + 6y = 9 \\ 3x + 6y = 9 \end{cases}$ $\rightarrow 3x + 6y = 9$
8.	$2x + 3y = 5xy$		✓	Ex: $x = 0$ $y = 1$ $2x + 3y = 3 \neq 0 = 5.0.1$
9.	$\lim_{x \to 0} \frac{f(3h + x) - f(3h)}{x} = f'(3h)$	✓		
10.	If two lines are perpendicular, so $m_1 \times m_2 = -1$ (note: m is gradient)	✓		

Figure 2. One of the answers of teachers who worked indenpendently

In phase (5) Analyze the reasons: experimental / exploratory students how to reason they answered right or wrong answer.

Data Analysis

The results obtained in the classroom there are a lot of mistakes early construction of mathematical concepts that are described in the following table:

Table 3. Results The True-False Math Problem

No.	Nama	School	Nationality	Score
1	T1	Phan Huy Chu Hight School	Vietnam	60
2	T2	ESGP 1 Maliana	Timor Leste	60
3	T3	Chiangrai Vidhayakome School	Thailand	60
4	T4	Solotsolot national High School	Philippines	40

5	T5	Bacarra National High School	Philippines	40
6	T6	Basic Education High School	Myanmar	50
7	T7	Kolej Vokasional Keningau Sabah	Malaysia	50
8	T8	Teacher of Sekong Secondary School	Lao PDR	80
9	T9	Pothisat High School	Cambodia	70
10	T10	SMAN 1 Pasui , Sulawesi	Indonesia	50
11	T11	SMKN 5 Bandar Lampung	Indonesia	40
12	T12	SMAN 3 Merlung	Indonesia	10
13	T13	SMA Pembina Palembang	Indonesia	70
14	T14	SMAN 4 Wira Bangsa, Aceh	Indonesia	50
15	T15	SMAN 1 Cilacap, Jateng	Indonesia	70
16	T16	Madrasah Mu'allimaat Muh. Yogyakarta	Indonesia	40
17	T17	SMKN 1 Nanggulan, DIY	Indonesia	50
18	T18	SMAN 2 Temanggung	Indonesia	70
19	T19	SMAN Titian Teras H. Abdurrahman	Indonesia	70
20	T20	SMAN 1 Rendang, Bali	Indonesia	40
21	T21	SMKN 1 Kulisusu	Indonesia	40
22	T22	SMAN 2 Kec. Harau	Indonesia	40
23	T23	SMKN 2 Pasuruan	Indonesia	40
24	T24	SMKN 1 Bula	Indonesia	50
25	T25	SMAN 7 Surakarta	Indonesia	50
26	T26	SMAN 1 Wedi	Indonesia	70
27	T27	SMAN 1 Candipuro	Indonesia	40
28	T28	SMAN 6 Prabumulih	Indonesia	40
29	T29	SMKN 3 Banjarmasin	Indonesia	40
			AVERAGE	**51,0345**

On the question no (1) Students are as much as 68.97% of representatives of all teachers answered incorrectly. This shows that teachers fixated on one side only, ie a positive number. They do not focus on the negative numbers so that these activities make teachers more creative in their thingking.

In question No. (2) Students of 41.38 % of teachers representative answering true. This suggests that teachers find it difficult when it is included in the math symbols. They do not have a meaningful learning so that the use of symbols only limited procedural and stored in short-term memory.

On the question no (3) teachers as much as 62.07% of representatives of all groups answered incorrectly. This shows that teachers glued to the prevailing concept in general that equation ax + by = c. They do not focus on the requirements of two systems of linear equations is there are two equations that not all of the coefficients x and y in both zero.

On the question no (4) teachers as much as 37.93% of representatives of all groups answered incorrectly. This shows that teachers glued to the prevailing concept in general that indirectly teacher explained that cm^2 obtained from cm x cm. They do not focus on that area unit derived from one unit or a unit square is not the result of multiplying cm x cm, please note that the vast extent of the bias expressed triangular, rectangular area and others.

On the question no (5) Teachers are as much as 34.48% of representatives of all groups answered incorrectly. This suggests that teachers only focused on results (fog) (x), namely (fog) (x) = x. They do not focus on the numbers below the roots should be non-negative. Therefore, √ (-2) is undefined so that should not be processed again.

In a matter of no (6) teachers as much as 55.17% of representatives of all groups answered incorrectly. This shows that teachers get hung up on the definition of the terms without regard to the logarithm base and number sought his rank, namely the required base and the number who sought his rank should be non-negative numbers. They do not focus on the negative numbers so that these activities make students more extensive or developed in their thingking.

On the question no (7) teachers as much as 31.03% of the representatives of the group answered correctly. But among the true answer turned out to have the wrong reasons, namely the two equations can not be eliminated or substituted. Though the fault is not in situ, but in writing the wrong solution. This activity is useful to write down the correct mathematical symbols as well as the completion of the infinite exists.

On the question no (8) teachers as much as 96.55% of representatives of all teachers answered True. But they still saw that it was just a letter variable is nothing more than that. Teachers should because the x and y expresses a different number, not because x and y something different objects. This indicates that the learning process should be correct.

On the question no (9) teachers as much as 75.86% of the group's representative answered correctly. But among the true answer apparently did not have a reason, and when pressed the answer is "substantially".

In question No. (10) students as much as 6.9% of representatives of all groups answered incorrectly. This shows that students fixated on one side only, the number m. They m_1 x m_2 = -

1. Usually teachers rarely give an example that there is a line perpendicular to that line of $x = 0$ and $y = 0$ are mutually perpendicular, but $m_1 \times m_2 \neq -1$.

Table 4. Results The True-False for each number

No.	Nama	Score	NO.1	NO.2	NO.3	NO.4	NO.5	NO.6	NO.7	NO.8	NO.9	NO.10
1	T1	60	0	1	1	0	1	1	0	1	1	0
2	T2	60	0	1	1	0	1	1	0	1	1	0
3	T3	60	1	1	0	0	1	1	0	1	0	1
4	T4	40	0	0	1	0	0	1	1	1	0	0
5	T5	40	1	0	1	0	1	0	0	1	0	0
6	T6	50	0	0	1	1	1	1	0	0	1	0
7	T7	50	1	0	1	0	1	1	0	1	0	0
8	T8	80	1	1	0	0	1	1	1	1	1	1
9	T9	70	1	1	1	0	1	1	0	1	1	0
10	T10	50	1	0	1	0	0	1	0	1	1	0
11	T11	40	0	0	1	0	0	1	0	1	1	0
12	T12	10	0	0	0	0	0	0	0	1	0	0
13	T13	70	1	1	1	0	0	1	1	1	1	0
14	T14	50	1	0	1	0	0	1	0	1	1	0
15	T15	70	1	1	1	0	0	1	1	1	1	0
16	T16	40	0	1	0	1	0	0	1	1	0	0
17	T17	50	1	0	0	1	0	0	1	1	1	0
18	T18	70	1	1	1	0	1	1	0	1	1	0
19	T19	70	1	1	1	0	0	1	1	1	1	0
20	T20	40	1	0	0	1	0	0	0	1	1	0
21	T21	40	1	0	0	1	0	0	0	1	1	0
22	T22	40	0	0	1	1	0	0	0	1	1	0
23	T23	40	1	0	0	1	0	0	0	1	1	0
24	T24	50	1	1	1	0	0	0	1	1	0	0
25	T25	50	1	0	0	1	0	0	1	1	1	0
26	T26	70	1	1	1	0	1	1	0	1	1	0
27	T27	40	1	0	0	1	0	0	0	1	1	0
28	T28	40	0	0	1	1	0	0	0	1	1	0
29	T29	40	1	0	0	1	0	0	0	1	1	0
		51,0345	**68,97**	**41,38**	**62,07**	**37,93**	**34,48**	**55,17**	**31,03**	**96,55**	**75,86**	**6,9**

Results of interviews with teachers there are a lot of misconceptions, where students only understand the concepts procedurally without understanding factually. In addition, teachers pay less attention to the initial construction of a mathematical concept that many answered incorrectly. It is concluded that this study has been implemented well and get serious attention from the participants.

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Conclusions And Recommendations

This paper is limited only to know how the error occurred construction beginning math concepts in teacher who attending at Course on Utilization and Development of IT-based Mathematics Learning for Senior High School on 21 october- 3 November 2015 for general in several topics are scattered in algebra, geometry, and number operations. It is expected that the analysis of student misconceptions can become a model of teacher development in the regions, with broader support from all stakeholders and observers of education in Indonesia.

We recommend that teachers be trained in advance of the entry into force of a sufficient condition of a concept and explain how the concept of bias occurred. At stages in this study in order to get optimal results, as there is proper adjustment is necessary to discuss with friends, so make the atmosphere of the class to be competitive academically.

Finally, the authors asked for criticism and constructive suggestions to perfect the scientific work and may be useful for researchers to come. Good luck and strive for the benefit of the wider community especially in education.

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Innovation of Mathematics Education through Lesson Study Challenges to Energy Efficiency on STEM on Statistics and Saving Electricity

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Abstract

This collaborative research with the experts aimed to provide necessary scientific and practical knowledge on energy efficiency and statistic topics through integration of energy efficiency, security, and resiliency into vocational schools' mathematics curriculum. As a collaborative research, this research aimed to provide scientific and practical knowledge necessary on energy efficiency and statistic topic through integration of energy efficiency, security and resilience into vocational schools mathematics curriculum. Report on how the teacher develop teaching and learning model on statistics integrated with electricity saving is depicted in this research. The implementation processes of this learning model started with data collection, continued with data processing, data presentation, and data analysis of the electricity billing payment of the students' houses. Through this learning process, the students were expected to be more aware on Saving Electricity problem and uses energy more wisely. The analyses were done qualitatively using triangulation including combining documents, observations, photographs, and videos. The findings revealed that this learning model sharpened the students' ability on reasoning, processing, presenting and analyzing the data from contextual problems. Students can also define the factors affecting the electricity used, and then identify the ways to save electricity used. Further, the students created simple video and poster for saving energy campaign in the classroom, school environment, and social media.

Keywords: *education, innovation, international, mathematics, research*

Introduction

Science and Mathematic teaching emphasizing on reasoning, logical thinking and scientific ability is very important in preparing the young generation in order to deal with the problem in daily life and to address successfully the challenge of globalization era. Globalization era is characterized with the rapid technology development, and the increased energy demand in the world. On the one hand, the development of information technology is very helpful to various life aspects. However, when it is not utilized wisely, its presence will affect negatively the future of young generation. On the other hand, the increased energy demand at global scale is an inevitable problem. The energy demand in economic and industrial sector is currently still met by fossil energy that will be used up soon. Meanwhile, renewing it will take millions years. This condition presents natural disaster, energy crisis, and economic crisis threats that should be dealt with later by young generation in the future. The increased energy demand at global scale can be seen from in the figure below.

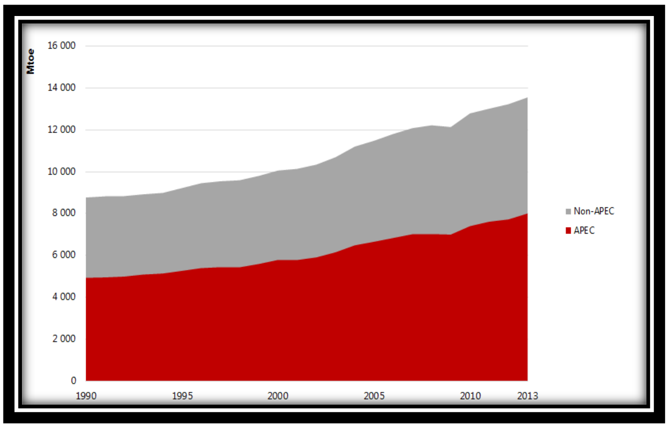


Figure 1. Global energy demand (Ishii 2016)

The diagram above (Ishii, 2016) shows us the comparison between APEC and Non-APEC economies regarding the energy demand. In the 2014 Energy Ministerial Meeting, issues were addressed as follows. “The world’s energy supply and demand pattern has been changing. Global energy demand continues to rise steadily. The Asia-Pacific assumes a more prominent role as the center of world energy demand. At the same time, political environment and economic situation, fluctuations in the energy market, climate change, and public perception and acceptance exert huge impacts on energy policy making in the Asia-Pacific. Moreover, energy costs are crucial to the competitiveness of energy intensive industries in the region (Beijing Declaration - Joining Hands towards Sustainable Energy Development in the Asia-Pacific Region, 2014 Energy Ministerial Meeting). Considering this fact, the globalization challenge requires the present generation to think logically, creatively, innovatively, and skillfully using technology. Life success and sustainability in the future lie on the next generation’s (the students’) hands. For that reasons, it is important for education realm to make innovation to prepare the young generation for such the challenge and threat, particularly related to Energy Efficiency.

Teachers, through their contribution in education realm, are very important to prepare the young generations by equipping them with science, skill, life competency, and character education. Care about natural, social environments and society should be developed in education. Global issues concerning natural disaster, energy crisis, and economic crisis threats should be introduced earlier in education realm. The students should be accustomed to think critically and logically, to reason and to innovate in a response to the natural disaster, energy crisis, and economic crisis threat issues. National Council of Teachers of Mathematics (NCTM, 2000:11) suggests that student’s must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. Meaningful learning should refer to process standard, by emphasizing on reasoning and sense making (NCTM, 2009:5). That means that every learning process should be related to the process standard that should be supported by infrastructures, learning media, and contextual issues addressed in learning. A learning process will be less meaningful if the learning media emphasizes only on the procedural skills. The lesson will be easily forgotten by the students and potentially lead to the misconception and difficulty in learning process (Kilpatrick, Swafford, & Findell, 2001: 123).

The integration of science, technology, engineering and mathematics (STEM) or integrated thematic learning is very possible to do to design innovative learning by raising energy efficiency, security and resiliency issues. In this lesson study, research activity focused on STEM, with an understanding that without practice in real life, it is difficult for the students to understand naturally the saving-energy issue raised in this learning (Wang, Moore, Roehrig, & Park, 2011). For that reasons, the writer is interested in making innovation in mathematic learning by raising saving-electricity energy in STEM and in joining the Lesson Study team established by Seameo Qitep in Mathematics (SeaQim).

The STEM content areas are taught as though they were one subject. Integration can be done with a minimum of two disciplines but is not limited to two disciplines. The lines indicate the various options in which integration could be achieved.

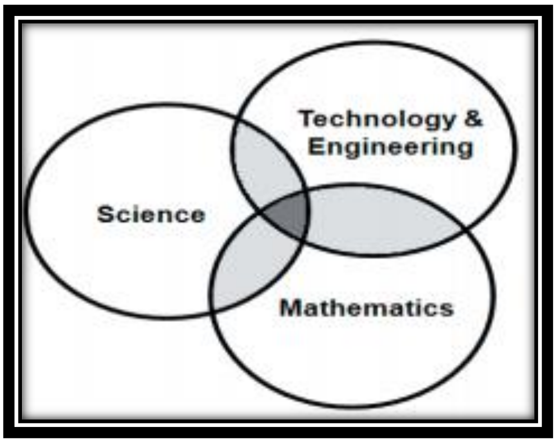


Figure 2. Integration approach to STEM education.

Wang et al. (2011) explain *interdisciplinary integration* begins with a real-world problem. It incorporates cross-curricular content with critical thinking, problem-solving skills, and knowledge in order to reach a conclusion. M*ultidisciplinary integration* asks students to link content from specific subjects, but *interdisciplinary integration* focuses students' attention on a problem and incorporates content and skills from a variety of fields. The real problem in this saving-electricity statistic learning is whether the electricity usage in the student families tends to be above/below the mean electricity use of student family in one classroom in several last months. The students are told to estimate its cause. In this problem, mathematics is integrated into Physics related to Energy material. Energy material studied includes definition, types of energy, renewable energy, and non-renewable energy. Meanwhile, the statistics involve describing and presenting data in displaying data in the form of appropriate table or chart or plot to communicate information of a data series through comparative analysis on varying data displays. The students are allowed to use both Excel program and calculator to facilitate the calculation, so that the time is spent more on reasoning and analyzing data. As suggested by Afrial, & Rohmah (2014), using Ecel can enhance understanding of content within a graphic presentation; it provides a visual representation of data that makes it easier to analyze. In addition, excel reduces the difficulty of plotting data and allow students a means for interpreting the data.

Sharing and growing together in lesson study group with themed energy efficiency, security and resiliency is, of course, very beneficial and provides new experience to students, teacher and school where the research is conducted. Teachers, before joining this activity, have attempted to connect the learning material to saving-energy theme, despite less optimality. This activity helps open the teachers' insight in order to explore further. In addition, it encourages them to share and collaborate with each other. They can be open to constructive critique and suggestion to accomplish the developed learning plan. This activity gives the students the more contextual experience during learning process. The students formerly not thinking of saving energy become care more about and more motivated to use energy more wisely. In addition, the students feel more benefit of mathematics in daily life. This learning development is related to collecting, processing, displaying, and analyzing data of student families' personal electricity payment receipt.

The objectives of research through Lesson study are generally to: (1) increase the awareness students on the issue of energy efficiency in real life, (2)facilitate students with the scientific and practical knowledge regarding energy efficiency and related skills to reduce or minimize the use of energy,(3) increase students' beliefs on the importance of mathematics, science, and technology that will motivate them having good attitudes toward mathematics, science, and technology, (4) increase the awareness students to predict and anticipate the energy efficiency issue.

METHOD

This research employed a descriptive qualitative method. The objective of research was to develop an integrated learning in STEM (Science, Technology, Engineering and Mathematics) through Lesson Study. This research focused on the material of statistics related efficiency energy.

The questions riset can be aroused among others are: (1) how to help students to learn *statistics related efficiency energy* meaningfully, (2) how to help students to learn statistics related efficiency energy joyfully, (3) how to help students to learn to think, and, (4) how to help students to learn statistics related efficiency energy by/for themselves or to be independent learners?

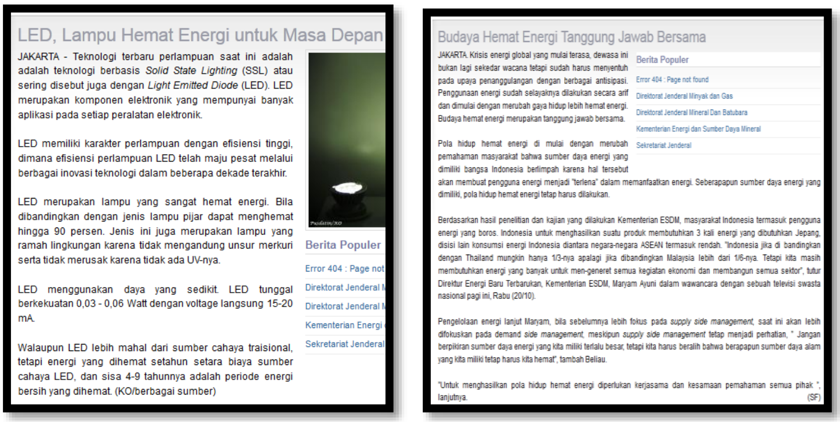
Overall, the procedure of lesson study used was Plan Do-See. Plan involves: Seeing a curriculum based on the theme of energy, Choose the material that matches the theme of energy, Developing lesson plans, Peer teaching, Revising the lesson plans. Do involves: Scheduling, Discussions with partner teachers as well as observers, Implementation of the lesson plan 1 – reflection, Implementation of the lesson plan 2 – reflection, Implementation of the lesson plan 3 – reflection, Collecting data. See involves: The reflection with observers from SEAQiM and teachers in SMK N 2 Wonosari, and Evaluation.

The cycle passed through in this research involves: (1) designing and developing learning activities, (2) Peered teaching, (3) Field experiment, (4) observation, (5) reflection, (6) report, and (7) dissemination.

RESULT AND DISCUSSION

Lesson Study Activity was initiated with workshop on learning set development. This workshop was held in seameo Qitep in Matematics. After the learning set has been developed completely, it was continued with peer teaching, including presentation and teaching practice based on the set developed. This activity is conducted to elicit constructive critique, suggestion, and input to make the developed set better and ready to apply. The next stage was the implementation in the classroom. Teacher, as the author, conducted the activity corresponding to the revised learning plan. Then, observation and reflection are conducted by Qitep in Mathematics. This activity is intended to evaluate and to analyze the result of classroom practice, to be reported and disseminated later.

The Mathematic learning implemented was related to basic statistic material involving describing and displaying data in the form of appropriate table, chart or plot to communicate information of data series through comparative analysis on varying data displays. After taking pre-test, the students were asked to submit their families' electricity and water billing, and to record the fuel use for their personal motorcycle to go to and to go back from school. In addition, the students were assigned to browse news or articles related to efficiency, resilience and saving energy. Those articles would be beneficial to give insight into efficiency, resilience and saving energy. Here are some articles browsed by students.



LED, Lampu Hemat Energi untuk Masa Depan

JAKARTA - Teknologi terbaru perlampuan saat ini adalah adalah teknologi berbasis *Solid State Lighting* (SSL) atau sering disebut juga dengan *Light Emitted Diode* (LED). LED merupakan komponen elektronik yang mempunyai banyak aplikasi pada setiap peralatan elektronik.

LED memiliki karakter perlampuan dengan efisiensi tinggi, dimana efisiensi perlampuan LED telah maju pesat melalui berbagai inovasi teknologi dalam beberapa dekade terakhir.

LED merupakan lampu yang sangat hemat energi. Bila dibandingkan dengan jenis lampu pijar dapat menghemat hingga 90 persen. Jenis ini juga merupakan lampu yang ramah lingkungan karena tidak mengandung unsur merkuri serta tidak merusak karena tidak ada UV-nya.

LED menggunakan daya yang sedikit. LED tunggal berkekuatan 0,03 - 0,06 Watt dengan voltage langsung 15-20 mA.

Walaupun LED lebih mahal dari sumber cahaya traisional, tetapi energi yang dihemat setahun setara biaya sumber cahaya LED, dan sisa 4-9 tahunnya adalah periode energi bersih yang dihemat. (KO/berbagai sumber)

Berita Populer
Error 404 : Page not
Direktorat Jenderal
Direktorat Jenderal
Kementerian Energi
Sekretariat Jenderal

Budaya Hemat Energi Tanggung Jawab Bersama

JAKARTA. Krisis energi global yang mulai terasa, dewasa ini bukan lagi sekedar wacana tetapi sudah harus menyentuh pada upaya penanggulangan dengan berbagai antisipasi. Penggunaan energi sudah selayaknya dilakukan secara arif dan dimulai dengan merubah gaya hidup lebih hemat energi. Budaya hemat energi merupakan tanggung jawab bersama.

Pola hidup hemat energi di mulai dengan merubah pemahaman masyarakat bahwa sumber daya energi yang dimiliki bangsa Indonesia berlimpah karena hal tersebut akan membuat pengguna energi menjadi "terlena" dalam memanfaatkan energi. Seberapapun sumber daya energi yang dimiliki, pola hidup hemat energi tetap harus dilakukan.

Berdasarkan hasil penelitian dan kajian yang dilakukan Kementerian ESDM, masyarakat Indonesia termasuk pengguna energi yang boros. Indonesia untuk menghasilkan suatu produk membutuhkan 3 kali energi yang dibutuhkan Jepang, disisi lain konsumsi energi Indonesia diantara negara-negara ASEAN termasuk rendah. "Indonesia jika di bandingkan dengan Thailand mungkin hanya 1/3-nya apalagi jika dibandingkan Malaysia lebih dari 1/6-nya. Tetapi kita masih membutuhkan energi yang banyak untuk men-generet semua kegiatan ekonomi dan membangun semua sektor", tutur Direktur Energi Baru Terbarukan, Kementerian ESDM, Maryam Ayuni dalam wawancara dengan sebuah televisi swasta nasional pagi ini, Rabu (20/10).

Pengelolaan energi lanjut Maryam, bila sebelumnya lebih fokus pada *supply side management*, saat ini akan lebih difokuskan pada demand *side management*, meskipun *supply side management* tetap menjadi perhatian, " Jangan berpikiran sumber daya energi yang kita miliki terlalu besar, tetapi kita harus beralih bahwa berapapun sumber daya alam yang kita miliki tetap harus kita hemat", tambah Beliau.

"Untuk menghasilkan pola hidup hemat energi diperlukan kerjasama dan kesamaan pemahaman semua pihak ", lanjutnya. (SF)

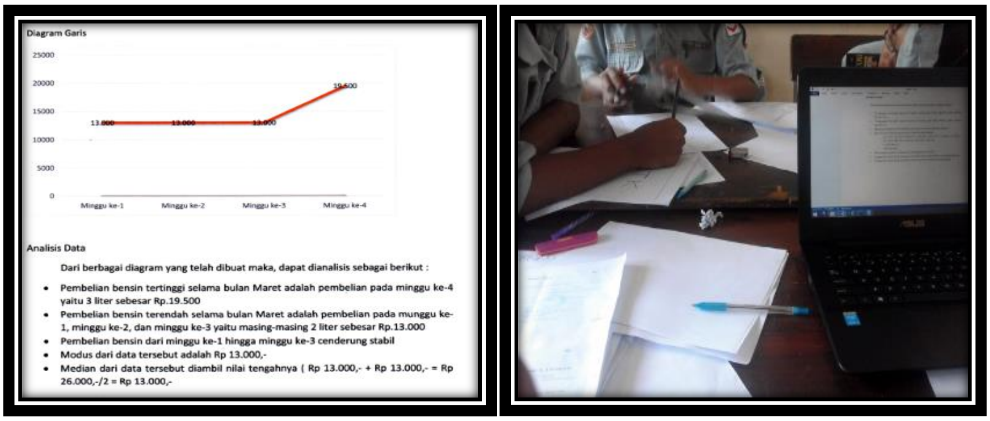
Berita Populer
Error 404 : Page not found
Direktorat Jenderal Minyak dan Gas
Direktorat Jenderal Mineral Dan Batubara
Kementerian Energi dan Sumber Daya Mineral
Sekretariat Jenderal

Figure 3. Reading Source Article
Source: http://www.esdm.go.id/berita

In the first meeting, as an opening activity, a student is voluntarily asked to reveal his/her perspective/thinking about article/news on efficiency, resilience and saving energy they obtain. Furthermore, the students were told to observe a set of electricity billings they carry with them. Teacher told the students to recall the statistic material they had ever learnt in Junior High School. Then, the students were stimulated with the following questions: "What can be done to process and to display data of electricity billing set?" and what can we analyze and conclude from the activity of processing and displaying the data of personal electricity billing?". Some students answered that the data can be ordered and displayed in the form of table and chart. The data can also be analyzed and process to see mean, median, mode, highest data, lowest data, and etc.

In the main (core) activity, teacher gave the students the freedom of displaying and analyzing the data of electricity billing, and connecting it to information on efficiency, resilience, and saving-energy. The students were allowed to use excel program, calculator, and other aids.

In the first meeting, in 2 x 45 minutes duration: (1) the students discuss the group to collect, to process, to display, and to analyze the data of their family electricity payment receipt, (2) the students were allowed to count manually, or using calculator, excel and other programs/aids, and (3) the students presented their work voluntarily.



Analisis Data

Dari berbagai diagram yang telah dibuat maka, dapat dianalisis sebagai berikut :

- Pembelian bensin tertinggi selama bulan Maret adalah pembelian pada minggu ke-4 yaitu 3 liter sebesar Rp.19.500
- Pembelian bensin terendah selama bulan Maret adalah pembelian pada munggu ke-1, minggu ke-2, dan minggu ke-3 yaitu masing-masing 2 liter sebesar Rp.13.000
- Pembelian bensin dari minggu ke-1 hingga minggu ke-3 cenderung stabil
- Modus dari data tersebut adalah Rp 13.000,-
- Median dari data tersebut diambil nilai tengahnya (Rp 13.000,- + Rp 13.000,- = Rp 26.000,-/2 = Rp 13.000,-

Figure 4. Documentation of the 1st meeting

In the second meeting, in 2 x 45 minutes duration: (1) the students discussed in group to compare and to analyze the data of mean monthly family electricity need and the mean family electricity need of entire class, (2) the students were allowed to count manually or to use calculator, excel, and other programs/aids, and (3) the students presented their work voluntarily.

Figure 5. Documentation of the 2nd meeting

In the 3rd meeting in duration 2 x 45 minutes: (1) the students made saving-electricity campaign in the classroom using poster/brochure/simple video constituting the product of their group work at home, and (2) the students delivered their plan of saving-electricity campaign at home, in school environment and social media they have.

In closing activity, the students gave critique, input and suggestion (recomendation) to other groups. Then, teacher gave reinforcement (confirmation) and drew a conclusion along with the students from the statistic and saving electricity- related activity. The activity ended by facilitating the students to reveal the benefit of statistic learning with saving-electricity theme.

After the learning activity has been completed, posttest and reflection activity were conducted. The posttest questions given were as same as the pre-test one in the beginning of activity, related to energy efficiency in real life. From the data of pretest, it can be found that belief and awareness of the students on the issue of energy efficiency in real live is 7.71 (medium), and already increases to 10.32 (high) on post test. The percentage increase from pretest to posttest is 33.9%. The detail of analysis is presented in the table 1 below.

Table 1. Percentage Belief and Awareness of the Students on The Issue of Energy Efficiency In Real Live

Rubric	Notes	Pre Test	Post Test
1	To help increase the awareness of Indonesian students on the issue of energy efficiency in real live	62%	67%
2	To help and facilitate Indonesian students with the scientific and practical knowledge regarding energy efficiency and related skills to reduce or minimize the use of energy	66%	66%
3	To help increase Indonesian students' beliefs on the importance of mathematics, science, and technology that will motivate them having good attitudes toward mathematics, science, and technology	79%	98%
4	To help increase Indonesian students' beliefs on the importance of mathematics, science, and technology that will motivate them having good attitudes toward mathematics, science, and technology	24%	73%

REFLECTION

The strength of this research is, among others, that the theme of statistics is consistent with the 2013 curriculum of old spectrum for the 10th grade of Vocational Middle School with material profundity including: "*displaying data actually in the form of certain table and chart/plot consistent with the information to be communicated*'. The students are expected to display the data in the form of table and diagram (chart), and to analyze and to communicate it. The contextual issue raised is Saving Electricity. The students were asked to process and to analyze the data of their families' electricity use. Learning activity emphasizes on reasoning and the contextual problem solving is more meaningful to the students, compared with the activity focusing only on sharpening mathematic skill with problem drill. In addition, this learning activity utilizes the technology the students have mastered, using excel program or using calculator to process and to display data. It facilitates the students to explore the problem by reasoning.

This learning activity also utilizes the students' ability of preparing simple poster and video. The students' ability of using technology should be directed and accustomed to develop the positive things. In this research, the students are told to prepare simple poster and video for campaigning Saving Electricity, Water, and Fuel in the classroom, school environment, and society, either directly or indirectly. Indirect campaign can be carried out using social media. Through this activity, the students are expected to be accustomed to using technology and social media wisely and positively.

In learning process, there is an activity of analyzing the factors affecting the volume of energy use; it is very good to the students to reason and to reflect on daily activity in the student family related to habit and decision in using energy. Through discussing and arguing corresponding to the students' experience, the students are expected to use energy more wisely.

On the other hand, this research has some weaknesses as well, one of which is that learning has started with contextual problem, but the packaging of student worksheet is still less good so that the contextual problem touches less. This learning is more meaningful and joyful, invites the students to think and to be independent compared with previous statistic learning conducted by the author, but it still less perfect and more improvement is required. In the beginning of learning, the discussion concerning energy should be conducted in not too long time.

Teacher has given stimulus question when starting the lesson: "*What can we do to process this data based on your previous experience*?" It is good; but the teacher does not need to emphasize or to direct the students to calculate immediately or to calculate the mean only. The students should be given discretion to determine what will be done. Teacher does not give impose limit/standard to conclude whether electricity use belongs to high or low category. Teacher should raise the error one group makes in drawing line chart, to be the deeper source of exploration. However, it is better for entire group to present its work. Or if possible, Presentation Gallery can also be used to display and to discuss all of jobs.

The most appropriate technology integration is the one using Excel. But, in one activity, Excel has not been utilized optimally yet. Teacher should position himself/herself or serve as facilitator. The closing activity should be emphasized on what can be developed after learning this material.

CONCLUSION AND RECOMMENDATION

This lesson study activity is really beneficial to teachers, students, and schools joining it and the school surrounding. From this activity, the following conclusions can be drawn: (1) It is very important to start the learning with contextual problem, to make the students understand more about the benefit of mathematic learning, (2) The main activity of learning should start by giving the students the discretion to reason and to solve the problem using the knowledge they have, thereby the learning is more meaningful to the students because the students construct their own knowledge, (3) The appropriate media choice highly supports the successful learning, in this case Excel utilization to process and to present the result of data analysis, (4) The students' IT skill (e.g. video/poster/social media) can be used to improve their communicating skill and positive character development, (5) Teacher should actually serve as facilitator, focusing on the students' activity to make them independent and creative, (6) In closing activity, teacher should give feedback and reinforcement, and emphasize on what the students can develop by learning this material.

Considering the conclusion above, the teacher or the author interested in developing this material are recommended to take the following into account: (1) Varied contextual problem design with open-ended question. Thus, the students can determine their own way of solving the problem. This design allows for the varying solution corresponding to the students' experience and creativity, (2) In data collection process, the students should attempt to make an experiment related to energy efficiency, so that the learning process will be more meaningful and impacts on the change of energy saving habit.

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The Effect Of Inquiry Training Models Using Lectora And Formal Thinking Ability Toward Students Achievement

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Abstract

This paper presents the results of researching the use of lectora multimedia as innovation in science education teaching and learning toward measurement knowledge of students when applied to inquiry training instructional models and formal thinking ability in Harapan Senior High School Academic Year 2014/2015 in grade X. The research aims : (1) to explain the students achievement that applied inquiry training instructional models based on lectora multimedia better than conventional instructional models, (2) to explain the students achievement that had high formal thinking ability better than students that have low formal thinking ability, (3) to explain the interaction between inquiry training instructional models based on lectora multimedia and formal thinking ability toward students achievement. The instruments used in this research are formal thinking ability instrument consist of 10 questions and student achievement instrument consist of 20 questions stated valid and reliable. To test the hypothesis used analysis of variance two tails (2 x 2 factorial design) with a significant level $\alpha = 0.05$. The results of research are : (1) the students achievement that applied inquiry training instructional models based on lectora multimedia better than conventional instructional models; (2) the students achievement that have a high formal thinking ability better than students that have low formal thinking ability, (3) there was interaction between inquiry training learning models based on lectora multimedia and formal thinking ability toward students achievement.

Keywords: *innovation, education, lectora, research*

INTRODUCTION

The 21st century is the century of science and technology, therefore we are faced with a situation where science and technology is changing and evolving very rapidly. To deal with these changes very rapidly then the required human resources (HR), which has high competitiveness, creative, reliable and quality to be able to handle the various possible solutions to a problem and change. But the expected demands of human resources do not match the reality. According to the data version of the UNDP Human Development Report in 2013, level HDI (Human Development Index) of Indonesia is on ranked 121 of 187 countries. Meanwhile the results of votes Trends in International Mathematics and Science Study (TIMSS), an international study which is held every four year in order to know the development of the ability of science to students in various countries, in 2011 put Indonesia level 38 from 42 countries. This indicates that the quality of human resources and the ability of Mathematics and Sscience students in Indonesia is still far below the average of other countries.

Omosewo (1999) stated the several factors that causes low students achievement: the limited number of professionally trained teachers, inadequate laboratory facilities, poor background of science students are currently learning ability in middle level. Various efforts have been made by the government to find solution to low learning outcomes such as by changes in the curriculum, facilities and infrastructure, improving the quality of teachers, and the renewal of the learning process. But has not shown results satisfactory because until now the results of studying math in high school students are still low. This indicates that these efforts have not been able to improve student learning outcomes. Students still have not been able to apply the concepts well understood form of knowledge, skills and attitudes in a real situation.

Based on preliminary study in Harapan High School performed in Medan, data showed the average value of class X student from the second semester of 2011/2012 was 53.62, year of 2012/2013 was 56.86 and years of 2013/2014 was 62.14 . The average value are still below minimum completeness criteria (KKM) where the KKM 's is 75.00. It makes a lot of students have to undertake remedial program to increase their value. In addition, based the results of a questionnaire distributed among 40 students found that as many as 70.0 % of students do not like math, 12.5 % stated that math was mediocre and only 17.5 % of students stating liked math. This is due to the many formulas that further highlight the form of a math equation rather than the concept should be applied. As a result, students difficulties in resolving issues related to science.

In the use of computer technology, as much as 100 % of the students understand how to operate a computer program for as many as 95 % of students have a computer/notebook at home and the rest understand computer programs because there are subjects computer in their school and also played notebook from their friends who sometimes bring notebook to school. Then many students who also have a ipad or tablet. However, the use of multimedia is also rare because many teachers do not understand how to create a formula or equations using the equation on toolsbar. In fact, teachers are the spearhead that affect the quality of the learning in class. Students also lacked confidence in his friends to shows the results of the experiments conducted so that 100 % of students never presented the results of his work in front of their friends. This is what makes unheard of scientific debate among students.

Teachers should also consider the different of formal reasoning with each student. This can be performed well when information about the formal reasoning of students already owned a teacher. This statement supported by a research conducted by Amien (in Eka, 2005) showed that formal thinking can be measured on secondary high school students. Through the class (and by age) there is a significant increase in terms of how the 15 year olds have a level think formal higher than children aged 14 years. It was also found differences in terms of gender, place of residence (urban and rural), and the education level of the family. The urban environment and the family environment that parents have higher education more stimulating children in developing formal reasoning.

According to Piaget in (Corebima, 2013), the development of formal reasoning is essential to understand the concepts for conceptual knowledge because it is a constructive process and reasoning are a necessary tool in the process. Furthermore, Piaget in Day (1981) stated that a child formal operational level is able to construct and concluded a hypothesis against certain phenomena and incorporate systematically a set of elements to create every possible combination and control the variables. This opinion is relevant to the purpose of inquiry learning model training. According Suchman (2007) inquiry can be applied to various ways such as observing nature, estimates the condition that would happen, manipulate variables, analyze the situation and give a statement. In addition, through research conducted by Ali (in Eka, 2005) found that the formal thinking ability positively correlated with the formal achievement either individually or in together, though do the control of the creativity variable and achievement motivation. This suggestion showed that the ability to think formal consistently positively correlated with the results achievement. It also means if students have high formal thinking ability, it will increase achievement of the students.

To repair the quality of processes and achievement of the students need a serious effort, one of them by applying the inquiry training learning which this learning can help to understand the concepts and solve problems of learning. According to Joyce, et al (2003) this learning is focused on the students' ability to observe, collate the data, understand the information, forming concepts, using symbols of verbal and nonverbal and solve the problems. The views of these research results also indicate that the inquiry training is proven to improve students achievement. According to Pandey, et al (2011) the inquiry training is more effective than learning by using conventional learning. Similar statement also delivered by Khalid & Azeem (2012) which states that the inquiry training that gave by the teachers can help achievement of the students where students can formulate and test their ideas, draw conclusions and pass on their knowledge in a collaborative learning environment.

Learn by using inquiry training model can also be applied using multimedia technology. According Gillani (2010) the development of web and hypermedia make opinions on inquiry learning training can be implemented by using technology in different conditions. According to Hayati (2013) stated there is significant effect by using inquiry training based on multimedia toward student achievement. In addition, research that conducted by Nurhafni (2010) stated the achievement taught using multimedia more higher than the achievement taught without using multimedia. The results of the same research also showed by Fatmi (2013) which states that there is significant effect of using multimedia in inquiry toward students achievement.

According Ismaniati (2010) the process and learning resources that can be designed and developed by learning technology based on student characteristics and is based on with learning theories can definitely will become more qualified because the learning process of each student will get optimum services according to their characteristics. So that students will be more active, more fun and easier to learn. Based on the description of the background issues and the above theory, the formulation of the problem with this research: (1) the result of comparing students learning outcomes when use inquiry training instructional models than use conventional instructional models in Harapan Senior High School Academic Year 2014/2015 in the measurement material ? (2) the result of student that have high formal thinking ability with students that have low formal thinking ability in Harapan Senior High School Academic Year 2014/2015 in the measurement material? (3) the interaction between inquiry training instructional models based on lectora multimedia and formal thinking ability toward students achievement in Harapan Senior High School Academic Year 2014/2015 in the measurement material?

The purpose of this study: (1) to determine students achievement that more better between using inquiry training instructional models based on lectora multimedia compared with students using conventional instructional models in Harapan Senior High School Academic Year 2014/2015 in the measurement material, (2) to determine students achievement that more better between student who have high formal thinking ability with students who have low formal thinking ability in Harapan Senior High School Academic Year 2014/2015 in the measurement material, (3) to determine the interaction between inquiry training instructional models based on multimedia lectora and formal thinking ability toward students achievement in Harapan Senior High School Academic Year 2014/2015 in the measurement material.

RESEARCH METHODS

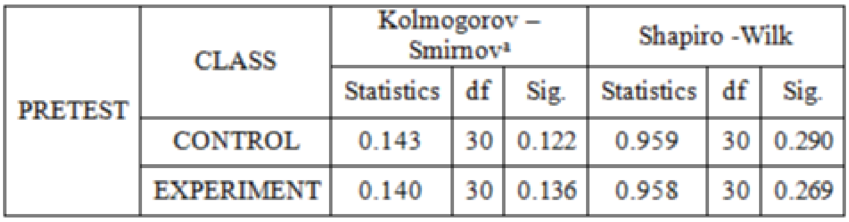
This research was conducted in Harapan Senior High School of Medan grade X in first period (first semester) on September Academic Year 2014/2015. The population of this study consist of all the students grade X in Harapan Senior High School of Medan. The sample of this study divide by two classes, namely control class and experimental class. The study involved two classes given a different treatment, for class control using conventional instructional models and experimental class using inquiry learning instructional models based on lectora multimedia. Research designs is ANOVA 2 x 2. To test the hypothesis of this research used analysis techniques data by analysis of variance (ANOVA) two tails (2 x 2 factorial design) with a significant level $\alpha = 0.05$ or 5%.

RESULTS AND DISCUSSION

Results

Description of the data onto the results of this study consists of scores of cognitive achievement and formal thinking skills. Pretest scores with using inquiry training instructional models based on lectora in the experiment class obtained the average value 5,10 and conventional instructional models obtained the average value 4,97. Testing by using SPSS 20.0 with samples free *t* test. Then both the data were tested for normality and homogeneity in advance. The normality test is shown in Table 1.

Table 1. The Normality Test

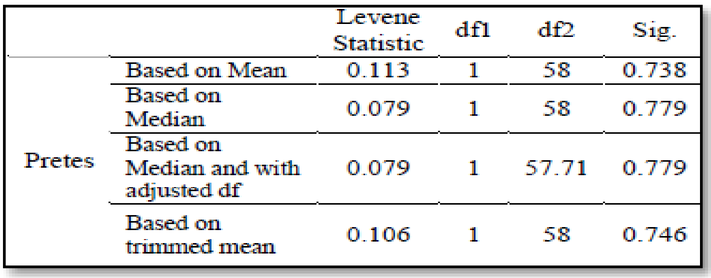


PRETEST	CLASS	Kolmogorov – Smirnov^a			Shapiro -Wilk		
		Statistics	df	Sig.	Statistics	df	Sig.
	CONTROL	0.143	30	0.122	0.959	30	0.290
	EXPERIMENT	0.140	30	0.136	0.958	30	0.269

Based on *kolmogrof-Smirnov*^{a} obtained value the significance of the results of experiment class is 0.122. These results indicate that *kolmogrof-Smirnov*^{a} greater than 0.05, then data on the experiment class is normal distributed

The significant value of classroom learning outcomes Control of 0.136. These results show that *kolmogrof-Smirnov*^{a} greater than 0.05, then the data onto the control class is normal distributed. Test for equality of variance and the average the value pretest is done with a Test of Homogenety of Variance using SPSS 20.0 with the test results in Table 2.

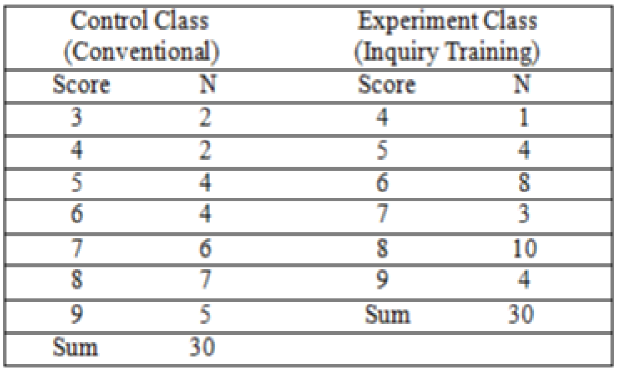
Table 2. Test of Homogenety of Variance



		Levene Statistic	df1	df2	Sig.
Pretes	Based on Mean	0.113	1	58	0.738
	Based on Median	0.079	1	58	0.779
	Based on Median and with adjusted df	0.079	1	57.71	0.779
	Based on trimmed mean	0.106	1	58	0.746

The test results are contained in Table 2. shows the value of F for the pre-test learning outcomes of significance 0.113 < 0.738 (F table = 4.12 , α = 0.05). Based on the results, the value of F < F table and significant count greater than α = 0.05, so data can be concluded pretest data of control class and experimental class have the same variance or homogeneous. Formal thinking ability tests given the control class (conventional) and class experiment (inquiry training) can be seen in Table 3.

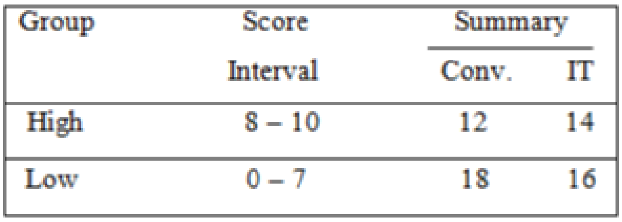
Table 3. Data of Formal Thinking Ability Control and Experiment Classroom



Control Class (Conventional)		Experiment Class (Inquiry Training)	
Score	N	Score	N
3	2	4	1
4	2	5	4
5	4	6	8
6	4	7	3
7	6	8	10
8	7	9	4
9	5	Sum	30
Sum	30		

By using SPSS 20.0, the data formal thinking ability of two classes then group into 2 groups each group of high (score interval 8 – 10) and low (score interval 0 – 7) seen in Table 4.

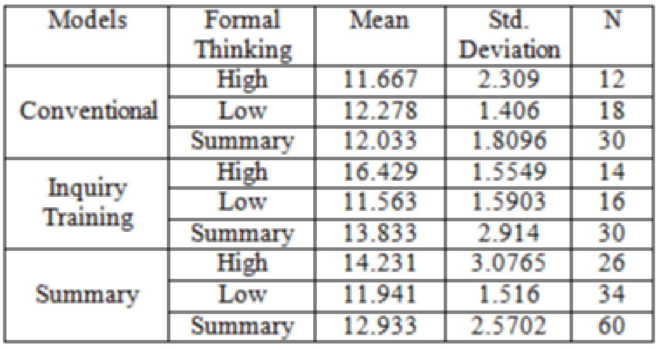
Table 4. Distribution of Formal Thinking Group High and Low Based on Interval



Group	Score Interval	Summary	
		Conv.	IT
High	8 – 10	12	14
Low	0 – 7	18	16

The average result of students achievement by using conventional instructional models for the students that have high formal thinking ability obtained 11.67 and student that have low formal thinking ability obtained 12.28, whereas average results by using inquiry training instructional models based on lectora multimedia for groups of students that have high formal thinking ability obtained 16.42 and for groups of students who have low formal thinking ability obtained 11.56. The data presented in Table 5.

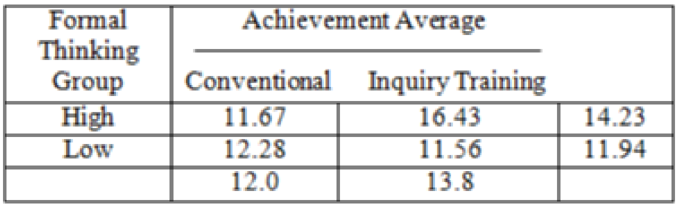
Table 5. Statistics of ANOVA



Models	Formal Thinking	Mean	Std. Deviation	N
Conventional	High	11.667	2.309	12
	Low	12.278	1.406	18
	Summary	12.033	1.8096	30
Inquiry Training	High	16.429	1.5549	14
	Low	11.563	1.5903	16
	Summary	13.833	2.914	30
Summary	High	14.231	3.0765	26
	Low	11.941	1.516	34
	Summary	12.933	2.5702	60

The Anova 2x2 design data onto the average students achievement against high and low formal thinking ability presented in Table 6.

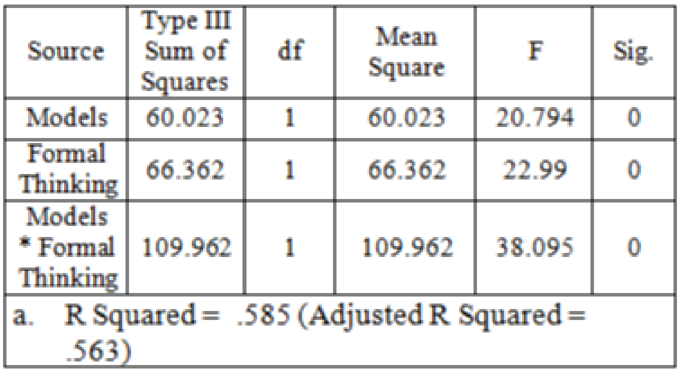
Table 6. Design of ANOVA Data 2x2



Formal Thinking Group	Achievement Average		
	Conventional	Inquiry Training	
High	11.67	16.43	14.23
Low	12.28	11.56	11.94
	12.0	13.8	

Output 2x2 ANOVA test results of research can be seen in Table 7.

Table 7. Output Calculation ANOVA 2x2



Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Models	60.023	1	60.023	20.794	0
Formal Thinking	66.362	1	66.362	22.99	0
Models * Formal Thinking	109.962	1	109.962	38.095	0
a. R Squared = .585 (Adjusted R Squared = .563)					

Based on Table 7, the calculations based on the significantly models obtained significant results 0.00 and this significantly smaller than α = 0.05. So there are significant differences in students achievement between using inquiry training instructional models based on lectora multimedia compared with students who used the conventional instructional models in Harapan Senior High School Academic Year 2014/2015. The calculations based on the significantly formal thinking obtained 0.00 and this significantly smaller than α = 0.05. Then there are differences students achievement between students who have high formal thinking skills with students who have low formal thinking ability in Harapan Senior High School Academic Year 2014/2015. The calculations based on In a significant models* derived formal thinking obtained 0.00 and this significantly smaller than the significant α = 0.05. The interaction between inquiry training instructional models based on multimedia lectora and formal thinking ability toward students achievement in Harapan Senior High School Academic Year 2014/2015 can be seen in Figure 1.

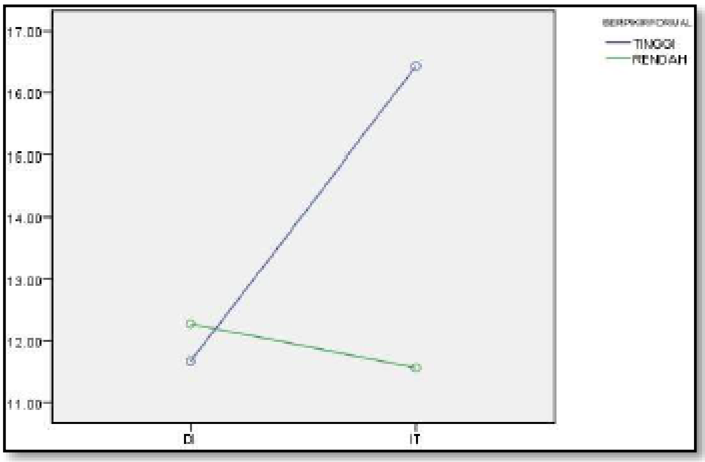


Figure 1. Graph Interaction Model Learning and Thinking Formal

Graph of high formal thinking ability and low formal thinking ability on inquiry training instructional models based lectora multimedia and conventional instructional models intersect at one point. The intersection shows the interaction between both of the models and formal thinking ability toward students achievement.

Discussion

Learning activities such as questions or problems, formulating hypotheses, collecting and analyzing data and concluded performed by students. Students are active in conducting such experiments to investigate the characteristics of the measuring instrument. Students who presented the problems of measurement able to identify what is the right instrument for measuring an object geometry, how to read the measurement results, identify errors in measurement and formulate a response while on the issues presented. The students carefully to measure the objects and do the measurement results correctly. The data were analyzed by a question and answer in the group. The students also asked to the teacher about the results that they conclude.

In digital eras, the development of information technology is fabulous especially in the field of e-learning. It has become a necessity to study independently and quickly manage many knowledges. When lectora multimedia was the basis of the inquiry training instructional models of learning, its impact on learning in the classroom. Lectora supports various common media types including text, images, audio, video, animation, and internet technology even popular file types such as Shockwave, Flash, HTML, Java, Javascript, ASP .NET and Cold Fusion. Lectora becomes a interactive learning multimedia, and the laboratory can be presented in different condition, namely a computer device. Students become better to interact with learning, and the students achievement became better. Theories that support the inquiry training instructional models is Vygotsky's theory. There are two very important concepts in Vygotsky's theory, namely Zone of Proximal Development (ZPD) and scaffolding. ZPD is the distance between the actual development level that defined as the ability to solve problems independently with the development level of potential which is defined as the ability to solve problems with adult guidance or in collaboration with peers who have higher capacity.

CONCLUSION

Based on the results of research and discussion it can be concluded:

1. There is a difference in student achievement that learn by inquiry training instructional models based on lectora multimedia and conventional instructional models. Students achievement who applied inquiry training instructional models based on lectora multimedia better than conventional instructional models.
2. There is a difference in student achievement that have formal thinking high and low formal thinking ability. The students achievement that have high formal thinking ability better than students that have low formal thinking ability.
3. There is an interaction between the instructional models toward students achievement. Students achievement taught with inquiry training instructional models based on lectora multimedia influenced by formal thinking ability, while students achievement taught by conventional models is not affected by the formal thinking ability.

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STRUCTURES OF STUDENT'S THINKING IN SOLVING PROBLEM OF DEFINITE INTEGRAL APPLICATION ON VOLUME OF ROTATE OBJECTS

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Abstract

This paper is based on the analysis of answers given by the 65 students in solving problem of definite integral application on volumes of rotate objects. The problem is given to students of 3rd and 5th semester academic year 2016/2017. After examine the results of students' answers and conduct interviews to some subjects, it turns out that the structure of their thinking can be grouped into four different. In this paper, it names, the structure of thinking type 1 where student gives the wrong answer which is preceded by an error in understanding the problem, the structure of thinking type 2 where student gives the wrong answer that begins with errors in deciding plan (correct in understanding the problem), the structure of thinking type 3 where students give a wrong answer which is preceded by an error in implementing the strategy (to understand the problem and devise a plan are correct), and the structure of the thinking type 4 are students who really understand the issues, plan the correct strategy and implement the strategy. The study on the structure of thinking is based on Polya's steps, namely to understand the problem, devise a plan, carry out the plan and look back. This research is important to be done to determine the representation of the thinking process of students in solving problems from the stand point of Polya's steps.

Keywords: *structure of thinking, error of thinking, problems solving, definite integral, and Polya's step.*

Introduction

Integral is one part of the Calculus (Tall, 1975; Yost 2009; Varberg, Purcell, and Rigdon 2010). For Indonesia, the Integral material has been introduced and taught at the high school class of 12. In the K-13 stated that the basic competencies integral to the material of course is 1) understand the concept Riemann of sum and definite integral of simple functions, 2) use the theorem fundamental calculus to find the relationship between integral in definite integral and in the indefinite integral, 3) process the data and create a model of a simple function non-negative and real and interpret the problem in the picture and solve problems by using the concepts and rules of definite integral, and 4) filed a real problem and identify the fundamental nature of the definite integral calculus simple function and apply it in problem solving. The fourth basic competitions are translated into several sub-material objectives which math problems involving definite integral concept can be solved by students. This study is conducted on students who have already learned the material definite integral ranging from high school grade 12 until the second semester at the University (Malang State of University). In voting student intended to look at the complexity and variation do problem solving and any errors or reinforce their understanding of the underlying issues related to the settlement of certain integral material.

Many researchers in the field of mathematics education said that the integral material is very important (Kiat, 2005; Yee & Lam, 2008; Yost 2009; Dorko, 2011; Serhan, 2015). On the other hand, many researchers revealed that this material is a material that is difficult for learners, so that the problems they encountered regarding this material (Machin, 2003; Yasin, et. al., 2007; Rosken, et. al., 2007 ; Mbokane & Kedibone, 2011; Sugiman, 2012;). In the context of the application of integral course on the volume of solid of rotate difficulties experienced by students include, 1) the difficulty in drawing a graph, 2) the difficulty in formulating the volume of the object around in the form of definite integral 3) the difficulties in determining the outcome of definite integral and 4) difficulties in applying the definite integral to determine the volume of rotating object. Difficulties occurred is needed to be revealed, how their thought process when solving problems. It will become even more interesting if the instruction about not mention directly that student solves problems using definite integral.

The process of thinking is a mental activity, which in this study focused on students when solving a problem. New problems and challenges that will make students become complex mental activity. It can also be seen as a given problem can be interpreted differently from each student according to their ability and understanding possessed associated with the concepts needed to solve the problems faced.

The thinking process of students that occur in the brain can be revealed through the writings and interviews. Researchers can create a representation of the thought process students through the structure of thinking. Variations in the process of thinking of students can be seen through the structure of thinking. Thus, it becomes important to reveal how the structure of thinking of students in solving a problem, in this case the problem is certainly in the application of definite integral in volume of rotate objects.

RESEARCH METHODS

This study is a qualitative study that investigated a social phenomenon or a human problem (Creswell, 2007). The research findings are not obtained through statistical procedures. In this type of study, researchers created a complex picture, studying words, a detailed report on the views of the subject of research, and conduct studies in a natural situation. Bogdan & Taylor (in Moleong, 2007) states that qualitative methods is a research procedure that produces descriptive data in the form of written and spoken on the subject of research related to the behavior observed ang. Yin (2011) states that qualitative research to further highlight the process and meaning in the perspective of the subject.

The research was conducted at the State University of Malang on 3rd semester student who was 23 people and 5th semester student who totaled 42 of the school year 2016/2017. In his vote of mathematics education student who has been studying the concept of integral since high school and pursue back at his lectures, is assumed to have a more complete structure of thought and depth so that the process of exploration conducted by researchers associated with the search structure thinking based on Polya's steps will be more visible.

Instrument in this study is the researchers themselves and assisted with job sheets and sheets of semi-structured interviews. The problem posed is as follows:

Table 1. Adoption Instruments Task Sheet

Problem from book: "How to Solve Word Problem Calculus; Proven Techniques from the Expert"	**The instrument of this research assignment sheet**
A hole of radius 2 is drilled through the axis of a sphere of radius 3. Compute the volume of the remaining solid	 A company wants to produce a gold necklace latest model with two beads that looked like the picture on the side. Ways of making gold beads were first (before carved) with a solid ball-shaped gold pierce diametrically use the drill radius 5 mm. For the sake of aesthetics radius specified solid ball 2 times larger than the radius of the drill bit. The company wanted to know how the remaind of the two volume of solid ball used in the necklace (before carved). Help companies to solve.

Based on input from three experts 2 experts mathematics education and 1 expert pure mathematicians) obtained some improvements to the instrument, including: 1) the context of the problem must be included so that the motivation of candidates subject to complete, 2) context that is made must be the context that makes sense, 3) create sentences are easy to understand and clear of the term.

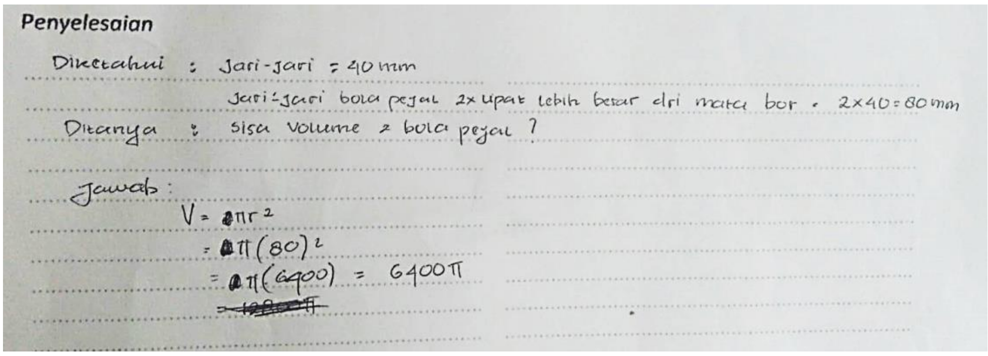
The uniqueness of the matter is there is no explanation is directly linked with the concept or procedure that can be used. However, student response associated with the given problem is a matter of understandable, context / story makes sense, because challenging, and have never found or do the problems are the same.

RESULTS

There are 65 answers obtained from two classes in this study, with details: 23 answers from the 3rd semester students and 42 answers from students from fifth semester. Of the 65 answers obtained, there are 2 answers (3.08%) were not considered because they do not support this research. Thus, there are 63 (96.92%) were considered and analyzed further. Here are examples of answers that are not considered.

Examples of answers that are neglected and do not support the study.

example 1

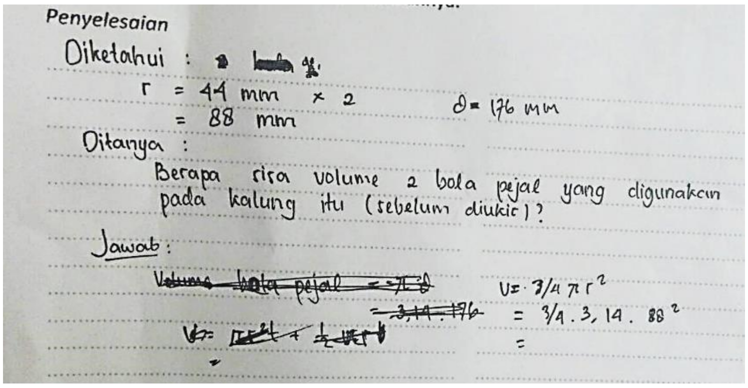


Penyelesaian

Diketahui : Jari-jari = 40 mm
Jari-jari bola pejal 2x upat lebih besar dri mata bor . 2x40 = 80 mm
Ditanya : sisa volume 2 bola pejal ?
Jawab :
$V = \pi r^2$
$= \pi (80)^2$
$= \pi (6400) = 6400\pi$

Figure 1. The work of students who do not support research

Example 2



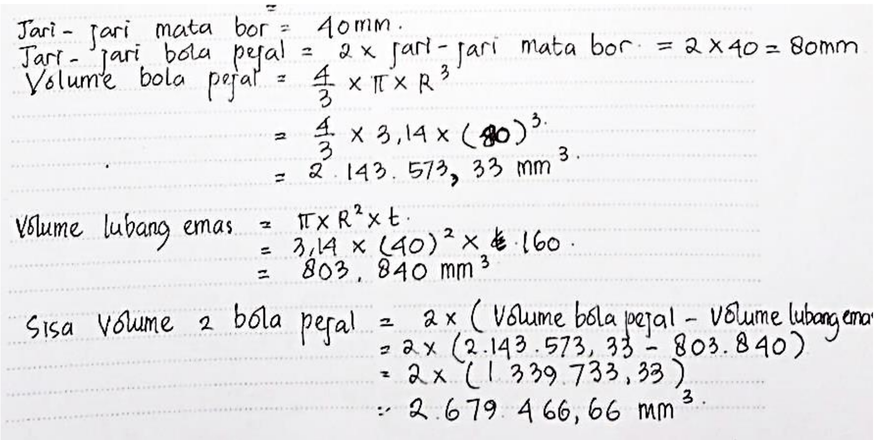
Penyelesaian

Diketahui :
r = 44 mm x 2
= 88 mm
d = 176 mm
Ditanya :
Berapa sisa volume 2 bola pejal yang digunakan pada kalung itu (sebelum diukir)?
Jawab :
~~Volume bola pejal = π d~~
~~= 3,14 . 176~~
$V = 3/4 \pi r^2$
$= 3/4 . 3,14 . 88^2$
=

Figure 2. The work of students who do not support research

For the initial distribution of problem solving in which students, researchers grouped into 3 groups. 1) students do not write or work their 3-D shape irregular (students determine the volume of balls left by subtracting the volume of a sphere with the volume of the tube), 2) students write (realize) their 3-D shape irregularly through the images but not to the use of definite integral to solve the problem, and 3) students write (realize) their 3-D shape irregular and using definite integral to solve it. The purpose of the irregular waking is waking up that does not have a standard formula, such as tubes, balls, blocks, cones, etc. (in determining the volume).

Examples of student answers in group 1



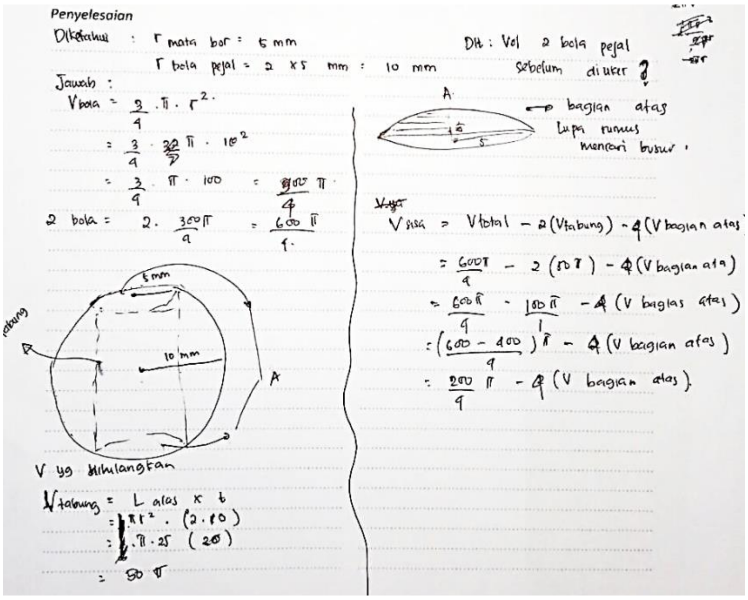
Jari - jari mata bor = 40mm.
Jari - jari bola pejal = 2 x jari - jari mata bor = 2 x 40 = 80mm
Volume bola pejal = $\frac{4}{3} \times \pi \times R^3$
$= \frac{4}{3} \times 3{,}14 \times (80)^3$.
$= 2.143.573{,}33$ mm^3.

Volume lubang emas = $\pi \times R^2 \times t$.
$= 3{,}14 \times (40)^2 \times 160$.
$= 803.840$ mm^3

Sisa volume 2 bola pejal = 2 x (Volume bola pejal - Volume lubang emas
= 2 x (2.143.573, 33 - 803.840)
= 2 x (1 339.733, 33)
= 2.679.466,66 mm^3.

Figure 3. The work of students in the group 1

Examples of student answers in group 2



Penyelesaian
Dikětahui : r mata bor = 5 mm
r bola pejal = 2 x 5 mm = 10 mm
Jawab :
$V_{bola} = \frac{3}{4} \cdot \pi \cdot r^2$.
$= \frac{3}{4} \cdot \frac{22}{7} \pi \cdot 10^2$
$= \frac{3}{4} \cdot \pi \cdot 100 = \frac{300}{4} \pi$.
2 bola = $2 \cdot \frac{300\pi}{4} = \frac{600\pi}{4}$.

V yg dihilangkan
V_{tabung} = L alas x t
$= \pi r^2 \cdot (2 \cdot 10)$
$= \pi \cdot 25 \; (20)$
$= 50\pi$

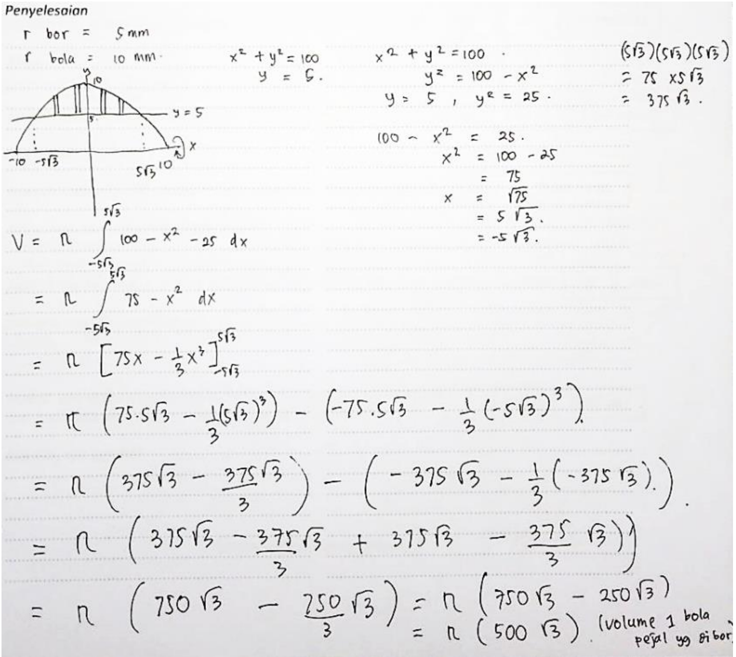
Dit : Vol 2 bola pejal sebelum diukir ?

→ bagian atas
Lupa rumus mencari busur.

$V_{sisa} = V_{total} - 2(V_{tabung}) - 4(V$ bagian atas$)$
$= \frac{600\pi}{4} - 2(50\pi) - 4(V$ bagian ata$)$
$= \frac{600\pi}{4} - \frac{100\pi}{1} - 4(V$ baglas atas$)$
$= \left(\frac{600 - 400}{4}\right)\pi - 4(V$ bagian atas$)$
$= \frac{200}{4}\pi - 4(V$ bagian atas$)$.

Figure 4. The work of students in the group 2

Examples of student answers in group 3



Penyelesaian

r bor = 5 mm

r bola = 10 mm.

$x^2 + y^2 = 100$

$y = 5.$

$x^2 + y^2 = 100.$

$y^2 = 100 - x^2$

$y = 5 , y^2 = 25.$

$100 - x^2 = 25.$

$x^2 = 100 - 25$

$= 75$

$x = \sqrt{75}$

$= 5\sqrt{3}.$

$= -5\sqrt{3}.$

$(5\sqrt{3})(5\sqrt{3})(5\sqrt{3})$

$= 75 \times 5\sqrt{3}$

$= 375\sqrt{3}.$

$$V = \pi \int_{-5\sqrt{3}}^{5\sqrt{3}} 100 - x^2 - 25 \, dx$$

$$= \pi \int_{-5\sqrt{3}}^{5\sqrt{3}} 75 - x^2 \, dx$$

$$= \pi \left[75x - \frac{1}{3}x^3\right]_{-5\sqrt{3}}^{5\sqrt{3}}$$

$$= \pi \left(75.5\sqrt{3} - \frac{1}{3}(5\sqrt{3})^3\right) - \left(-75.5\sqrt{3} - \frac{1}{3}(-5\sqrt{3})^3\right).$$

$$= \pi \left(375\sqrt{3} - \frac{375\sqrt{3}}{3}\right) - \left(-375\sqrt{3} - \frac{1}{3}(-375\sqrt{3}).\right).$$

$$= \pi \left(375\sqrt{3} - \frac{375\sqrt{3}}{3} + 375\sqrt{3} - \frac{375}{3}\sqrt{3}\right)$$

$$= \pi \left(750\sqrt{3} - \frac{250}{3}\sqrt{3}\right) = \pi \left(750\sqrt{3} - 250\sqrt{3}\right)$$

$$= \pi (500\sqrt{3}).$$

(volume 1 bola pejal yg dibor

Figure 5. The work of students in the group 3

There are 52 students (82.56%) were answered as described in group 1, which distribute 16 students (69.57%) of the 3rd semester, and 36 students (90%) of the 5th semester. For the students answer as described in the second part, there are 7 students (11.11%) of them, 3 students (13.04%) of semesters 3 and 4 students (10%) of the semester 5. in the third part, there are 4 (6.35%) were the answer as the description and 4 students from 3rd semester.

Based on 3 patterns of answers of students with a description that has been made can researchers reviewed again from Polya's step (4 Polya's steps). This review will produce exciting process that can represent the structure of thinking of students in solving problems in application of definite integral in volume of rotate objects. After analysis and interpretation, resulting there are four structural patterns of thinking that happens, 1) the structure of thinking type 1, in which the students started error when conducting phase to understand the problem, 2) the structure of thinking type 2, in which the students started error by strategizing or devise a plan, 3) the structure of thinking type 3, students begin to implement the strategy mistake (to understand the problem and devise a plan were correct), and 4) the structure of thinking type 4, students correctly understand problems, develop strategies (devise a plan), implement the strategy, and look back.

The Structure of Thinking Type 1

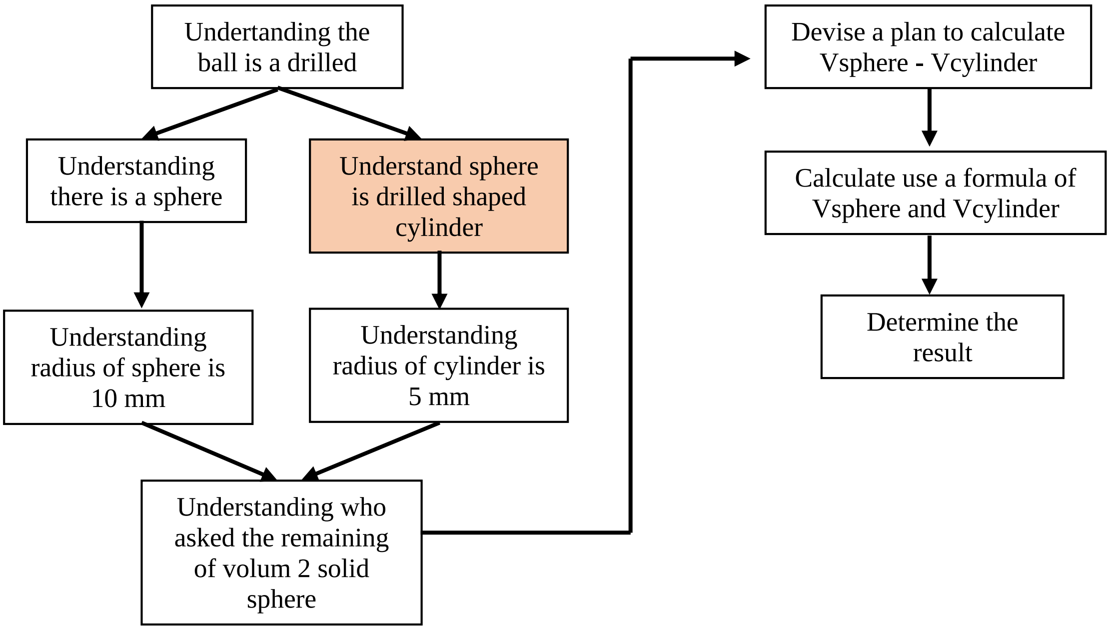


Diagram 1. Structure thinking of students who answered incorrectly and errors that occurred in the beginning of understanding the problem (structural thinking type 1).

Students (call it S1 or subject 1) revealed that during work on the problems, the first thing you think about is to understand there is a ball drilled. S1 imagine that the ball is drilled tubular. Then, based on the known of the matter, S1 reveals that the radius of the tube (drill) 5 mm and the radius of the sphere 10 mm. S1 also understand that being asked volume remaining 2 solid ball. S1 devise a plan for settlement, which calculates the volume of the sphere is reduced cylinder volume. Once calculated using the formula, S1 ensure the results.

Completion conducted by S1 wrong, and errors that occur stems from S1 mistake in understanding the problem. S1 to think so quickly without careful analysis associated with 3-D shape formed after solid ball drilled. Errors in understanding this problem, resulting in S1 wrong in planning and implementing the plan. Completion conducted by S1 in the group 1.

The Structure of Thinking Type 2

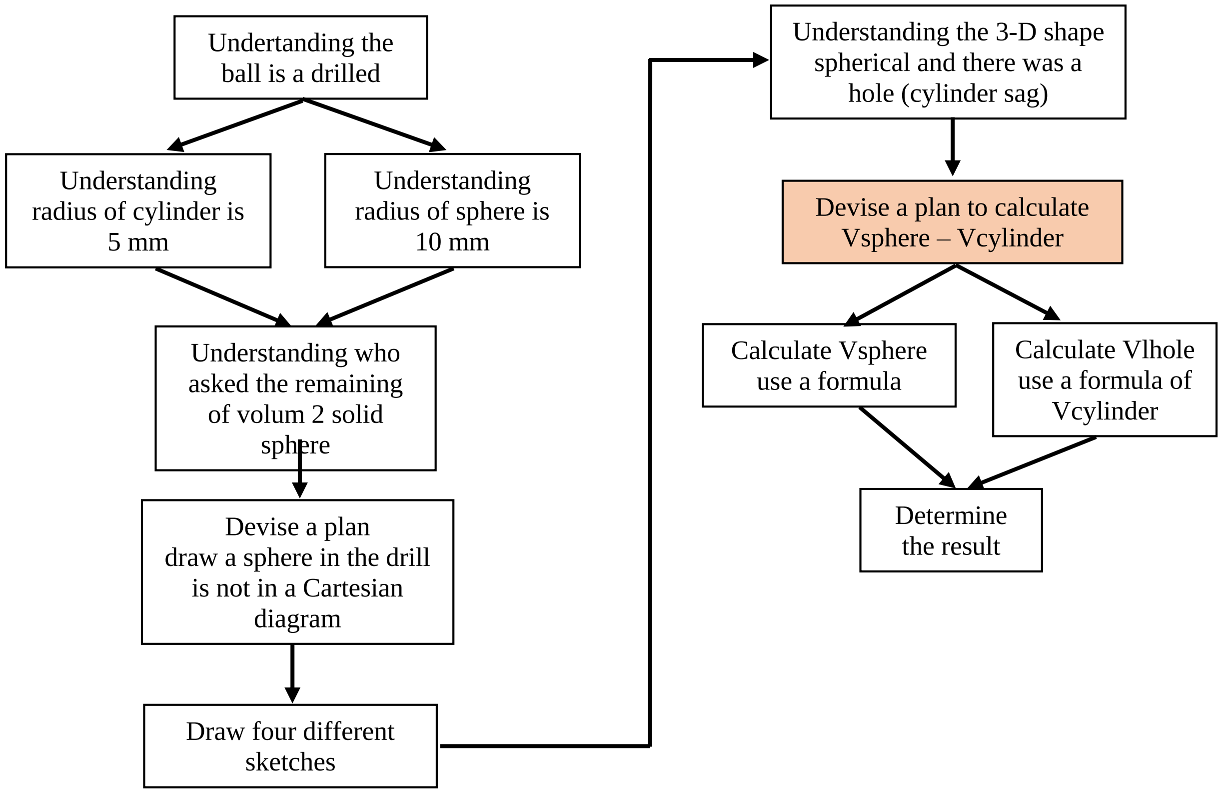


Diagram 2. Structure of thinking of students who answered incorrectly and errors that occurred beginning of the planning strategy (structural thinking type 2).

Students (call it S2 or the subject 2) revealed that during work on the problems, the first thing you think about is to understand there is a ball drilled. Based on what you know:, S2 reveals that the radius of the drill 5 mm and the radius of the sphere 10 mm. S2 also understand that being asked volume remaining 2 solid sphere. S2 then devise a plan to draw. By the plans, S2 produces 4 pictures (sketches) are different. S2 understand that there is a sphere and the hole, where the hole in question is the sag cylinder. S2 then devise a plan for the settlement, which calculates the volume of the sphere is reduced tube volume. Once calculated using the formula, S2 ensure the results.

Completion conducted by S2 wrong, and errors that occur S2 stems from an error in planning strategy. S2 thinks that the strategy is to calculate the remaining volume by subtracting the volume of solid sphere the sphere to the volume of the tube. Though S2 has been able to draw and understand that the 3-D shape is formed tube sag. Errors in planning this strategy, resulting in S2 wrong in carrying out the plans that have been made, or settling does not correspond to a given question. Completion conducted by S2 in the group 2.

The Structure of Thinking Type 3

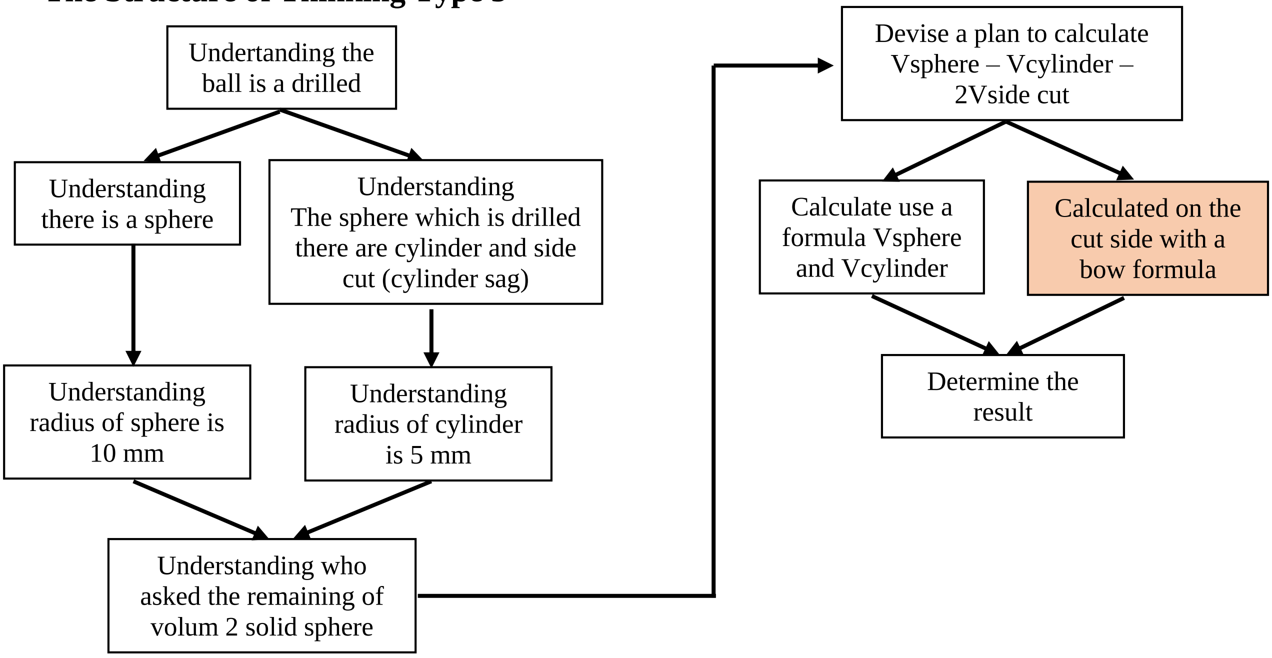


Diagram 3. Structure of thinking students who answered incorrectly and errors that occurred beginning of implementing the strategy created (structure thinking type 3).

Students (call it the S3 or subject 3) revealed that during work on the problems, the first thing you think about is to understand there is a ball drilled. Through the images created, S3 understand that the 3-D shape is formed sphere and cylinder that contained the upper side cut (cylinder sag). Then, S3 Understand that the radius of the drill 5 mm and the radius of the sphere 10 mm. S also understand that being asked volume remaining 2 solid spheres. Based on this insight, S3 plan by calculating the volume of a sphere minus the cylinder volume minus volume on the two sides cut off. S3 perform calculations to determine the volume of a sphere and a cylinder with a formula and calculate the volume of a truncated upper side of the formula arc. Once everything is found, S3 determines the results obtained.

Completion conducted by S3 wrong, and errors that occur when implementing a strategy that has been made. The formula used arc S3 does not have a solid foundation. In fact, S3 realize that to calculate the volume of a truncated side could not use a standard formula, only S3 is not able to choose a procedure or choosing the right concept to finish. S3 settling in the group 2.

The Structure of Thinking Type 4

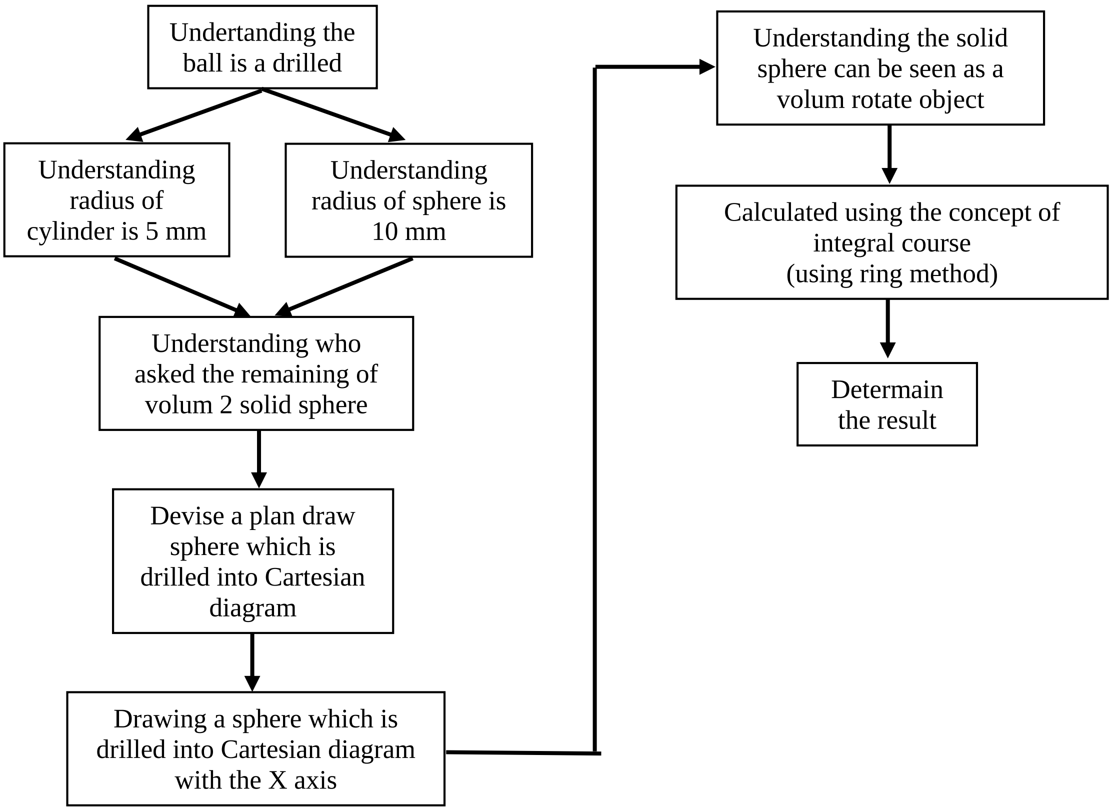


Diagram 4. Structure of thinking of students who answered correctly by using the ring to count the 3-D shape of irregular (structure thinking type 4).

Students (call it the S4 or subject 4) revealed that during work on the problems, the first thing S4 was thinking to understand there is a ball drilled. Then understand the known radius of the drill 5 mm and a radius of 10 mm sphere. S4 also asked understand that the volume of the remaining 2 solid sphere. After S4 understand their problems, S4 plan by drawing a ball drilled into the cartesian diagram. S4 draw a ball drilled through the X-axis Cartesian diagram. Based on the images created S4 understand that 3-D shape remaining can be seen as a rotating object volume. Then S4 calculates the volumes of solids of rotate objects using the concept of definite integral or with the ring method. After calculation S4 ensure the results.

Completion conducted by S4 true. S4 complete thought structure and even very complete, because when explored further, the S4 is able to resolve the issue with the skin tube method too. S4 settling in the group 3.

DISCUSSION

Problem solving is the process of resolving a situation that is faced by a person, which requires new solutions (resolutions) and the path / way towards such a solution is not immediately known (Posamentier & Krulik, 1998; Someren, 1994). Musser, burgers, and Peterson (2011: 4) states that a different problem with exercises in which only emphasizes the ways or procedures are routinely performed, while the issue is more emphasis on things that are not routine so that people who will solve it paused, to reflect on the possibility of using a creative way that he had never used before. In this case the problem is given to students in the form of a problem that is challenging, so that students feel interested in being able to solve it and find a solution. The given problem should match the cognitive state of the student, meaning that a given problem can be understood, it's just that the solution is not immediately known.

With the strategy of problem solving, students can understand the problem, develop a plan, and implement their plans. Furthermore, they can consider whether their answers make sense and are there any answers or other approaches, and finally they can deliver answers and their reasoning. The ability to accurately calculate very important in solving the problem, but thinking is at the core of teaching and learning mathematics.

Resolving a problem requires an understanding of the issue itself and the various strategies that can be applied. As a result, students develop a good answer or answers and content, as well as the skills necessary to solve the problem. With face a variety of problems with different challenges, students can develop the concepts, procedures, flexibility in thinking, and self-confidence in the face of the new situation. One of the key of solving the problem is the ability to think. The new and complex problems that can make students to think harder, because there is a process of understanding, a strategy / plan, implement the strategy / plan superbly made, up until the check back. These four processes can be determined through in-depth interviews. Through this disclosure, researchers can find out in full how the thought process students on solving problems, which is represented in the structure of student thinking.

In this study, the challenge is students are expected to remember the material definite integral which is taught in the 2nd semester and used to solve problems encountered. However, only 4 students among 65 students who can use and remember that to calculate the volume of irregular objects (do not have a standard formula) can use the concept of definite integral. Things that concern again is the 5th semester student none is able to utilize the concept of definite integral as a way to calculate the volume. Though some of them have been able to see (making representations) that are waking up irregularly. Sure to be a question, why did this happen?

Based on the four structures think that concluded as a result of research, it is of interest related to the construction that is incorrect or missing in the process of settling resulting in their weak connections between processes (structure) that occurred. Think about some separate information that has not been used (called to mind). Why did it happen? According to Gerace, W.J. (1992), there are some aspects of the information acquired, stored and deleted. It cannot be in full make someone's forgotten something and there is space in the memory to accommodate the other. That means there is one key or a missing word to be recalled information that is ever saved. Sternberg (2012) also stated that if the information does not get the intensity of attention, meaning that the information is often not remembered, have not been studied repeatedly, are not meaningful and memorable are therefore short-term memory information is not passed on to the long-term memory (passed means that the information will be created its own code making it easier for someone to call back). So we need a more in-depth disclosure related to the search for holes or pseudo understanding of resulting in a person difficult to remember information is ever stored, as well as find solutions how efforts should be made to avoid things like that.

CONCLUSION

Based on the research results and explanation, we can conclude that in solving mathematical problems by Polya’s steps, structural thinking of students can be divided into four, namely the structure of the thinking type 1, in which students answered incorrectly and errors start from understanding the problem, the structure of thinking type 2, students answered incorrectly and errors start from the plan, the structure of thinking type 3, students answered incorrectly and errors occur when implementing a plan that was made (to understand the problem and devise a plan correctly), and the last structure of thinking type 4, students answered correctly and all Polya’s steps is done correctly. The errors that occurred are in terms of mental processes which is a errors in imagining (thinking) 3-D shape formed from solid ball drilled, errors in drawing up a plan for drawing solid ball drilled, errors in positioning the solid sphere drilled Cartesian diagram, errors in choose a procedure that can be used to calculate the 3-D shape of irregular, errors in using the course to determine the volume integral rotary object. These mistakes, needs to be pursued further and deeper to uncover the hole construction or construction pseudo happened.

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Innovation of Mathematics Education through Lesson Study Challenges to Energy Efficiency on STEM on Proportional Reasoning and Wind Power

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Abstract

Developing activities to support student understand by working collaboratively with the researchers to provide scientific and practical knowledge needed on energy efficiency and related competencies through the integration of energy efficiency, security, and robustness of education in the school curriculum. This paper reports how the teachers learned to develop teaching and learning models regarding to energy efficiency, security, and resiliency and implemented it in situated environment. It was found that, learning carried out in the various disciplines, through science and mathematics. In teaching and learning science, it was easy to find the context related to energy, especially the changes in energy. In science class core activities, the teacher presents a problem of photo on a beach and students has to react or find solutions. Learning is followed by an assignment of manufacture and test the propeller on a windmill. The teacher related that activity with proportional reasoning. On learning performance test propellers, mathematical topic on proportional reasoning was not quite suitable for this activity. In fact, the comparison and the proportion cannot always be made. However, the most interesting part activities and discussions sessions is when the students adjust the position of the blades. The students found that in order to produce electrical energy, the device is influenced by other factors.

Keywords *: STEM, Energy Efficiency, Proporsional Reasoning*

Introduction

Changes in the school curriculum in Indonesia become a new challenge for teachers in schools. Not only for those who teach in schools pilot project for the implementation of the curriculum in 2013, but also for teachers who still using 2006 curriculum need to learn an initiate change in learning toward learning suitability to be expected on the new curriculum.

One of the goal in mathematics curriculum in 2013 is solving the problem that involves understanding the concept, explaining the relation among concepts and applying them in activities of problem solving. The ability to think and to reason are very important for students. Mathematics can be seen as a language that describes patterns (NCTM, 2000; De Lange, J., 2004). Based on that statement, during mathematics teaching and learning process, students can learn to think, to solve problems, to reason, and to comunicate. The meaning of solving problem in this context is, to solve problems when learning math, and to solve real world problems. However, solving math problems in real world often occurs in research, especially science research.

The problem is lately attracting experts is a matter of energy issues. In this regard, many research about alternative energy reveal that there is a potential development of Wind Power on the south coast of Java island (Musyafa, A., 2011). It can be used as a topic while developing activities to support student understand to provide scientific and practical knowledge needed on energy efficiency and related competencies through the integration of energy efficiency, security, and robustness of education in the school curriculum.

STEM education is used to address real-world situations through a design-based problem-solving process, much like reviews those used by engineers or scientists (Williams, 2011). STEM becomes the choice of appropriate learning to apply the learning to raise the issue of energy. It should be prepared is to collaborate on research in natural atmosphere with learning. The study of science itself is conducted based on the observed facts so that students see for themselves a problem, gathering data on the existing problems, planning research, then do research and draw conclusions from the research conducted. This is in accordance of the interpretation that without practice related to STEM students often fail to understand the integration roomates occurs naturally between STEM subjects in the real-world (Breiner, Harkness, Johnson, & Koehler, 2012).

The questions can be aroused: 1) what should be done and how to help and facilitate students to learn mathematics meaningfully and joyfully, 2) what should be done and how to help and facilitate students to learn to think an be independent learners, 3) what should be done and how to design STEM education related to energy efficiency and mathematical topic.

Metodology

This research is a descriptive research. The research raises the issue of energy. Subjects to be studied is the class VI SDIT Assalaam Sanden many as 22 students. This research was done in two stages. The first phase aims to facilitate students to observe the problems and find alternative ideas packaged solutions to problems through science teaching. At this stage, students are given factual examples of the beach with the potential to be developed into a tourist area but do not yet have electricity supply. Students are involved in discussions and gallery walk in order to find sources that enable transformed into a source of electrical energy. Assessment in the first phase of the research activities are based on a worksheet that has been validated. Assessment of the results set forth in the discussion group worksheets are used as one of the achievements of student competence assessment.

In the second phase, carried out research activities vane windmill with lesson study involving the team as a documenter. Video documentation obtained is used as a reference reflecting on learning activities. In addition to involving the team, in the second phase of this activity using propeller installation media. Media for the installation of a propeller prepared by the teacher with input from the team during the lesson study activities at the planning stage, and also take into consideration the observations and input from experts in the development center of Hybrid Power Plants Poncosari Srandakan. It was observed in the second stage mainly on how students draw conclusions through a process of comparing the numbers in the electrical voltage generated from the model propeller with each other from each group based on experiments and observations.

Pre-test and post-test is also given to the students with the analysis item for about 4 things are: 1) To help Increase the awareness of students on the issue of energy efficiency in real live. 2) To help and facilitate students with the scientific and practical knowledge regarding energy efficiency and related skills to reduce or minimize the use of energy. 3) To help increase of students' beliefs on the importance of mathematics, science, and technology that will motivate them having good attitudes toward mathematics, science, and technology. 4) To help increase of Indonesian students' beliefs on the importance of mathematics, science, and technology that will motivate them having good attitudes toward mathematics, science, and technology.

Data Collections

From the data the results of pre-test that belief and awareness of the students on the issue of energy efficiency in real live is 9.45 (medium), and already Increase Become 11.73 (high) on the post test. The detailed analysis is presented in Table 1 below.

Tabel 1. Persentase Belief and Awareness of The Students On The Issue Of Energy Efficiency In Real Live

Rubric	Item	Pre Test Score	Post Test Score
1	To help increase the awareness of students on the issue of energy efficiency in real live	45%	68%
2	To help and facilitate Indonesian students with the scientific and practical knowledge regarding energy efficiency and related skills to reduce or minimize the use of energy	95%	100%
3	To help increase students' beliefs on the importance of mathematics, science, and technology that will motivate them having good attitudes toward mathematics, science, and technology	95%	100%
4	To help increase students' beliefs on the importance of mathematics, science, and technology that will motivate them having good attitudes toward mathematics, science, and technology	66,5%	88,5%

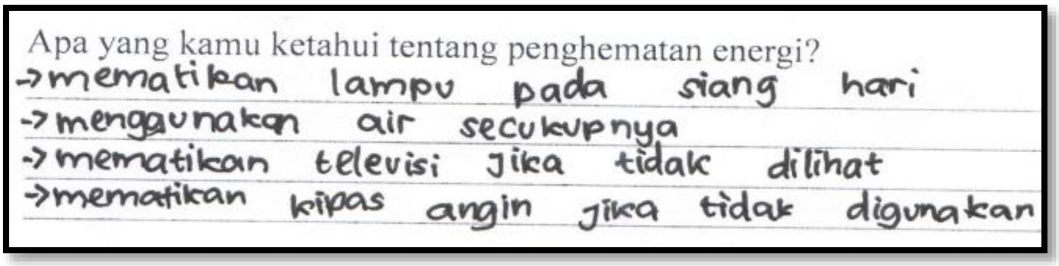
The first phase of the learning outcomes obtained some idea of students. The students found three natural resources can be converted into electrical energy including wind energy, solar energy and wave energy.

In the second phase, carried out observations of the efforts of students in practical activities to get optimum power. Several attempts were made among other things, paying attention to installation position propeller blades adjustable wind direction, changing the slope of the propeller, bringing windmills to a higher place, comparing the quality of generators on teaching and belt on props they use compared to the use of other groups, and rotating the belt by hand instead of wind to get a large number voltage.

Findings of practical activities include, 1) there is a propeller that does not rotate, 2) demonstration that was broken, 3) propellers generate electricity but has no comparison, 4) numbers in the generated voltage can be compared, but in the form of a decimal number. In the second stage, in addition to lab activities are also conducted presentations. Presentation of activities carried out as a final step before students make conclusions related to the research conducted.

Findings dan Discussion

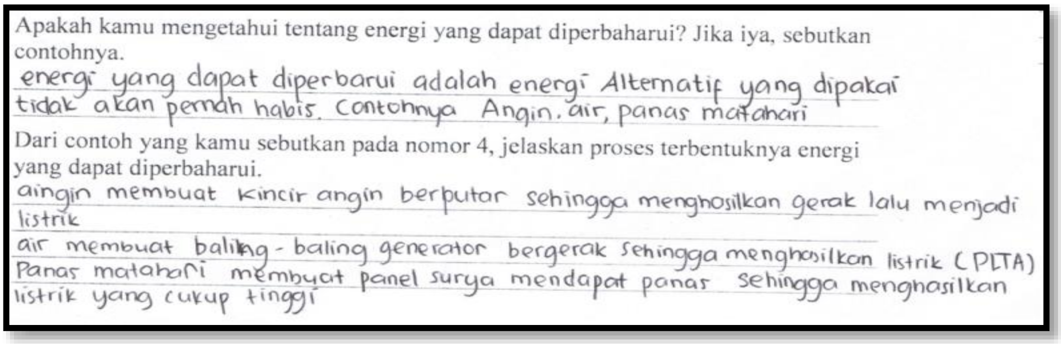
From the results of the data pre-test and post test known that students can identify renewable energy very well. However, there are only 9% of students who associate energy savings with the use of alternative energy. This finding shows that during learning, still less emphasized that the use of alternative energy as an important part of the energy savings.



Apa yang kamu ketahui tentang penghematan energi?
-> mematikan lampu pada siang hari
-> menggunakan air secukupnya
-> mematikan televisi jika tidak dilihat
-> mematikan kipas angin jika tidak digunakan

Figure 1. Student Answer

In addition, it is known that in general the students have to save electricity. But, there is no offensive about using gadgets.



Apakah kamu mengetahui tentang energi yang dapat diperbaharui? Jika iya, sebutkan contohnya.

energi yang dapat diperbarui adalah energi Alternatif yang dipakai tidak akan pernah habis. Contohnya Angin. air, panas matahari

Dari contoh yang kamu sebutkan pada nomor 4, jelaskan proses terbentuknya energi yang dapat diperbaharui.

aingin membuat kincir angin berputar sehingga menghasilkan gerak lalu menjadi listrik

air membuat baling - baling generator bergerak sehingga menghasilkan listrik (PLTA)

Panas matahari membuat panel surya mendapat panas sehingga menghasilkan listrik yang cukup tinggi

Figure 2. Student Answer

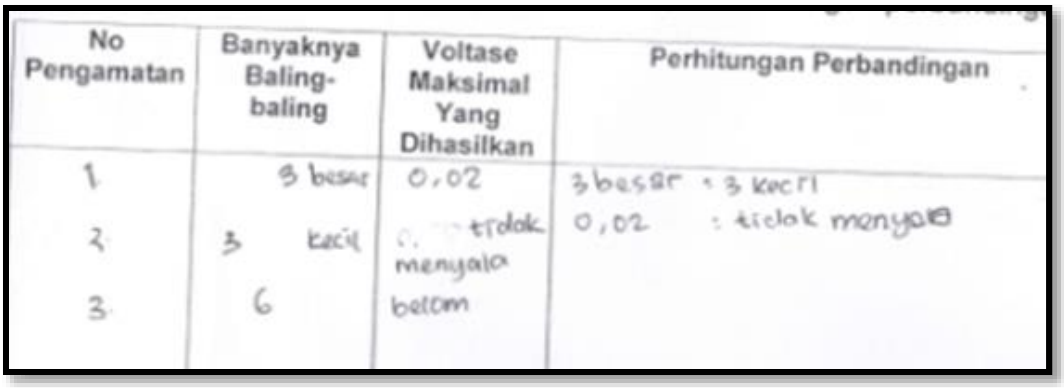
The students also have to have a good understanding of alternative energy. Some alternative energy sources results of students such as wind, water and solar heat. The process of formation of electrical energy from renewable energy sources, also have been understood.

The second stage of learning is an innovative math learning. Not only to observe the ability of students perform on arithmetic operations comparisons, but also to reveal the students' ideas.

Figure 3. Students' Activity

The enthusiasm of the students participating in these learning activities encouraged them find a lot of knowledges and experiences. Adjust the position of the propeller installation is part of the activities of the students on applying math learning outcomes. Some of the groups decided to change the position of the propeller. The students being active learner.

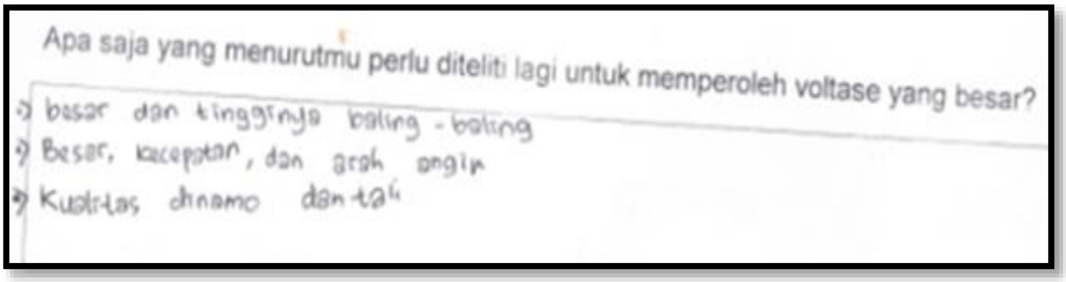
Data collected on student worksheet shows that there are some groups that the amount of voltage can not be compared. They found several blades is not proper. It draws from this case are, they find themselves that the propellers were successfully tested with a fan not necessarily work well when tested at the beach.



No Pengamatan	Banyaknya Baling-baling	Voltase Maksimal Yang Dihasilkan	Perhitungan Perbandingan
1	3 besar	0,02	3 besar : 3 kecil 0,02 : tidak menyala
2	3 kecil	0, tidak menyala	
3	6	belom	

Figure 4. Student Answer

There was a group who find the magnitude of the voltage with three propeller size to produce 2.73 volts. They wrote many comparisons propeller with a large voltage is generated which is 3: 2.73 and simplifying to 1: 0.91. Activities substantiation ratio of 3: 2,73 = 1: 0.91 which is the only activity calculates the ratio of numbers. But with one chance calculating this ratio, students have the opportunity to share about how students find ways to simplify comparison of integers with the decimal fraction. Although they found the numbers that can be compared, they concluded that the ratio of the number of vanes is not associated with a ratio of the amount of electricity produced by windmills.



Apa saja yang menurutmu perlu diteliti lagi untuk memperoleh voltase yang besar?

→ besar dan tingginya baling - baling
→ Besar, kecepatan, dan arah angin
→ Kualitas dinamo dan tali

Figure 5. Student Answer

Students have found that the ratio of the number of vanes is not associated with a ratio of the amount of electricity produced by windmills. They found also the other factors such as the magnitude and direction of the wind, the size of the propeller, generator quality, quality belt and the weather condition. The students' findings indicate that this mathematical topic on proportional reasoning was not quite suitable for this activity. By this activity, the comparison and the proportion can not always be made.

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Joyful Supervision by Video Call Increase the Teacher's Competence

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Abstract. Joyful Supervision by Video Call Increase the Teacher's Competence

Abstract. Supervisors will determine the quality of education in schools. Supervision of education are important elements need to be done more intensively to improve the competence and performance of teachers in learning. By carrying out academic supervision is programmed, it will achieve continuous service quality learning process. Learning led by professional teachers will enhance learners' achievements. Many variations are used instructional teacher to help learners to achieve these outcomes, one of which the role of communications technology in education is a positive influence in the planning and learning process. However, in the era of globalization that has entered into many sectors including education, until now still not many teachers who use communication media such as facebook, phone, computer, internet, e-mail, and so on in the learning process. Along with the development through globalization in education, information technology plays an important role as a means to obtain information related to the source of the material being taught.

This activity is motivated by the performance of teachers demonstrated in the learning process that is less take advantage of learning media to motivate students to think critically, logically, and not give up as expected of the students related to core competency-2 is "showing an attitude logical, critical , analytical, consistent and conscientious, responsible, responsive, and does not easily give up ". Supervision over the Internet with technical innovation and Email (Two in One) as an effort that is deemed appropriate school superintendent also be applied in carrying out the supervision of education so that teachers feel comfortable because the coaching methods that are innovative, creative and effective. The learning process is fun and meaningful interaction between teachers and students not only done through face to face relationship but also done using media technology as a preparation to compete in the era of global so educate generations to meet Indonesia Indonesia gold 2045 gold.

Effective learning process is fun and meaningful requires a media or resources that support the absorption of as much information as an effort to educate and deliver learners have basic competencies "shows the attitude logical, critical, analytical, consistent and conscientious, responsible, responsive, and not easily give up "as the character of the civilized face of global competitiveness. The results show that the activity through technical innovation by "Video Call" the above objective can be achieved. Increased harmonious relationship shown by the increasing willingness of teachers to consult with a supervisor. The increased performance demonstrated better teachers in the planning and process of creative learning and innovative and positive impact on learners' achievements.

Keywords: *Global Competitiveness, Joyful Distance Education, Technology and Innovation.*

1.1 Introduction

The role of information technology is very important for human activity in the era of globalization at this time that has been entered into many sectors including education. Globalization through education needs innovatif teachers in mathematics class by using communication media such as internet, electronic mail (e-mail), Face book, online discussions or conferences, bulletin boards, Twitter, Friendster, Myspace, Skype, chatrooms, mailing list and so on. Teacher is a manager of the class and as change of agents should encourage the students learning in the real life contexts. Mathematics classroom needs technology as a tool to build conceptual knowledge and thinking mathematically than the tradiotionally displayed in textbooks. Mathematics teachers need innovation in teaching process to create the class fun, entertaining, interesting, joyful and meaningful learning by using the internet very appropriate in mathematics curriculum 2013 and can be linked to mathematics class. The internet has become popular and potential as a medium for learning in mathematics class joyful and meaningful but also motivate the students to think critically.

In this case the role of a supervisor as a teacher partner provide a major contribution to the performance and competence of teachers. This paper provides an alternative example of learning math joyful and meaningful by using the internet through brainly.co.id. hopes can help the students get the widest source of information relating to the subject matter being taught.in the basic competence on the second core competencies in Permendikbud number 68 of curriculum 2013 states that one of the basic competencies of a junior high school mathematics expected of learners related to core competency-2 is a "show attitude logical, critical, analytical, consistent and conscientious, responsible, responsive, and does not easily give up ". Many variations are used teacher learning in helping students to achieve these outcomes, one of which is the role of communication technology by internet activities for joyful and meaningful in education that specifically influence the mathematics learning is the target of a profesional supervisor and a profesional teacher to realize the importance of the quality of learning that is joyful and meaningful.

In order to build the teaching professiol, these conditions require the teaching profession. Increased activity of the learner and teacher competence in State Junior High School 4 Metro City becomes the target of achieving writers, as with significantly increased teacher competence learners produce quality and dignified. Here presented an overview of the results of activities at State Junior High School 4 City Metro based on initial observations of the impact of the performance and competence of teacher.

1.2 Problem Formulation

The problem can be formulated as follows: How can increase the activity of students in Junior High School 4 Metro ?; How to improve the competence of teachers in the learning process in State Junior High School 4 Metro ?.

1.3 Problem Solving Approach

In addressing the above problems there are several ways that can be implemented, namely: Supervisors conduct training activities in order to improve the competence of learners and teachers in Secondary Schools 4 Metro; Supervisors conduct training so as to improve the learning process fun and meaningful in State Junior High School 4 Metro with internet-based learning through “Video Call” Moor and Zaskis (2000) "The interactive aspect of the internet holds the attention of the students much longer than a regular page of information such as is found in a textbook. This suggests that internet-based learning can motivate students to learn mathematics so that more meaningful and enjoyable.

1.4 Purpose

Coaching aims to: Increase activity learners through fun and meaningful learning; Improve the competence of teachers in the learning process fun and meaningful in State Junior High School 4 Metro with internet-based learning through “Video Call”.

1.5 Benefits

Some of the benefits that can be derived from this paper as follows: To provide information to other supervisors to be able to improve the competence of teachers in elementary schools to use information technology (IT) as a means or effort in the learning process fun and meaningful; Institutions and teachers improve the quality of the learning process through the use of information technology (IT) to improve the activity of learners so as to create the learning process fun and meaningful.

II. DISCUSSION / ANALYSIS

2.1 Steps

Many variations are used teacher learning in helping students to achieve competency, one role of communication technologies that influence the activity of the students in the learning process. Along with the development of globalization through education, the information technology played an important role as a means to obtain the widest source of information relating to the subject matter being taught. The learning process is fun and meaningful interaction between teachers and students is not only done through face-to-face relationships but also done using the media of information technology that supports the absorption of as much information with Internet-based learning through “Video Call”.

2.2 Basis Theory

The development of information and communication technology (ICT) has an impact on education, especially in the learning process. According to Rosenberg (2001), with the widespread use of ICT there are five shifts in the learning process, namely: From training to performance; From the classroom to where and at any time; Of paper to the "on line" or channel; Physical facilities to network facilities; From time to time the cycle is real. Communication as a medium of education is done by using communication media such as telephone, computer, internet, e-mail, and so on. Interaction between teachers and students is not only done through face-to-face relationships but also done using these media (Rosenberg, 2001).

III. RESULTS AND IMPACT ACHIEVED

Coaching by supervisors are not given instruction or order, but human relations atmosphere full of warmth, closeness and openness; so that teachers have a sense of security, and a willingness to accept the repair. The context of process improvement coaching is done as supervisory tasks. The teacher as a supervisor needs to get treatment target measured as prescriptive efforts towards improving the performance of teachers so that the teachers make learning improvement efforts, as suggested by the supervisor guiding in an effort to improve the quality of the learning process fun and meaningful to increase the activity of students and as an effort to increase performance.

IV. CONCLUSION AND RECOMMENDATIONS

4. 1 Conclusion.

Taking into account the results that have been obtained from the regulatory guidance can be concluded that: There was an increase of activity in the learning process by using the internet through “Video Call” By using Internet-based learning model one through “Video Call” students can bridge the basic competence on the second core competencies in the curriculum, 2013; The learning process is fun and meaningful interaction between teachers and students is not only done through face-to-face relationships but also done by using Internet-based media in this case through "Video Call" Effective learning activities and fun requires a media that supports the absorption of as much information to deliver learners have basic competencies in the curriculum 2013.

4.2.Suggestion

From the conclusion of this paper, it is suggested some of the following:

For Teachers: The need for improved competence by using technology in the learning process through a variety of media approaches technology (internet) to increase student activity; **For Supervisors:** The need for academic supervision to improve the competence of teachers in learning through a collaborative approach; **For Schools:** Need to motivate teachers to always communicate with the supervisor in the form of guidance related to the quality of learning in schools; **For Institution** Office of Educator training and workshops should be conducted on an on going basis of the concepts of supervision for school inspectors.

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Profiling Self-Regulated Learning in Online Mathematics Teacher Training

Case Study : GeoGebra Course

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Abstract

Online training now becomes an alternative of continuous professional development to improve teacher's competencies. The benefits of online training are flexibility of access, cost efficiencies, ease of content updates, and uniformity in contents. However, there are many aspects so teachers can be successful in online training, one is self-regulated learning. This study aims to profiling self-regulated learning in online mathematics teacher training that was held by PPPPTK Mathematics, using GeoGebra Course as case study. Using survey method, this study describes how the participant's self-regulated learning profile when they accomplished nine tasks in GeoGebra Course.

Keywords: *online training, self-regulated learning, geogebra*

Introduction

The changing of education requires teachers to constantly improve their professionalism. As a profession, "teaching" cannot be separated from the technology itself. There are many evidences about the impact of technology to increase teaching and learning. Educational technology applications produce a positive effect on mathematics achievement (Cheung & Slavin, 2013). Eisinger (2000) also mentioned that by combining traditional learning characteristics with the unique environment available on-line, elements that emerge would differentiate excellent e-learning, namely the sharing of knowledge. Currently, there are a lot of teachers who are familiar with computers and Internet for student learning media. However, teachers would often ignore utilize information and communications technologies to improve their own competencies.

Application and digital technologies enhancement in education has always been one priority by the Indonesian government. Therefore Indonesia Ministry of Education and Culture through PPPPTK Matematika (Center for Developing and Enhancing Competencies of Mathematics Teachers and Education Personnel) facilitates mathematics teachers to improve their competencies as a route to their continues professional development. Many teacher trainings has been implemented by PPPPTK Matematika through various modes. In 2015, one of the training conducted is an online training for mathematics teachers, including junior high school, senior high school and vocational high school.

There are many hesitation about how online training could produce effective change in teacher practice. Online training requires participants to have positive self-regulated learning so they can get the benefits of the training. Even though some essential abilities improve with age, self-regulated learning competence is neither automatic nor fast and it can take advantage of suitable teaching and practice (Delfino, Dettori, & Persico, 2011). Therefore, understanding self-regulated learning profile of the participants is an important issue.

Online Training and Self-Regulated Learning

The Organization of Economic Cooperation and Development (OECD) defines professional development as "a body of systematic activities to prepare teachers for their job, including initial training, induction courses, in-service training, and continuous professional formation within school settings" (Burns, 2011). Thus with the number of teachers in Indonesia, online training is being embraced by in-service programs as a route to teacher continues professional development. Indonesian Central Bureau of Statistics (BPS) reported the number of teachers in Indonesia at the Ministry of Education and Culture as follows (Badan Pusat Statistik, 2016).

Table 1. Number of teachers in Indonesia at the Ministry of Education and Culture

Level	Number of teachers
Primary school	1 539 819
Junior high school	596 089
Senior high school	278 711
Vocational high school	186 401

This number represents a very large number compared the ability of government to facilitate teachers through conventional training. Therefore, the implementation of online training becomes breakthroughs that provide the massive solution, efficient, and overcome geographical constraints (Perraton, 2002). Distance education has grown in significance as an educational tool just like technology has developed and progressed over the years. The 21st century has seen rapid progress with such things as the Internet and online learning (El-Seoud, Taj-Eddin, Seddiek, El-Khouly, & Nosseir, 2014).

Moreover, online training provides many benefits:

- flexibility of time and place
- reduce the cost required to put teachers and students in the same room,
- allow facilitators update the course material anytime so participants can access the latest material as soon as possible
- participants can control the learning process themselves
- uniformity in content, each participant obtain the same subject matter, thus reducing the possibility of misinterpretation, and
- easier data management with greater capacities than traditional classroom learning.

However, does not mean there are no weaknesses and challenges in the implementation of online training as follows.

- online training participants are required to have basic computer literacy skills. This become a challenge, especially when teachers are not familiar with the Internet,
- Internet network, bandwidth, reliable provider are a primary needs in order to participate in online training. Unfortunately, there still are so many areas in Indonesia with poor Internet network,
- online training requires a culture of self-regulated learning, the ability to learn independently. Success in the online training process highly depends how the teacher as the training participants foster a culture of self-regulated learning,
- although the distance education cost-saving, online training providers must issue a sizeable initial investment in the application, either for the provision of infrastructure, design, and maintenance. Furthermore, the training participants are also expected to pull out all of the costs arising from the online activities.
- Material delivering that are less constructive will hamper the transfer of knowledge through online training.

Among those challenges, self-regulated learning has an important role in determining the success of online training participants. According to Zimmerman (1990) self-regulated leaning involve three features: their use of self-regulated learning strategies, their responsiveness to self-oriented feedback about learning effectiveness, and their interdependent motivational processes. Meanwhile, Zumbrunn, Tadlock, and Roberts (2011) describe self-regulated learning as a process that assists persons in managing their thoughts, behaviors, and emotions in order to successfully navigate their learning experiences. Hence it can be concluded that self-regulated learning is a process when an individual foster learning by him/her self, including planning, implementation and evaluate the learning objectives gained. Zimmerman defines three phases in self-regulated learning are planning, implementation and reflection (Zumbrunn, Tadlock, and Roberts, 2011).

Figure 1 Phases of Self-Regulated Learning

Self-regulation is not a mental ability or an academic performance skill; rather it is the self-directive process by which learners transform their mental abilities into academic skills (Zimmerman, 2002). According to Chen (2002), self-regulation is neither a measure of mental intelligence that is unchangeable after a certain point in life nor a personal characteristic that is genetically based or formed early in life. So, there still options to continue developing self-regulation for any individual who wants to learn. Chen (2002) also cited Pintrich and

Zimmerman research that learner with high academic achievement is known better in implementing self-regulated learning than learners with low learning achievement.

Research Study

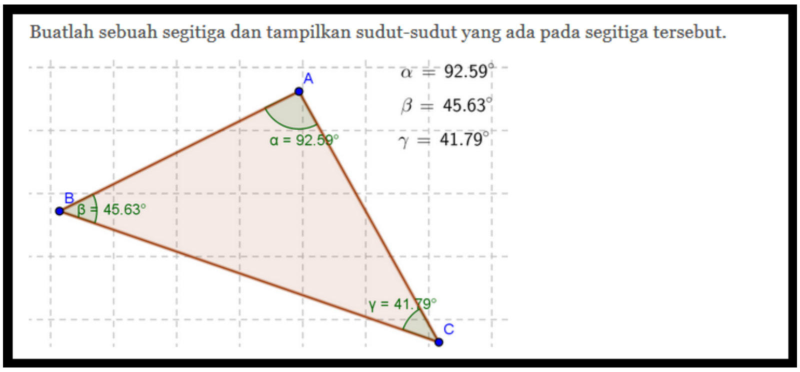
The considered experience was carried out within a course designed and run for mathematics teacher training of PPPPTK Matematika in 2015. The course could be accessed in http://diklatonline.p4tkmatematika.org/ with specific username and password. The course was GeoGebra. Aim of the course was to acquaint the participants with GeoGebra as learning and teaching tools so they became able to improve teaching and learning by using it. The course lasted 3 weeks full online. The participant cohorts was segmented into virtual workgroups, each supported by two tutors. There were three big cohorts, each with 100 participants.

Context and Methodology

The aim of this study was to profiling the participants self-regulated learning in online mathematics teacher training held by PPPPTK Matematika in 2015 using GeoGebra course as case study. A total of 219 teachers as the subjects of the study. Their background varied with regard to their genders, teaching experience, technology literacies, and levels of teaching.

During the course, nine tasks were sequentially proposed in three different discussion spaces created on purpose. The task were: Basic of GeoGebra (1-6), GeoGebra for System of Linear Equation topic (7) and GeoGebra for Geometry Transformation Topic (8-9). At the end of every task, the teachers were asked to respond a simple questionnaire consisted of 5 item statements to profile their self-regulated learning.

Furthermore, self-regulated learning to be surveyed in this study is restricted to three aspects, which are motivation, strategies, and self-evaluation. For the aspects of the strategy, there are three strategies: 1) allocating time for the sake to complete the task 2) find another learning resource and 3) collaborative with other participants. Questionnaire survey were compiled using a Likert scale of 1-5, with 1 expressed very disagree with the statement and 5 expressed very agree with the statement.



Buatlah sebuah segitiga dan tampilkan sudut-sudut yang ada pada segitiga tersebut.

Figure 2 One of GeoGebra Task in Online Training

Study Outcomes

Statistical analysis was done to ensure that the questionnaire was adequately valid and reliable. Mean and standard deviation for each scale were calculated to profile teachers' self-regulated learning. The value of Cronbach alpha scores for all 5 item statements is 0,65 implies that the questionnaire was adequately good in measuring teachers' self- regulated learning in online training.

Table 2. Average Item Mean and Standard Deviation Scores

Task	Motivation	Strategies	Ease (self-evaluation)	Mean	STD
Task 1	4.68	3.83	3.69	3.97	0.65
Task 2	4.67	3.81	3.42	3.89	0.64
Task 3	4.59	3.77	3.46	3.87	0.65
Task 4	4.70	3.85	3.48	3.94	0.64
Task 5	4.64	3.77	3.53	3.89	0.65
Task 6	4.65	3.76	3.44	3.88	0.63
Task 7	4.59	3.84	3.21	3.86	0.60
Task 8	4.57	3.77	3.33	3.84	0.67
Task 9	4.51	3.75	3.09	3.77	0.67
Mean	**4.62**	**3.79**	**3.41**		

Overall, teachers held positive trends on all aspects of self-regulated learning measured. The highest mean score is motivation, which is very good, may imply that teachers somewhat expected the benefits of GeoGebra as dynamic software for teaching mathematics. Out of 219 teachers, 73 teachers (33.3%) said that they have zero knowledge about GeoGebra before participated in this course. So it is understandable why the score on easiness was lower than others.

Motivation

Motivation is a critical factor that determines self-regulated learning. Motivation drives individuals to exert their best efforts to accomplished learning activities (Zumbrunn, Tadlock, & Roberts, 2011). All motivation mean scores of the participants to complete the tasks were above 4 which was very good category.

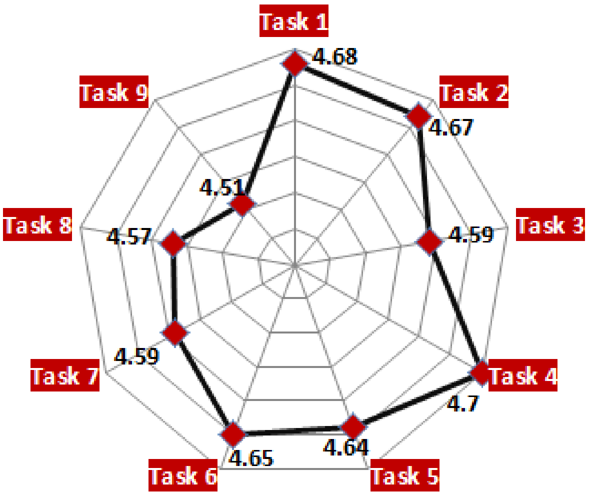
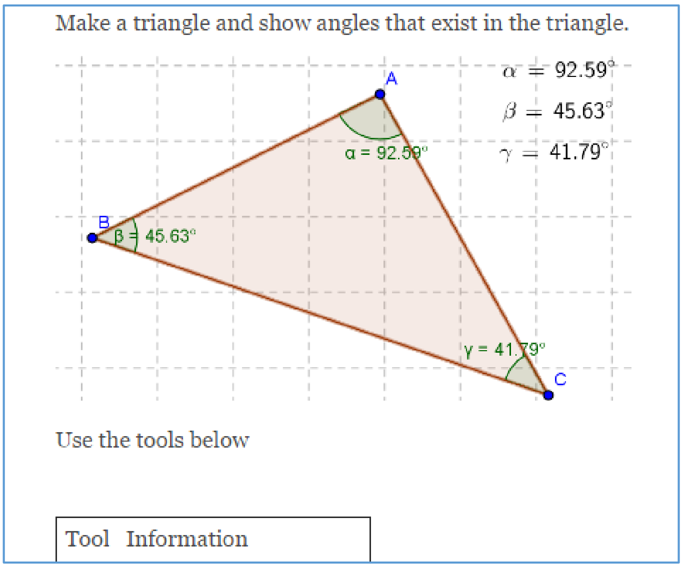


Figure 3 Motivation Score Diagram of GeoGebra Course Tasks

Highest motivation was when accomplished Task 4 (mean 4.70). Task 4 aims to draw simple triangle and its angles. A teacher said he was really helped with GeoGebra that he often draw such triangle to develop learning material.



Make a triangle and show angles that exist in the triangle.

Use the tools below

Tool Information

Figure 4 Task 4 in GeoGebra Course (diklatonline.p4tkmatematika.org)

Strategies

As discussed above, there are three strategies of self-regulated learning measured in this study: 1) allocating time for the sake to completed the task 2) find another learning resource and 3) collaborative with other participants. All strategies mean score of the participants to complete the tasks were between 3 to 4 which was good category. The result as shows in Figure 5 below.

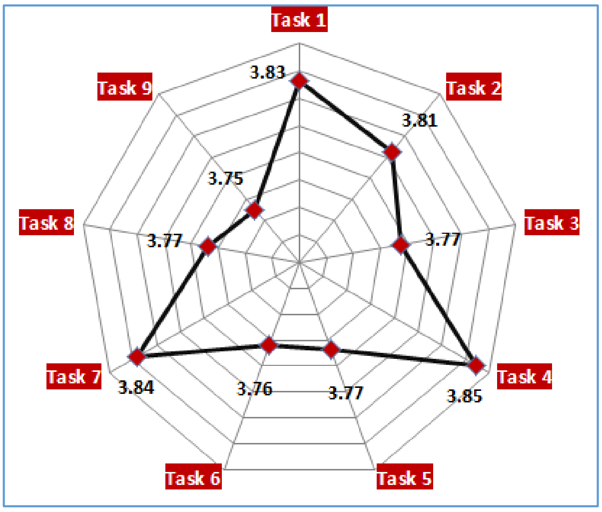


Figure 5 Strategy Score Diagram of GeoGebra Course Tasks

Time management involves scheduling, planning, and managing one’s study time (Chen, 2002). Online environment not only gives benefits because participant can learn anytime, but also could be neglectful when someone did not well-planned their schedule. Our tasks require learners to expand on the information presented in course materials. These type of activities encourage the learners to be more self-directed as they reflect on their learning processes. Furthermore, participants should not rely on facilitators or reading material given. Online environment makes it easy for participants to seek other learning resources using search engines. Moreover, online training allows participants to develop social constructivism through discussion in forums and chats. We also provided groups in social media channels to facilitate the participants discuss with other participants.

Self-evaluation

Last aspect of self-regulated learning of this survey is an evaluation. Delfino, et.al (2011) revealed that the self-regulated learning indicators of self-evaluation on computer-supported collaborative learning includes evaluation of cognitive and evaluation of emotion. This survey question is restricted in cognitive evaluation about level of ease of each task. When someone finds the material interesting but finds it somewhat difficult to understand, it will reminds him / her about needing a plan for better strategies to accomplish the tasks (Harris , Lindner, & Pina, 2011). All ease level mean score of the participants to complete the tasks were between 3 to 4 which was good category. The higher the value of response then the easier the task according to participants.

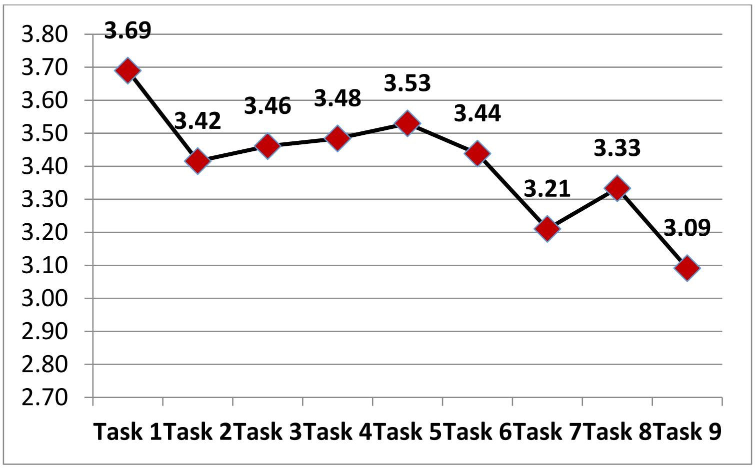
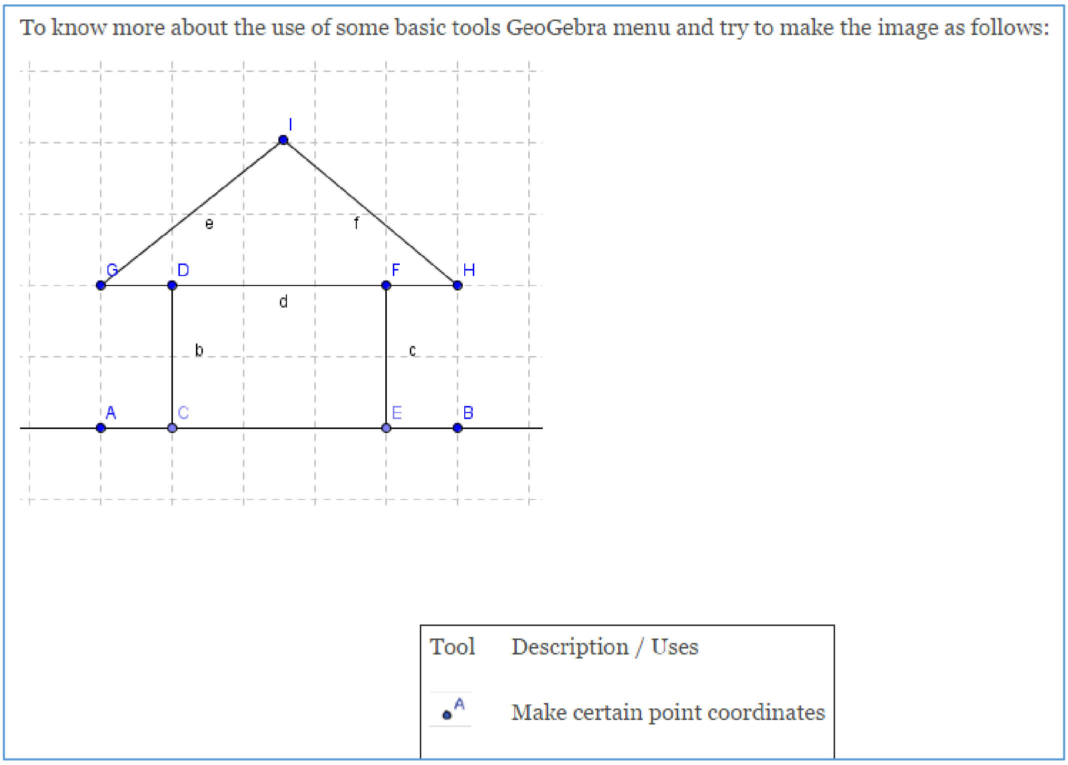


Figure 6 Ease Score Diagram of GeoGebra Course Tasks

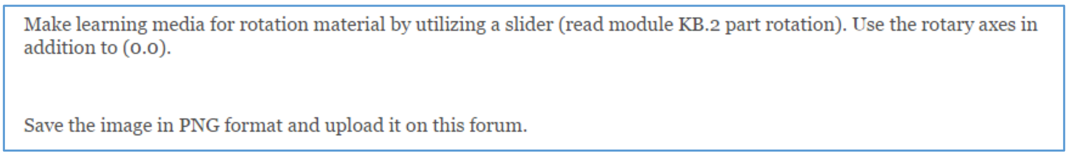
Figure 6 shows that Task 1 is the easiest task according to participant responses and Task 9 is the most difficult one.



To know more about the use of some basic tools GeoGebra menu and try to make the image as follows:

Tool	Description / Uses
•A	Make certain point coordinates

Figure 7 Task 1 in GeoGebra Course (diklatonline.p4tkmatematika.org)



Make learning media for rotation material by utilizing a slider (read module KB.2 part rotation). Use the rotary axes in addition to (0.0).

Save the image in PNG format and upload it on this forum.

Figure 8 Task 9 in GeoGebra Course (diklatonline.p4tkmatematika.org)

Task 1 was the easiest because it was the most basic for beginner in using GeoGebra. This task asks participants to draw any objects using GeoGebra drawing tools. While Task 9 was about creating learning media using GeoGebra to teach about rotation with non O(0,0) as center of rotation. Task 9 might become the most difficult for participants because they have to use their previous knowledge from previous tasks and no examples given in course materials.

Overall, participants' self-regulated learning profile can be seen in Table 3 below.

Table 3. Participants' Self-Regulated Learning Profile

Self-regulated learning category	Score	Frequency	%
Very Good	$180 \leq$ score ≤ 225	93	42.46
Good	$135 \leq$ score <180	116	52.96
Average	$90 \leq$ score < 135	9	4.10
Poor	$45 \leq$ score < 90	0	0

Out of 219 teachers, 93 teachers (42.46%) were in very good criteria, 116 teachers (52.96%) were in good criteria, while only 9 teachers (4.10%) were in average criteria of implementing self-regulated learning. There were no participant with poor self-regulated learning given fact that participants sign up for online training in PPPPTK Matematika by their personal initiatives.

Conclusion

This study reviews the issue of understanding the nature of the tasks proposed in GeoGebra online training affected the way teachers practiced self-regulated learning during the activities carried out to accomplish the tasks. Almost 96% teachers have positive self-regulated learning profile during GeoGebra online training, including motivation, strategies and self-evaluation as shows in Figure 9 below.

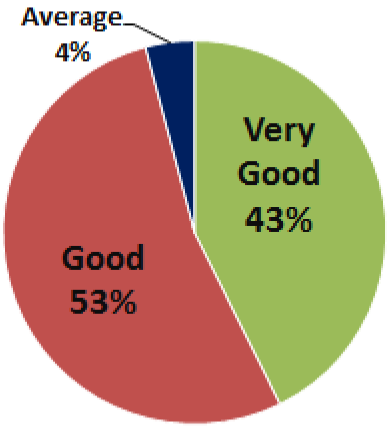


Figure 9 Diagram of Teachers Self-regulated Learning Profile

Out of three aspects of self-regulated learning measure in this study, participants have been very good in motivation, while they are good in processing strategies and self-evaluation.

Among the various obstacles for teachers to join online training, still they maintain the spirit to accomplish the tasks. Even though the GeoGebra course is still far away meet the ideal of online course, these study shows that these teachers were able to implementing self-regulated learning as a mean for teacher professional development.

As for the future, online course should be designed by considering self-regulated learning aspects. Online training should become a way to foster self-regulated learning for teacher's professional development. Positive self-regulated learning profile will encourage teachers to give their best efforts in teaching and learning.

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IMPROVING STUDENTS' MATHEMATICAL LITERACY ABILITY OF JUNIOR HIGH SCHOOL THROUGH TREFFINGER TEACHING MODEL

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Abstract

Mathematics is a queen and servant of sciences, it means that the development of mathematics is not depend on other science and mathematics as a basic for other science. The characteristics of mathematics are as a tool to solve problem and give a way to the real situation model. But oftentimes, mathematics is only looked as a theoretical science and the use of it cannot be perceived directly by student. It is looks from the PISA result in 2012, put Indonesian in 64th place from 65 countries in the world. That position, have not shown satisfactory progress, especially in mathematical literacy ability. By this ability, student can apply their ability to solve the problem. The mathematical literacy ability, creates the people be able to make decision based on constructive mathematical mindset. The mathematical literacy ability in this research is focused on level that student can perform the procedure properly, choose and apply the procedure, interprete on the different source and will communicate the result. The learning process still focused on the curriculum content achievement. In fact, there is not much exercise to measure mathematical literacy, also affect the student mathematical literacy ability at this time. One of the learning model that gives opportunity to develop the creativity in solving the problem freely is Treffinger's model. By This model, student can present some solutions from the problem and able to communicate the idea, so the student's mathematical literacy ability will improved.

Keywords : *Mathematical literacy ability, Treffinger's model*

Introduction

Mathematics, it is widely understood, plays a key role in shaping how individuals deal with the various spheres of private, social and civil life. Moreover, the scientist stated that "Mathematics is a queen and servant of sciences" it means that the development of mathematics is not depend on other science and mathematics as a basic for other science. In line with this, UU Nomor 20 Tahun 2003 stated that mathematics as one of must lesson for student in elementary and intermediate school. The goals to be achieved in the process of learning mathematics according to "Permendiknas No.22 Tahun 2006" in order the student be able to : 1) understand the mathematical concept, explain about interconnected within mathematics and applying the concept flexibly, correctly, efficiently and appropriately in solving problem; 2) use a reasoning in pattern and characteristic, doing manipulation in mathematics to made generalization, arrange proof or explain an idea and mathematical expression; 3) solving problem that consist of the ability to understand the problem, design the mathematical models, solving the models and interpreting the solution; 4) communicate an idea by symbol, table, diagram or other media to explicated the problem; 5) appreciated the useful of mathematics in life that is curiosity, attention and interest in mathematics, with persevere and confident in problem solving; 6) thinking logically, analytically, systematically, critically, creatively and able to working together in a group.

The goals in mathematics subject also defined by National Council of Teachers of Mathematics (NCTM, 2000), the objective of mathematics learning is to develop the ability of mathematical communication, mathematical reasoning and proof, mathematical problem solving, mathematical connection and mathematical representation. The combine of the fifth competences need to be owned in order to employ mathematics in everyday life. The ability that involved five competencies called mathematical literacy. When student know about mathematical literacy, they can apply their ability to solve their problem, since the center of mathematics is problem solving (NCTM, 2000). In addition, the ability of mathematical literacy, create the people be able to make decision based on constructive mathematical mindset (Yurika, Noviani and Wijayanti, 2015).

But, in reality, the ability of mathematical literacy has not been practiced to the fullest (Pulungan, 2014). It is supported from the PISA result in 2012. Program for International Student Assessment (PISA) is conducted by the OECD (Organization for Economic Co-operation and Development). PISA aims to examine the ability of children aged in 15 years in the regular reading (reading literacy), mathematics (mathematics literacy) and science (science literacy). Indonesia is one of the PISA participating countries that have joined since 2000. In 2012, Indonesia get 375 point (International score average is 500) and put Indonesia in 64th of 65 countries (OECD, 2014).

According to draft assessment framework PISA 2012 (OECD, 2010), mathematical literacy is an individuals' capacity to formulate, employ and interpret mathematics in a variety of context. It include, reasoning mathematically and using mathematical concept, procedure, facts and tools to describe, explain and predict phenomena. Mathematical literacy assist individuals to recognize the role or useful of mathematics in everyday life and use it to make well-founded judgments and decisions as a constructive, and reflective citizens (Jufri, 2015).

One of factor, the lack of mathematical literacy because of the learning process that occur was still focus on reaching the curriculum content, there is not much exercises to measure mathematical literacy (Yurika, Noviani and Wijayanti, 2015). The exercise only emphasise in retention and understanding, the learning process has not been able to develop mathematical thinking skills to a higher level (Rohaeti, Priatna and Dedy, 2013). Moreover, at this time, the learning process not fully connected to the everyday life. In line with this, Edo, Hartono and Putri (2013) investigated the difficulties of student in mathematical literacy, it showed that students is difficult to :1) formulating situations mathematically, such as for representating a situation mathematically, recognizing mathematical structure in problems; 2) evaluating the reasonableness of a mathematical solution in the context of a real-world problem. This situation showed that mathematical learning still to be result oriented. Learning process which needed by student for future has not to construct and develop optimally.

The mathematical learning that only result oriented and rule out the learning process will give the systemic effect, then cause the learning objective not reached

optimally. In general, conventional mathematics learning begins with an explanation of the topic, begins with explaining the learning material followed by showing the students examples and exercise and demonstrating how to answer those exercise. Such learning makes student tend to be passive, thus, gain limited knowledge and creativity. Furthermore, the learning process has not been able to develop mathematical thinking skills to a higher level.

Therefore, one alternative learning to cover the situation above is Treffinger learning model. Treffingers' model which firstly developed by Donald J. Treffinger, emphasize cognitive and affective dimension, in order to encourage learning process. Furthermore, the learning is required for students to be more communicative, so that learning situation becomes fun and interesting (Ekawati, 2013). Treffingers' model is encourages students to think creatively, when students deal with the various problem and providing opportunities for students to freely explore (Huda, 2014).

By using Treffingers' model, the learning process directed to the development or improving of students' mathematical literacy ability, especially in level 3, that is students able to do the procedure well, including procedure that requires decisions consecutively. Students able to select and apply simple strategies for solving problems, they able to interpreted and use the representations based on different sources, and explain their arguments, in the end, they able to communicate the outcomes and their arguments. When students given a mathematical problem, they are asked to comprehend the problem and work on the solution in small group. This situation facilitating students to express and share their ideas to the whole class (Alhaddad, Kusumah, Subandar and Dahlan, 2015). Beside that, the one advantages of Treffingers' model is required for student to develop creativity to solve the problem based on the facts and situation in everyday life, so that students able to find many solutions.

Based on the introduction above, this paper supposed to study, theoretically, about " Improving The Students' Mathematical Literacy Ability of junior high school through Treffinger Teaching Model".

Theoritical Framework

Mathematical Literacy Ability

Mathematical literacy is the one of human rights and the basic for long life learning, include various aspect of life (Mahdiansyah and Rahmawati, 2014). One

of aspect that individual needed is mathematical literacy. The definition of mathematical literacy based on draft assessment PISA 2012 (OECD, 2010) is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. The categories to be used for reporting are as follows: 1) Formulating situations mathematically. The word formulate in mathematical literacy definition refers to

individuals being able to recognize and identify opportunities to use mathematics and then provide mathematical structure to a problem presented in some contextualized form. In the process of formulating situations mathematically, individuals determine where they can extract the essential mathematics to analyze, set up, and solve the problem. They translate from a real-world setting to the domain of mathematics and provide the real world problem with mathematical structure, representations, and specificity; 2) Employing mathematical concepts, facts, procedures and reasoning. The word employ in the mathematical literacy definition refers to individuals being able to apply mathematical concepts, facts, procedures and reasoning to solve problems, individuals perform the mathematical procedures needed to derive result and find a mathematical solutions. They work on a model of the problem situation, establish regularities, identify connections between mathematical entities and create mathematical arguments; 3) Interpreting, applying and evaluating mathematical outcomes. The word interpreting used in mathematical literacy definition focuses on the abilities of individuals to reflect upon mathematical solutions, result or conclusions and interpret them in the context of real-life problems.

There are seven competences that also underpins mathematical literacy in practice, are follows: 1) communication, to communicate the problems; 2) mathematizing, to transform a problems from the real world to mathematical models; 3) representation, to represent a problem or an mathematical object; 4) reasoning and argument, a competence to analyze the information to create the conclusions; 5) devising strategies for solving problems; 6) using symbolic, formal and technical language and operation; 7) using mathematics tools.

According to PISA 2012, mathematical literacy not same in every level. Every level indicated level of competence achieved by student. OECD stated that there are six level which describes mathematical literacy ability. Consider, level 1 and 2 is the basic level of mathematical literacy, so this paper only focus on Mathematical literacy level 3. In addition, based on OECD (2014) Indonesian student achievement in level 1 and 2 is quite good. Furthermore, mathematical literacy level 3 define that students able to do the procedure well, including procedure that requires decisions consecutively. Students able to select and apply simple strategies for solving problems, they able to interpreted and use the representations based on different sources, and explain their arguments. Finally, they able to communicate the outcomes and their arguments. By, Mathematical literacy level 3 achievements, it supposed to strengthen step by step of measurement mathematical literacy level. Hence, the goal to be achieved in mathematical learning, such as reasoning and solving problem ability will be reached.

Treffinger Model

Treffinger model involved two skills, both cognitive and affective, which is connecting and depending each others, in order to encourage teaching and learning process. Furthermore, the learning is required for student to be more communicative so that learning situation becomes fun and interesting. Ekawati (2013) stated that there are many characteristic of Treffinger model, there are: 1) involves students in a problem and create the students to be an active participant in solving a problem, 2) integrate cognitive and affective dimension for looking the solutions will be pursued to solve the problem, 3) students requires investigating by collecting and analyzing information to strength an idea.

Huda (2014) stated that the model of teaching by Treffinger developed was a development model which process became the main concern. In the first stage of Treffinger model, *basic tools,* student can think divergently without fear of being rejected. The next stage, *practice with process,* the student are given complex problem that creates cognitive conflict. This will enable student to use their potential to solve the problem. Finally, *working with the real problem,* which involves the students thinking in a real challenge and encourages them to find out their own solution for the problems given.

Conclusion

The main component of mathematical literacy are mathematical modeling and problem solving in the real world (Ozgen, 2013). Furthermore, mathematical literacy level 3 emphasize on student ability in selecting and applying strategies to solve problem. Then, interpreting and representing based on different information, and communicate the outcome. Therefore, to improve mathematical literacy level 3, need to be develop understanding problem, solving problem and communicating skills.

According to Ekawati (2013), Treffinger model requires student opportunity to develop student thinking abilities and finding the way independently. In addition, Treffinger model able to emerge students curiosity and sensitive to problem (Darminto, 2013), providing opportunities for students to collect and analyze information through data presented contextually and communicate the outcome (Alhaddad, Kusumah, Subandar and Dahlan, 2015).

Based on the explanation above, Treffinger teaching model, theoretically can be used as a alternative teaching to improve the ability of students mathematical literacy level 3.

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MCREST as An Alternative Learning Strategy for Students in Learning Algebra

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Abstract

This paper described how secondary school students in Cimahi learned about algebra by using MCREST. MCREST stand for Meaningful, Confidence, Relevance, Enjoyment, Social Relationship, and Targets. It's one of the learning strategies driven by affective aspects motive. A topic of algebra was chosen as the mathematics strands. In this study, there were two mathematics classrooms of a secondary school in Cimahi was chosen to use this strategy. This study was held for a month from Augusts until September 2016. The data of this study were learning artifacts from students. Using this strategy, students will have an experience to study algebra in a different way, not only rote learning or memorize like mathematics school doing but also doing mathematics by their hands. This way will lead the students to rich representations and meaningful learning. Finding of this study may provide others teachers who are interested in various learning to use this strategy to develop motivation and creative thinking.

Keywords: *MCREST Strategy, Motivation, and Creative Thinking.*

Introduction

One of standards that have an important role in supporting implementation of educational goals is a standard process. According to PP No. 32 in 2013 (Kemdikbud, 2013a) standard process is the criteria regarding the implementation of learning in the educational unit to achieve competency standards that include attitudes, knowledge, and skills. According to Sanjaya (2012) standard educational process or *standar proses pendidikan* (SPP) as a minimum standard that must be made to have a function as a controller for obtaining the educational process and the quality of the learning process. In other words, in order that educational goals can be achieved with good results, student learning in the educational unit should be organized well too, begins with a lesson plan, the implementation of the learning process and learning assessment. PP No. 65 in 2013 (Kemdikbud, 2013b) requires learning principles include: (a) from the students were told to the students to find out; (b) from the teacher as the only source of learning-based learning into a variety of learning resources; (c) from the partial learning towards an integrated learning; (d) from learning that emphasizes the single answer to the truth of learning to multi-dimensional answers; (e) on verbal learning towards applicative skills; (f) learning that takes place at home, at school and in the community; (g) learning to apply the principle that everyone is a teacher, student, and everywhere is a class; (h) the use of information and communication technologies to improve the efficiency and effectiveness of learning; and (i) the recognition of individual differences and cultural background of learners.

It can be concluded that the learning process is expected to hold with interactive, inspiring, fun, challenging, motivating students to actively participate and provide enough space for innovation, creativity, and independence in accordance with their talents, interests, and physical and psychological development of students. Learning effort above the other not to reach graduates and core competencies expected of each student.

Permendikbud No. 69 in 2013 (Kemdikbud, 2013a) outlines those mathematical core competencies students need to have:

1. Understanding knowledge (factual, conceptual and procedural) by curiosity about science, technology, art, cultural phenomena and events related to the visible.
2. Trying, process, and describes in the realm of the concrete (using, parse, compose, modify, and make) and the realm of the abstract (writing, reading, counting, drawing, and writing) in accordance with the learned in school and other sources in the same viewpoints/ theories.
3. Understand, implement, and analyze factual knowledge, conceptual, procedural sense of knowing based on science, technology, art, culture, and humanities with the insight of humanity, national, state, and civilization-related causes of phenomena and events, as well as applying procedural knowledge in the field specific studies according to their talents and interests to solve the problem.
4. Processing, reasoning, and describes in the realm of the concrete and the abstract realm associated with the development of the learned in school independently, and able to use the method according to the rules of science.

From the above, it is clear that students are not only expected to have good skills in cognitive but also should be able to apply them in everyday life. A mathematical concept that has many connections with the daily life is algebra. Therefore, it was not a strange thing if algebra occupies a significant portion in school mathematics, which amounts to 29% (Siregar, 2011). The scope of materials according to Wardani (2008) is the algebra and its elements, linear equations and inequalities and their resolution, sets, and operations, relations, functions and graphs, systems of linear equations and solutions. Greer (2008) in his article titled "Algebra for all?" revealed that most people did not have to learn algebra to live in a society, but students have to learn and engage in learning algebra because of its gate or the first step into the wide world of education and economic opportunities.

Students attempt to apply the capabilities mentioned above must emerge from within the students themselves. Students are motivated to learn show interest, enthusiasm, and persistence of high learning, without depending on a lot of the teachers. Deci and Ryan (1985) divide motivation into three forms, namely intrinsic motivation, extrinsic motivation, and amotivation. Motivation arising from within self was called intrinsic motivation. The concept of intrinsic motivation that identifies a person's behavior when feeling happy about something then it will be motivated to perform these activities without any coercion from outside. In general, students have the intrinsic motivation to be more engaged in learning than students who had the extrinsic motivation (Gage and Berliner, 1988). This will impact on their academic success (Mullis, 2013).

Although intrinsic motivation occupies a dominant role in the success of students, but not necessarily all of the students have a good intrinsic motivation in learning. Skinner states that the environment will determine students' motivation to learn. It can be said that the learning environment in the classroom directly related to the extrinsic motivation of students.

The technique used to motivate students varies depending on the subject, the personality of students and teachers, and the learning environment (Yelon and Grace, 1977). According to Deci and Ryan (1985) intrinsically motivated students in mathematics or science subjects viewed it as something fun and interesting. One of the processes of learning appropriate to enhance students' skills involving aspects of attitudes, knowledge, and skill is learning strategy by using MCREST. Learning with MCREST strategy is a learning strategy that involves six drive or motive, there are meaningfulness, confidence, relevance, enjoyment, social relationships, and targets.

Discussion

Learning with MCREST Strategy

MCREST strategy was developed by Yoong (Yoong, 2014) in Singapore schools. Based on theory and learning associated with motivation, formulate a strategy involving the study of mathematics by six theories of motivation there are meaningfulness, confidence, relevance, enjoyment, social relationships, and targets hereinafter referred to MCREST. Theories related to motivation are derived from several theories of motivation in education. Some of them are according to (a) the behavioral theory, motivation comes from outside as reward and punishment; (B) the theory of affective more emphasis on emotions from within which is a source of motivation itself, (c) cognitive theory emphasizes the significance and achievements, (d) the theory of socio-cultural, that requirement has on the social environment and the group, (e) humanistic theory of self-actualization and value, and the latter is (f) theoretical neuroscientists who put more emphasis on the psychological and social cognition.

MCREST itself does not have the raw sequence in use during the study, but the sixth driver in MCREST will produce a very good thing when applied together in learning compared to only partially. Here will be described a discussion of MCREST.

1. Meaningfulness. The essence of MCREST itself is emphasizing that students are able to make sense of mathematics in the study. This is of course related to the meaningfulness (meaningfulness). When students understand the concepts and procedures, as well as the relationship of what they should have learned, directly they will be motivated to engage in learning activities that are more meaningful.
2. Confidence. Meaningful learning for students will have a positive correlation with increasing their confidence. Students are not allowed to dissolve in the problems it face in completing a given task without providing meaningful input to improve his self-confidence. The teacher's role is very beneficial when giving feedback to students.
3. Relevance. The next motive is relevance. At this stage, students are expected to answer questions about how the material studied relationship with real life, why should learn the material, and other things related to life in the future.
4. Enjoyment. Student involvement in pleasurable activities (enjoyment) is an excellent motivator, especially those activities foster their curiosity.
5. Social relationships. Basically, humans are social creatures. The need to engage in the group is a very good thing when it is developed from the beginning.
6. Targets. Target in MCREST more focused on helping students to develop and achieve the target of what they were supposed to get in learning. There are long-term targets, medium term, and short term. By knowing what targets they are supposed to achieve, students will be motivated to learn better.

Consideration teacher motivation is needed to determine what should be emphasized during the learning process. It relies on the experience of students in the past, needs at this time, and the ability of the pedagogy of teachers themselves. According to Hamachek (as cited in Prayitno, 1989) in the world of education, motivating students are (a) a process of guiding students to enter a variety of experiences where the learning process is ongoing; (b) a process that raises the excitement and liveliness to the students so that they are really ready to learn; and (c) a process that led to the students' attention focused on one direction or goal at a time, which is the purpose of learning. So the strategy MCREST is clear being one option appropriate learning strategies to improve mathematical ability to think creatively, as well as the motivation of students.

From the description of the learning strategy MCREST above, the author defined strategy MCREST as a learning strategy that involves six factors driving (motive) of learning, namely meaningfulness, confidence, relevance, enjoyment, social relationships, and targets, in this case six motives can run together or separately and without sequentially during learning in the classroom. Use of each of these motives depending on how teachers see what students are needs more during learning.

Some activities that can be selected in the learning with MCREST strategy are:

Table 1. Activities in MCREST Strategy

Strategy	Activity
Meaningful	1. Share stories, inspiring figures associated with the material being studied. 2. Perform hands-on activities related to daily life and mathematical concepts learned. 3. Presenting material in a variety of models of representation, one of them with a board "Multi-Model" in which there is a drawing activity (visual), symbolic (algebraic), written communications (description), and examples of the application in daily life (tell)
Confidence	1. Presenting the problems in easy and simple solution. 2. Providing feedback in the form of feedback to the students' answers is less precise. 3. Provide praise and reward of the work of the students were rated as excellent
Relevance	1. Discuss the material studied with everyday life. 2. Resolve the problems associated with the real world and the level of development of students.
Enjoyment	1. Presenting an unusual problem to be discussed with the group's friends. 2. Singing about mathematical concepts. 3. Presenting some games related to the material being studied. 4. Presenting the results of the discussion of students in the class.
Social Relationship	1. Form students into heterogeneous groups. 2. Familiarize students give feedback on the presentations of other student groups. 3. Allowing students to ask questions to the teacher if there is anything that is poorly understood. Grouping students into a heterogeneous group.
Targets	1. Create targets short term, medium term and long term by writing. 2. Helping students assess the target has been achieved or that has not been achieved. 3. Helping students develop strategies to achieve the targets that have been made

Learning with MCREST Strategy in Algebra

The following are examples of learning tools such as lesson plans and worksheets of algebra in eighth grades students. The subject matter had been implemented in several schools in Cimahi, lesson plans are presented only for the introduction of algebra.

a. Introduction (15 minutes)

Confidence:

1) Learning begins with a greeting and a prayer led by students.
2) Students are reminded again of the operations on numbers, such as: "Students, you have been able to perform addition or subtraction of numbers haven't you?", "Do you know how many kinds of numbers?", "Let's make an example of multiplication of several numbers!"
3) Students are informed of the purpose of learning today, such as "Today we will learn to recognize elements of algebra; in fact, you have ever known this form when you were at primary school level. For example: Suppose there is something added five, the result is 10, what is something? "
4) Students are given the opportunity to express their opinions about the algebra that he knows.

Meaningful:

5) Students are introduced to one of the idols in algebra, Al-Khwarizmi. Master pointed out that the word algebra was first written by a Muslim scientist named Muhammad ibn Musa in the year 820 AD in Khwarizmi in his book Al-Jabr wa-al Muqabala. Then the teacher communicates the benefits of the learning objectives of which are to make the presentation of a form to be simpler.

Social relationship:

6) Students are conditioned in a group that consists of four people.

Targets:

7) Teachers lead students to make short-term targets to be achieved, such as: "Children, from this learning what is roughly what you want to achieve? Try to write each of your targets today on the target board, you must share it later." The teacher gives one example of a target: "I should be able to distinguish the coefficients, variables, constants, a kind of tribe, two tribes, and tribes so much."

b) Core (55 minutes)

Meaningful and Enjoyment

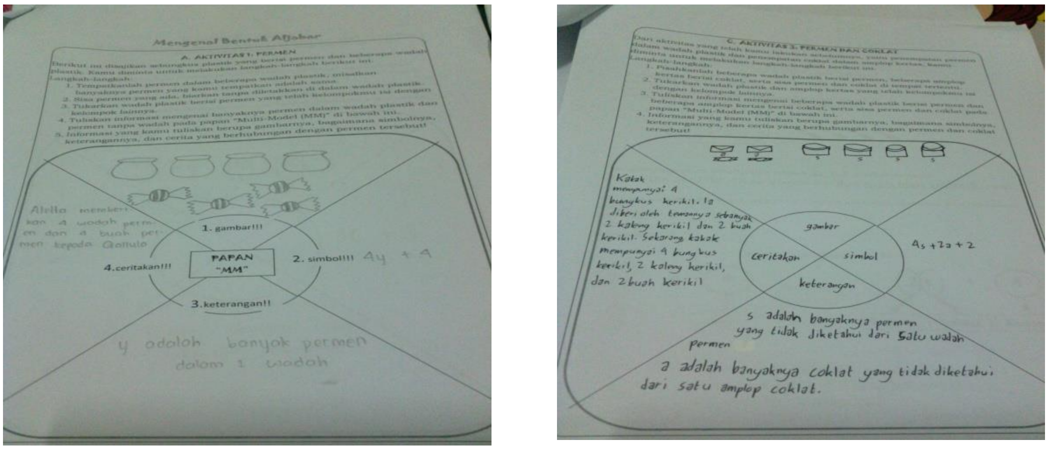
1) Students was given Worksheets Students (LAS), which contains some of the activities that should be undertaken students with their group, among these activities are related to (1)

Insert green candies into plastics, (2) Insert the chocolate into plastics, (3) Complete information, (4) Discuss the story.

For example in the following activity 1.

Each group of students was given several plastics and a large package contains green candy. Students were asked to enter some candies into a plastics.

Inform students to presuppose the number of candies in each plastic is the same. Then the rest of the green candies are not put into plastic. Students are required to write down the information on its activities to the board "Multi-Model (MM)". The information was written starting from describing, symbolize, provide information, and the situation related to the candy plastic. Here is an example of the board "MM".



Mengenal Bentuk Aljabar

A. AKTIVITAS 1: PERMEN

Alello memberikan 4 wadah permen dan 4 buah permen kepada Qollulo

1. gambar!!!

2. simbol!!! 4y + 4

3. keterangan!!

4. ceritakan!!!

PAPAN "MM"

y adalah banyak permen dalam 1 wadah

C. AKTIVITAS 3. PERMEN DAN COKLAT

Kakak mempunyai 4 bungkus kerikil. Ia diberi oleh temannya sebanyak 2 kaleng kerikil dan 2 buah kerikil. Sekarang kakak mempunyai 4 bungkus kerikil, 2 kaleng kerikil, dan 2 buah kerikil

gambar

simbol 4s + 2a + 2

keterangan

Ceritakan

s adalah banyaknya permen yang tidak diketahui dari satu wadah permen

a adalah banyaknya coklat yang tidak diketahui dari satu amplop coklat.

Figure 1. Sample Board "MM" Results of Student Work

2) The teacher noticed the students' work in groups, and provide assistance in the form of questions to provoke students to express their opinions, such as: "What symbols that you use for a plastic containing green candy?", "Suppose there are five plastics contain with green candies, how many green candies all you have?","What do you fill in the information? How about many candies? What the description of symbols?","How about the symbol? Is it amust always be in the form of letters? Can we use form of numbers or else? ".

3) In the next activity, students were given a large bag of chocolate and some plastic wrap. Then students are asked to perform the same activity in filling LAS. The difference lies in the provision of the symbol of the plastic contains green candy and chocolate.

4) Students are directed to perform activities such as incorporating three plastic containers containing green and brown plastic candy in one board "MM". Then the teacher work a way

around each group and give appreciation in the form of remarks such as "Wow, nice picture.", "Symbols that you made is obvious!", "Can you give another algebraic symbols?” Guiding groups of students who have difficulty to determine the information of the algebra, the teacher can give a direct example to them, such as "We suppose a plastic of green candy symbols x, a plastic of brown candy symbol y, can we give another symbol of something else?" "So if there are three green plastics contain green candy and two plastics contain chocolate, how are the symbols?", "Give me an example of the other!"

5) One group of students was asked to show his work in front of the class. Then another group of students directed to pay attention and give the question of what is shown.
6) Students were required to pay attention to the teacher's explanation about the kind of tribe, two tribes, and tribal plenty of examples of sweets and chocolates.

   Relevance:
7) Students were directed to fill out the information in LAS section (D) is the familiar form of algebra, which is a kind of ethnicity, different ethnic types, polynomials, coefficients, constants, and variables.
8) Students were directed to discuss the story presented in LAS section (E). There, students are directed to create the algebra of the story being presented. If students have difficulty making the story of algebra, the teachers provide assistance in the form of questions such as: "What is the symbol for a box of instant noodles?", "Amount of box can be symbolized by what?", "What difference?”

   Enjoyment
9) Students are given several options regarding the algebra games, such as: "Change the Symbol", "Fast Track" or "Math Bingo". All of the games associated with determining the coefficients, variables, constants, and polynomial.

   Change the Symbol: This game requires students to be able to make an algebraic form of a statement given by the teacher; it can be a short or long story. Students are asked to write symbols made on the board to be examined together.

   Fast Track: In this game, the students were asked to answer a few the question posed by the teacher as soon as possible, and then write the answers in front of the class. Students who have many correct answers are the winner.

   Math Bingo: Students presented boxes of four-by-four, in each box contains questions or statements that must be addressed by students. Students who can make a line horizontally, vertically or diagonally through three boxes is the winner.

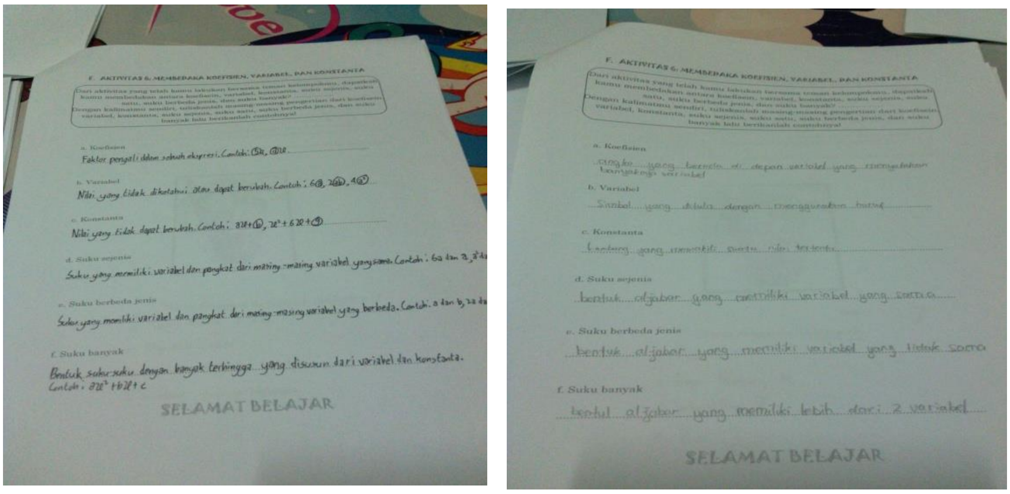
c. Closing (10 minutes)

Confidence

1) At the end of the learning, the student makes a difference in the coefficients, constants and variables in their own words (LAS section (F)), and one student was asked to express his opinion about the definition of the term.

Targets

2) Students were required to write down the target had been achieved today.
3) Students were reminded to recall today's lesson by reading students book or textbook about addition and subtraction of algebraic form.



F. AKTIVITAS 6. MEMBEDAKA KOEFISIEN, VARIABEL, DAN KONSTANTA

a. Koefisien
Faktor pengali dalam sebuah ekspresi. Contoh: ⑤x, ②x

b. Variabel
Nilai yang tidak diketahui atau dapat berubah. Contoh: [illegible]

c. Konstanta
Nilai yang tidak dapat berubah. Contoh: 3x+⑥, x² + 6x + ⑨

d. Suku sejenis
Suku yang memiliki variabel dan pangkat dari masing-masing variabel yang sama. Contoh: 6a dan a, a² da

e. Suku berbeda jenis
Suku yang memiliki variabel dan pangkat dari masing-masing variabel yang berbeda. Contoh: a dan b, 2a da

f. Suku banyak
Bentuk suku-suku dengan banyak terhingga yang disusun dari variabel dan konstanta.
Contoh: ax² + bx + c

SELAMAT BELAJAR

F. AKTIVITAS 6. MEMBEDAKA KOEFISIEN, VARIABEL, DAN KONSTANTA

a. Koefisien
angka yang berada di depan variabel yang menyatakan banyaknya variabel

b. Variabel
Simbol yang ditulis dengan menggunakan huruf

c. Konstanta
bilangan yang memiliki suatu nilai tertentu

d. Suku sejenis
bentuk aljabar yang memiliki variabel yang sama

e. Suku berbeda jenis
bentuk aljabar yang memiliki variabel yang tidak sama

f. Suku banyak
bentuk aljabar yang memiliki lebih dari 2 variabel

SELAMAT BELAJAR

Figure 2. Example Conclusion Students' Distinguishing Elements of Algebra

Conclusion

From the description of the learning strategy MCREST above shows that students have a great opportunity to develop their creativity, it can be seen from the presentation of algebra which is not only monotonous with symbols, but also with other representations such as images and verbal. The core of this strategy is the meaningfulness or meaningful learning. Mathematics courses understood, interpreted, conceptual, procedural and rational for students. They are motivated to keep learning until the end because it felt the significance of mathematics. When students feel that mathematics is meaningful, their confidence will increase. To maintain the confidence of students learning to be attentive needs of students, it means learning must have relevance to student life. Provide feedback with diverse forms such as flattery, gifts, or "penalty" that will preserve and even increase the confidence of students when they make sense of mathematics, making math a fun thing (enjoyment).

In addition, other things that need to be considered during the lesson was the relationship of students in a group (social relationships). The tendency of students in early adolescence pleased to make groups. Learning with MCREST facilitates student engagement strategy by forming a heterogeneous group as much as 4-5. By doing some given problem to be resolved within the time specified, the student will be awakened her motivation to accomplish their goals made (targets).

Acknowledgement

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ANALYSIS OF STUDENT'S MATHEMATICS COMMUNICATION SKILL THROUGH PISA-ITEM ON UNCERTAINTY AND DATA CONTENT

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Abstract

The mathematics communication skills of teacher student become an important things in the last few years. Students not only must have competencies to solve a problem but also have to share with the others. Share how to solve a problem can improve student's mathematics communication skill so their mathematics knowledge also can be enhance. Program for International Student Assessment (PISA) is an instrument to assess the students' mathematics literacy especially related to student's mathematics communication skills. Many efforts which have been spent by a country that joins this program to increase its PISA scores. This article studies the percentage of student's mathematics communication skill through PISA-item on uncertainty and data content.

Keywords: *PISA-item, Uncertaninty and Data, mathematics communication skills*

Introduction

Mathematics communication skill is one of the learning goals that line in Permendiknas No. 22, 2006 which declared about mathematics subject intended that studentshave skills of communicating ideas with symbols, tables, diagrams, or other media to clarify the situation or problem. In line with these, Curriculum 2013 states that student's mathematics communication skill should be enhance in order to prepare global competencies for students. Mathematics communication skill also one of the skills that observed by PISA.

PISA mathematics results show that start on 2000, 2003, 2006, 2009, and 2012, Indonesia occupied the position 39thof 41 participating countries, 38thof 40 participating countries, 50thof 57 participating countries, 61stof the 65 participating countries, and 64thof the 65 participating countries. PISA results show that Indonesia is still at the lower level. It means that mathematics skill of Indonesian students in solving problems that require the ability to review, giving reasons and communicating effectively, and solve and interpret problems in various situations is still lacking (Kamaliyah, Zulkardi, and Darmawijoyo, 2013).

Based on PISA result show that mathematics skill of Indonesian student is still in low level. It caused by student's mathematics communication skill is not effectively and interpret solution of the problems of PISA-item. This study will be categorise PISA-item in uncertainty and data content besed on mathematics communication skill.

Theoritical Framework

Mathematics communication skill

According to the National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics (2000), communication is an essential part of mathematics. The communication process helps build meaning. When students are challenged to think and reason and then communicate their ideas orally or in writing, true conceptual understanding develops. Listening to others' explanations provides students opportunity to clarify their understanding and consolidate mathematical ideas.(Kimberly Hirschfeld-Cotton, 2008)

The seven fundamental mathematical capabilities used in this framework are as follows: (1)*Communication,* (2) *Mathematishing,* (3) *Representation,* (4) *Reasoning and Argument* (5) *Devising Strategis for solving problems,* (6) *Using symbolic, Formal and Techical Language and Operation,* and (7) *Using Mathematics Tools* (PISA OECD, 2015).

Mathematical literacy involves communication. The individual perceives the existence of some challenge and is stimulated to recognize and understand a problem situation. Reading, decoding and interpreting statements, questions, tasks or objects enables the individual to form a mental model of the situation, which is an important step in understanding, clarifying and formulating a problem. During the solution process, intermediate results may need to be summarized and presented. Later on, once a solution has been found, the problem solver may need to present the solution, and perhaps an explanation or justification, to others. (PISA OECD, 2016: 70).

Sumarmo (2000) summarizedthe goal of learning mathematics language and symbols was to communicate mathematically so that students were able: (1) to reflect and explain their ideas mathematically; (2)to formulate definition of mathematics concepts and compile generalizationthrough invention method; (3)to express a figure, diagram, or a real situation into mathematical language,symbol, idea, or model;(4) to explain or clarify mathematical ideas, situation, or relation in daily languageorally or written; (5) to read, to clarify, and to examine mathematical presentation meaningfully; (6) to appreciate the beauty and the power of mathematical notations and used themaccurately and precicely. (Qohar dan Sumarno, 2013).

PISA (*Program for International Student Assesment*)

PISA is organized by the OECD in conjunction with a group of other participating countries, including Indonesia. The first survey took place in 2000, and then every 3 years since that time. PISA measures knowledge and skills of 15-year-olds, an age at which students in most countries are nearing the end of compulsory schooling. The focus is on areas that are important for life after school, including mathematics. PISA is a statistically rigorous programme to assess student performance and to collect data on the student, family and institutional factors that can help to explain differences in performance in countries around the world. (Stacey, 2011)

The OECD identifies the key features of PISA as: 1) policy orientation, with the major aim of informing educational policy and practice; 2) the PISA concept of "literacy" (see below) with a foundation of assessment of literacy for reading, mathematics and science; 3) its relevance to lifelong learning, so that assessment of knowledge is supplemented by reports on motivation to learn, attitudes towards learning and learning strategies; 4) its regularity, enabling countries to monitor improvements in educational outcomes in the light of other countries' performances on assessments held every 3 years; 5) measurement of student performance alongside characteristics of students and schools, in order to explore some of the main features associated with educational success; 6) breadth, with over 60 countries and economies participating by 2009, representing around 90% of the world economy. (Stacey, 2011)

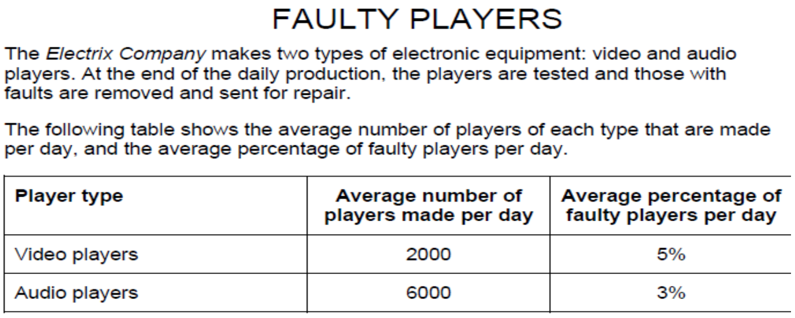
PISA 2012 will focus on mathematics as the main domain as in 2003. In each assessment, trend data is collected through an abbreviated test on the other two domains, using 'link items' from the earlier assessment to give results that can indicate trends. Students also answer a 30 minute background questionnaire, providing information about themselves, their attitudes to learning and their homes. School principals answer a questionnaire about their schools. These questionnaires provide baseline information about the conditions of schooling in different countries, and enable the examination of issues such as equity of schooling and effective practices. PISA is also developing other assessments. For example, it measures Information and Communication Technology skills and assesses the reading of electronic texts. In 2012, there will be an optional component on financial literacy. Since the science assessment of 2006, computer-based assessments have also been used to support a wider range of dynamic and interactive tasks. The mathematics assessment for 2012 will have an optional computer-administered component, which will provide new opportunities for presentations of items and may also test some aspects of doing mathematics assisted by a computer.(Stacey, 2011).

PISA has also conducted tests of general problem solving, and will do so again in 2012. The problem solving assessment taps students' "capacity to use cognitive processes to confront and resolve real, cross-disciplinary situations where the solution path is not immediately obvious and where the literacy domains or curricular areas that might be applicable are not within a single domain of mathematics, science or reading" (OECD 2013: 156).

The mathematics assessment also contains many items that might be considered problem solving, but they draw explicitly on mathematics content. The contents of PISA-item are: (1) *Change and Relationship,(2) Space and Shape;* (3) *Quantity; and* (4)*Uncertainty and data.* The *Uncertainty and data* content category includes recognising the place of variation in processes, having a sense of the quantification of that variation, acknowledging uncertainty and error in measurement, and knowing about chance. It also includes forming,interpreting and evaluating conclusions drawn in situations where uncertainty is central.

PISA-item on Uncertainty and Data Content

Problem 1



FAULTY PLAYERS

The *Electrix Company* makes two types of electronic equipment: video and audio players. At the end of the daily production, the players are tested and those with faults are removed and sent for repair.

The following table shows the average number of players of each type that are made per day, and the average percentage of faulty players per day.

Player type	Average number of players made per day	Average percentage of faulty players per day
Video players	2000	5%
Audio players	6000	3%

Question : FAULTY PLAYERS (*PM00EQ02 – 019)*

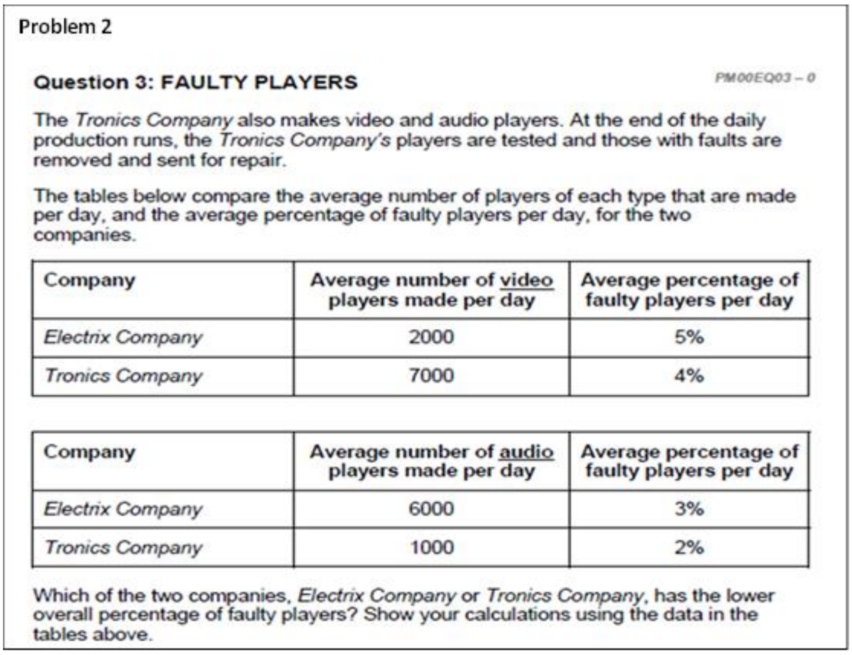
One of the testers makes the following claim:

"On average, there are more video players sent for repair per day compared to the number of audio players sent for repair per day." Decide whether or not the tester's claim is correct. Give a mathematical argument to support your answer.

....

(Source: PISA 2012 Released Mathematics Items 2013: 41-42)

Picture 1. PISA-item about faulty players



Problem 2

Question 3: FAULTY PLAYERS PM00EQ03 – 0

The *Tronics Company* also makes video and audio players. At the end of the daily production runs, the *Tronics Company's* players are tested and those with faults are removed and sent for repair.

The tables below compare the average number of players of each type that are made per day, and the average percentage of faulty players per day, for the two companies.

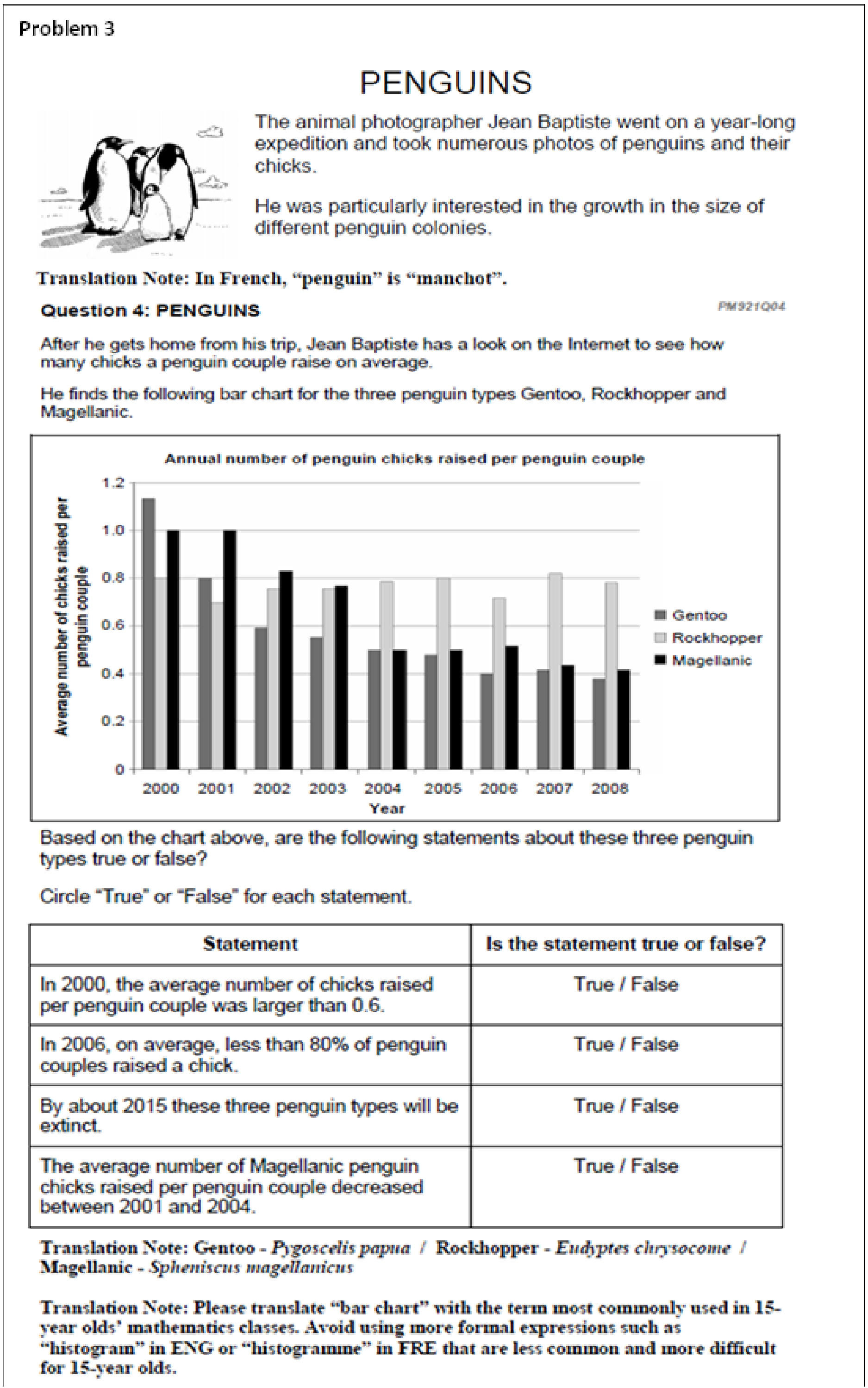
Company	**Average number of video players made per day**	**Average percentage of faulty players per day**
Electrix Company	2000	5%
Tronics Company	7000	4%

Company	**Average number of audio players made per day**	**Average percentage of faulty players per day**
Electrix Company	6000	3%
Tronics Company	1000	2%

Which of the two companies, *Electrix Company* or *Tronics Company*, has the lower overall percentage of faulty players? Show your calculations using the data in the tables above.

(Source: PISA 2012 Released Mathematics Items 2013: 41-42)

Picture 2. PISA-item about faulty players 2



Problem 3

PENGUINS

The animal photographer Jean Baptiste went on a year-long expedition and took numerous photos of penguins and their chicks.

He was particularly interested in the growth in the size of different penguin colonies.

Translation Note: In French, "penguin" is "manchot".

Question 4: PENGUINS PM921Q04

After he gets home from his trip, Jean Baptiste has a look on the Internet to see how many chicks a penguin couple raise on average.

He finds the following bar chart for the three penguin types Gentoo, Rockhopper and Magellanic.

Based on the chart above, are the following statements about these three penguin types true or false?

Circle "True" or "False" for each statement.

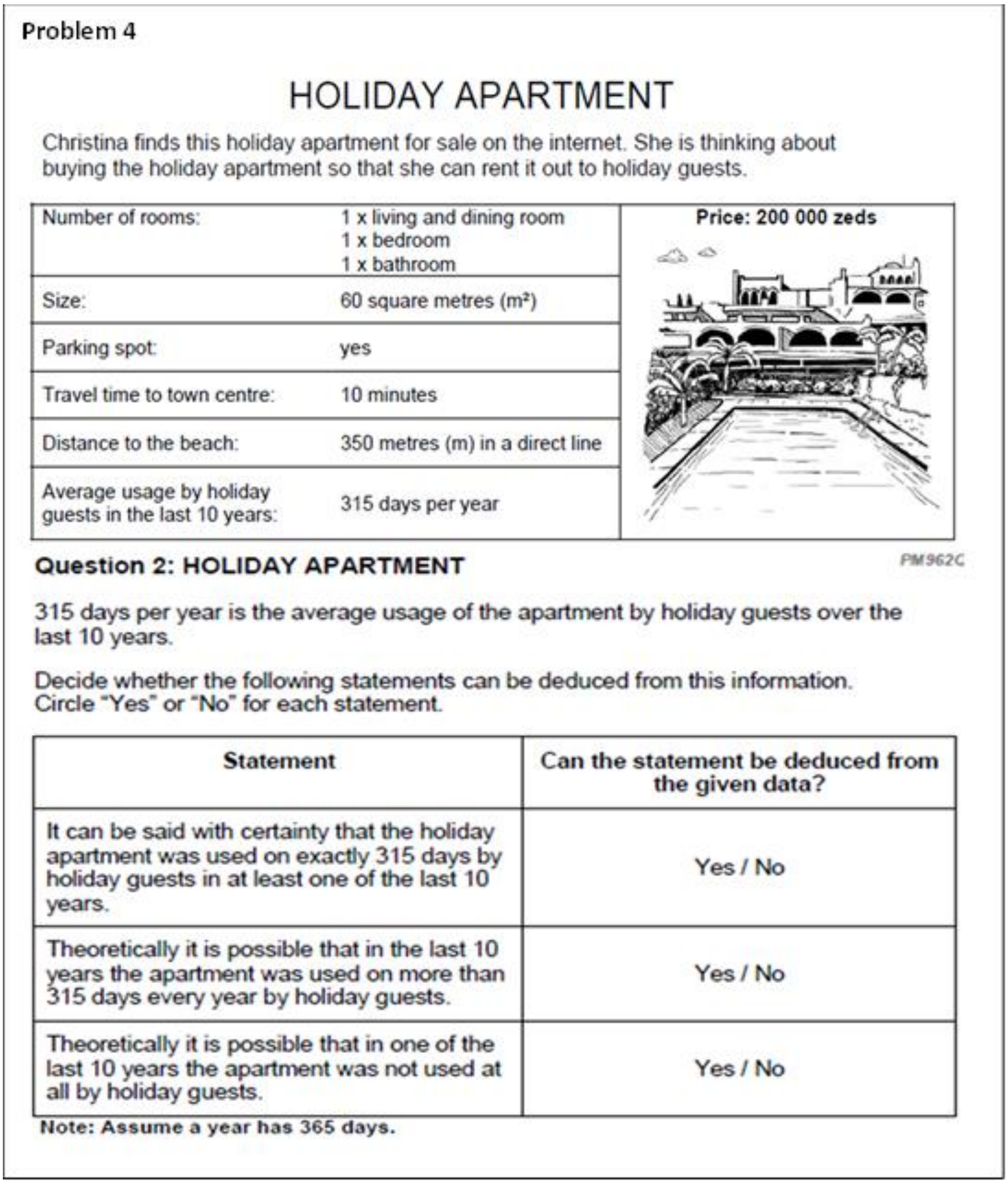
Statement	Is the statement true or false?
In 2000, the average number of chicks raised per penguin couple was larger than 0.6.	True / False
In 2006, on average, less than 80% of penguin couples raised a chick.	True / False
By about 2015 these three penguin types will be extinct.	True / False
The average number of Magellanic penguin chicks raised per penguin couple decreased between 2001 and 2004.	True / False

Translation Note: Gentoo - *Pygoscelis papua* / Rockhopper - *Eudyptes chrysocome* / Magellanic - *Spheniscus magellanicus*

Translation Note: Please translate "bar chart" with the term most commonly used in 15-year olds' mathematics classes. Avoid using more formal expressions such as "histogram" in ENG or "histogramme" in FRE that are less common and more difficult for 15-year olds.

(Source: PISA 2012 Released Mathematics Items: 57)

Picture 3. PISA-item about Penguins



Problem 4

HOLIDAY APARTMENT

Christina finds this holiday apartment for sale on the internet. She is thinking about buying the holiday apartment so that she can rent it out to holiday guests.

Number of rooms:	1 x living and dining room 1 x bedroom 1 x bathroom
Size:	60 square metres (m²)
Parking spot:	yes
Travel time to town centre:	10 minutes
Distance to the beach:	350 metres (m) in a direct line
Average usage by holiday guests in the last 10 years:	315 days per year

Price: 200 000 zeds

Question 2: HOLIDAY APARTMENT

PM962C

315 days per year is the average usage of the apartment by holiday guests over the last 10 years.

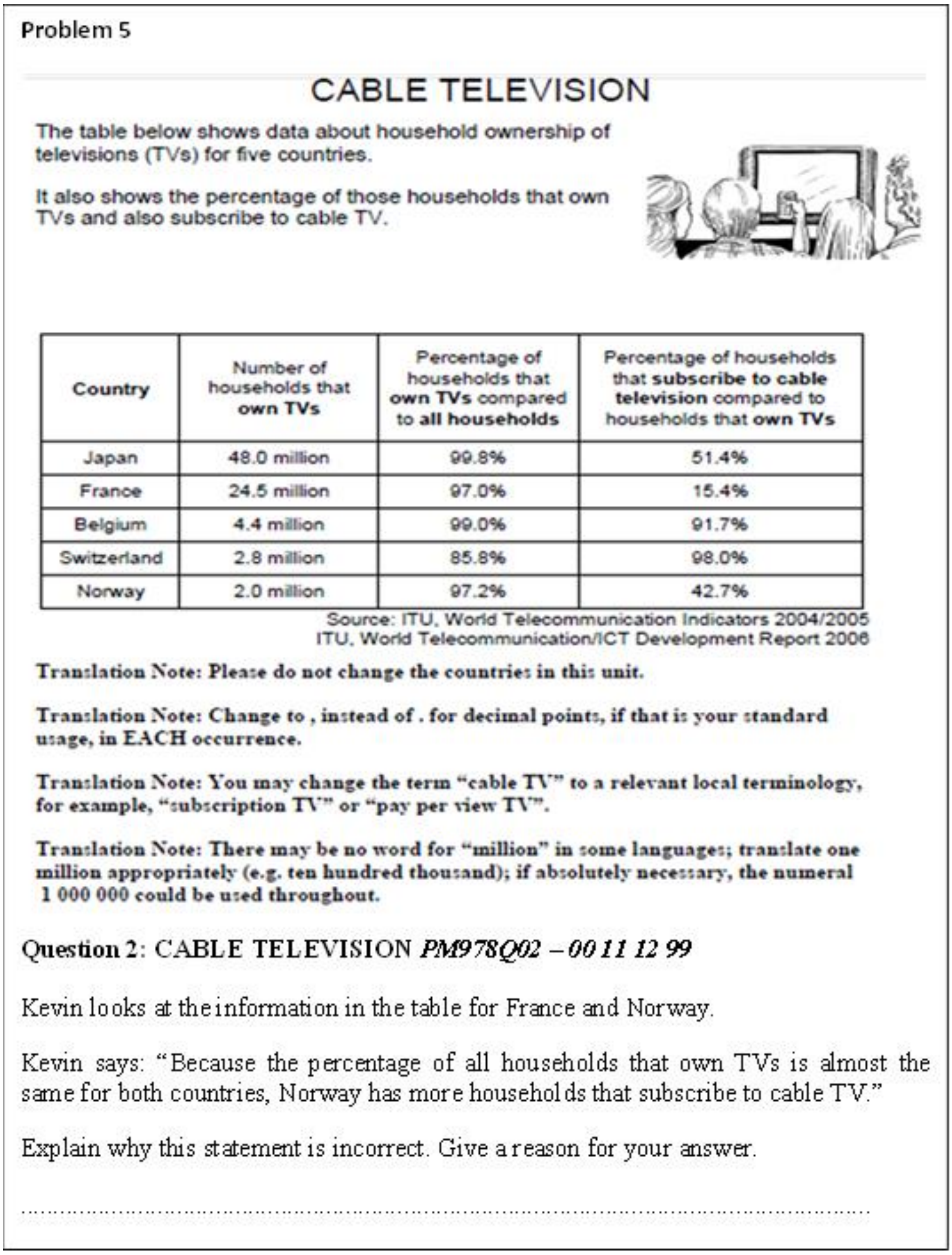
Decide whether the following statements can be deduced from this information. Circle "Yes" or "No" for each statement.

Statement	Can the statement be deduced from the given data?
It can be said with certainty that the holiday apartment was used on exactly 315 days by holiday guests in at least one of the last 10 years.	Yes / No
Theoretically it is possible that in the last 10 years the apartment was used on more than 315 days every year by holiday guests.	Yes / No
Theoretically it is possible that in one of the last 10 years the apartment was not used at all by holiday guests.	Yes / No

Note: Assume a year has 365 days.

(Source: PISA 2012 Released Mathematics Items: 57)

Picture 4. PISA-item about Holyday Apartment



Problem 5

CABLE TELEVISION

The table below shows data about household ownership of televisions (TVs) for five countries.

It also shows the percentage of those households that own TVs and also subscribe to cable TV.

Country	Number of households that own TVs	Percentage of households that own TVs compared to all households	Percentage of households that subscribe to cable television compared to households that own TVs
Japan	48.0 million	99.8%	51.4%
France	24.5 million	97.0%	15.4%
Belgium	4.4 million	99.0%	91.7%
Switzerland	2.8 million	85.8%	98.0%
Norway	2.0 million	97.2%	42.7%

Source: ITU, World Telecommunication Indicators 2004/2005
ITU, World Telecommunication/ICT Development Report 2006

Translation Note: Please do not change the countries in this unit.

Translation Note: Change to , instead of . for decimal points, if that is your standard usage, in EACH occurrence.

Translation Note: You may change the term "cable TV" to a relevant local terminology, for example, "subscription TV" or "pay per view TV".

Translation Note: There may be no word for "million" in some languages; translate one million appropriately (e.g. ten hundred thousand); if absolutely necessary, the numeral 1 000 000 could be used throughout.

Question 2: CABLE TELEVISION *PM978Q02 – 00 11 12 99*

Kevin looks at the information in the table for France and Norway.

Kevin says: "Because the percentage of all households that own TVs is almost the same for both countries, Norway has more households that subscribe to cable TV."

Explain why this statement is incorrect. Give a reason for your answer.

……

(Source: PISA 2012 Released Mathematics Items: 73-75)

Picture 5. PISA-item about Cable Television

Result and Discussion

Categorising mathematics communication skill in uncertainty and data content will be identified by indicators of mathematics communication skill which have arranged. Indicators of mathematics communication skill have made based on the definitions above. Indicator of mathematics communication skill can be modified by Sumarno's and OECD's definitions. These are: 1) formulate definition of mathematics concepts and compile generalization through invention method; (2) to express a figure, diagram, or a real situation into mathematical language, symbol, idea, or model; (3) to explain or clarify mathematical ideas, situation, or relation in daily language orally or written and may need to present the solution, and perhaps an explanation or justification to the others; (4) to read, to clarify, and to examine mathematical presentation meaningfully; (5) to appreciate the beauty and the power of mathematical notations and used them accurately, precicely and may need to be summarized and presented.

Then, indicators of mathematics communication skill will be identified on 5 PISA-items. Theme of these item are: (1) Faulty players; (2) Penguins; (3) Holiday Apartment; (4) Cable Television. The result of analysis mathematics communication skill on PISA-item seatled on table below.

Table 1. The result of analysis mathematics communication skill on PISA-item

No.	Indicators of mathematics communication skill on PISA-item	Problem 1		Problem 2		Problem 3		Problem 4		Problem 5	
		Yes	No	Yes	No	Yes	No	Yes	No	Yes	No
1.	PISA-item fasilitate student's to formulate definition of mathematics concepts and compile generalization through invention method			√						√	
2.	PISA-item fasilitate student's to express a figure, diagram, or a real situation into mathematical language, symbol, idea, or model			√		√				√	
3.	PISA-item fasilitate student's to explain or clarify mathematical ideas, situation, or relation in daily language orally or written and may need to present the solution, and perhaps an explanation or justification to the others	√		√						√	
4.	PISA-item fasilitate student's to read, to clarify, and to examine mathematical presentation meaningfully.			√		√		√			
5.	PISA-item fasilitate student's to appreciate the beauty and the power of mathematical notations and used them accurately, precicely and may need to be summarized and presented			√						√	

Based on Table 1, actually these PISA-item are involved mathematics communication skill to assess student's mathematics knowledge which are formulate definition of mathematics concepts and compile generalization through invention method; express a figure, diagram, or a real situation into mathematical language, symbol, idea, or model; explain or clarify mathematical ideas, situation, or relation in daily language orally or written and may need to present the solution, and perhaps an explanation or justification to the others; read, to clarify, and to examine mathematical presentation meaningfully; appreciate the beauty and the power of mathematical notations and used them accurately, precicely and may need to be summarized and presented.

PISA-item on problem 1,2, and 5 provide challanges students to make their symbol and decoding of mathematics to help their calculation easier. Students can express a figure, diagram, or a real situation into mathematical language, symbol, idea, or model based on problem that faced. Then students must make the relation in daily language orally or written and may need to present the solution, and perhaps an explanation or justification to the others. Explanation and present to the others also tested and make meaningfully presentations of the problem may also test some aspects of doing mathematics assisted by a computer.

PISA-item on problem 3 and 4 provide challanges to think about the interpretations based on data above. But the questions must be answered by Yes or No and there are not reason or summirezed the calculation why to answer Yes or No. It makes mathematics communication skill not be improve well but it provide student to develop their mathematical thinking.

PISA tested on 2012 for mathematics assessment have been an optional computer-administered component, which will provide new opportunities for presentations of items and may also test some aspects of doing mathematics assisted by a computer. That so, all of mathematics communication skill have been assessed on PISA-item and Indonesia should be facilitate opportunity for student to improve their mathematics communication skill not only state on paper test but also the presentation skill to share mathematics solution.

Conclussion

Based on categorising PISA-item that involved mathematics communication skill, it conclude that all PISA-item on uncertainty and data content prepared challenges for students to solve the problem not only to get the solution but also must be state how to made it solution easy to understand for the others. But, PISA test don't facilitate the students to discuss with other so the communication to work in team not be improve. In classroom, teacher must facilitate student's to face PISA-like item so that students learn more many kind of problems and based on PISA framework. Beside it, teacher can develop to immatiate items on PISA when they test their students. Teacher can develop PISA-like items which improve students' mathematics communication skill.

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Eight Grade Students Effectiveness Toward Web-Based Learning In Polyhedral

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Abstract

Student's difficulties in understanding polyhedral was one of the reason for developing web based learning media.The study was aimed at knowing the effectiveness of the implementation of web-based learning media for the students. The subject of this study was class VIII of SMPN 12 Mukomuko. Experimental method was adopted in this study. Samples were divided into control and experimental groups. Control group was taught by traditional method and experimental group with web-based learning. Pre-test and post-test was administered to both the groups. The findings showed that there is no significant mean difference between the student's achievements of pre-test and post-test taught by traditional method, and there is significant mean difference between the student's achievements of pre-test and post-test taught with web-based learning.

Keywords:*effectiveness, web-based, geometry ployhedral, learning media*

Introduction

Web Based Instruction is becoming more important in the field of education as the education paradigm is changing, shifting the classroom initiatives from teachers to students by HA Jin Hwang (2011:1-12). The Internet is very useful in the field of education, especially as a medium of learning that is packaged in the form of websites. The Internet has great potential to improve the quality of teaching in schools, particularly in mathematics. Many things that are difficult abstract or imaginative thought students can be presented through simulation internet, so the development of mathematical learning process to empower teachers to use the internet.

Based on observations at SMP Negeri 12 Mukomuko, it is known that the source of student learning provided only limited to textbooks, student worksheets, and the material presented in the classroom teacher. Learning resources to make minimal limitations of competence and knowledge of students. The use of facilities and infrastructure are in school as a learning medium to support the learning process is still limited to the blackboard and charts.

SMP Negeri 12 Mukomuko had own laboratory computers connected to the Internet network, had a wifi network facilities that can be accessed by teachers and students. From the results, it is known that the use of the Internet as a medium of learning of mathematics has not been optimally utilized. Computer laboratory facility is only used for learning the field of study of ICT (Information and Communication Technology). While the wifi network is available just additional facilities of schools so that teachers and students can access the internet, but its utilization as a source of learning and learning media is rarely used.

The results of interviews with students, it is known that the math still seems difficult and unpleasant. One of the factors that cause students feel math is difficult is the lack of teachers' creativity in making media so that learning does not seem interesting and boring. One of the components that influence the effectiveness of learning mathematics is with the selection of appropriate learning media. This is in accordance with the views expressed by Daryanto (2010: 5), the use of media in the learning process can clarify the message that is not too verbalize, overcoming the limitations of space and time, and excite student learning. So it can be concluded that the use of instructional media provides good stead in the learning process.

Viewed from the side of the learning process, the material geometry flat side have an important role in math. This is due to understanding the material three-dimensional space on a higher level takes a good grasp on the material side of the flat geometry. The fact is that this material is a material that is difficult to understand the concept of students. The material geometry of space is one of the subject matter of abstract mathematics. This can lead to the abstract nature of difficult students understand the material. Student difficulties in understanding this material resulting in the learning process only find numbers, formulas, charts, and pictures to die so as to make the students feel less interested and feel the material lessons boring.

Based on the above, is the development of instructional media on the material geometry flat side of a web-based valid and practical that can help students in understanding the math. This study examines the use of web-based learning media which have been in SMP 12 Mukomuko as a source of optimal learning to improve student learning outcomes.

Subject and Method

This research was conducted at SMP 12 Mukomuko. The subjects were 50 students of class VIII. Experimental method was adopted in this study using experimental design pretest-posttest control group design. Pre test was administered in order to find out the entry behavior of students toward web based learning. Sample consists of 50 students of which 24 of control group and another 24 for experimental group. Opportunity was given to the students to explore the websites at their own pace. Post test was conducted in order to find out the effectiveness of web based learning.

Table 1. Plan Design Research

Goup	***Pretest***	**Tratment**	***Posttest***
Experiment	O1	X1	O2
Control	O3	X2	O4

Flowchart of this study can be see at Figure 1.

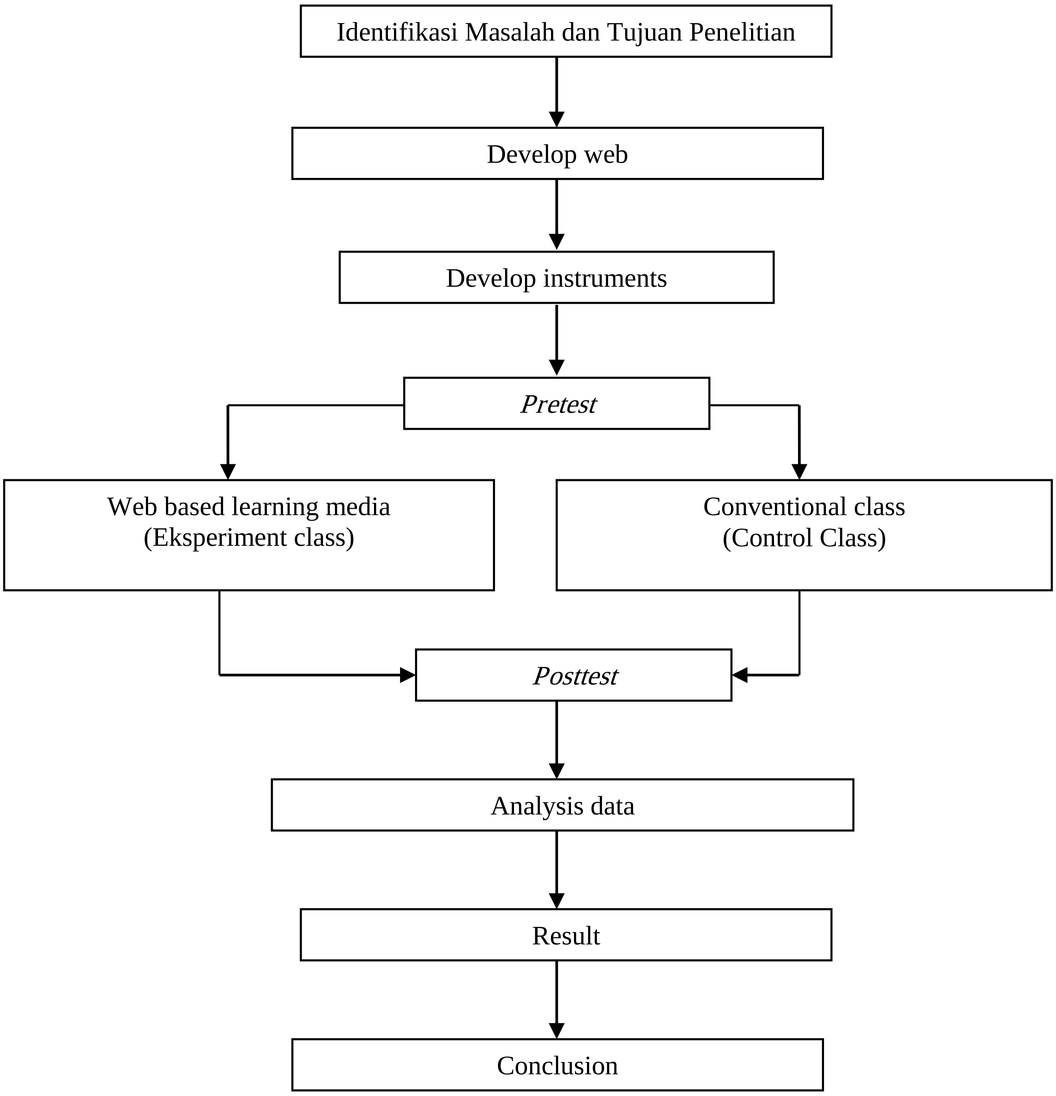

Figure 1. Flowchart

Data Analysis Technique

The data obtained were classified into two groups of data, namely quantitative and qualitative data. Technical data analysis performed using descriptive quantitative method, to process data pretest and posttest results. The effectiveness of web-based learning media on the material side room flat wake analyzed by adapting the theory Hake regarding normalized gain. The gain is the difference between the posttest and pretest. The gain indicates increased understanding or mastery of concepts students after learning process. Hake (1999), the value of the gain normalized formulated as follows:

$$g = \frac{skor\ Posttest - skor\ pretest}{Skor\ maksimum - Skor\ Pretest}$$

Large normalized gain is interpreted to declare the criteria of gain normalized by Richard R.Hakke (1999):

Table 2. Gain Score Classification

G Score	Interpretation
0,7 < g < 1	High
0,3 <g .7	Medium
0 < g < 0,3	Low

Result

Results showed that proven through analysis of statistical tests with SPSS 16.0 shows that early ability control class and experimental class is the same (homogeneous). It can be seen from the average value of both class and pretest results evidenced by t-test to see similarities two averages. The results show that there is no difference between the initial ability of the experimental and control classes. This is reasonable because the class is not getting the treatment and study materials. After the learning process carried out by treating with a medium of learning E-Learning in the experimental class and treatment with conventional learning media in the control class, study results show that both groups experienced the differences end. Differences in learning outcomes indicated by the average value of the experimental class 86.09, while the control class 80.34. From the average value posttest seen that the learning outcomes of the experimental class are higher than the control class. To determine the effectiveness of instructional media use web-based E-Learning in the experimental class and the use of conventional learning media also used the calculation of the normalized gain. The results of the test calculation using the normalized gain g values obtained for the control class is at 12:30, while the value of g for the experimental class is at 0:54. Based on the value of g above shows that the learning outcomes of the experimental class are higher than the control class.

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GeoGebra as a Means for Understanding Limit Concepts

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Abstract

Limits is a major concept in calculus that underpins the concepts of derivatives and integrals. The common misconception about limits is that students treat the value of a limit of a function as the value of a function at a point. This happens because the teaching of limits only leads to the procedural understanding without a proper conceptual understanding. Research suggest the importance of geometrical representations to a meaningful conceptual understanding of calculus concepts. In this research, GeoGebra as a dynamic software is used to support students' understanding of limit concepts by bridging the students' algebraic and geometrical thinking. In addition to this, Realistic Mathematics Education (RME) is used as a domain theory to develop an instructional design regarding how GeoGebra can be used to illustrate and explore the concept of limits so that students will have a meaningful understanding both algebraically and geometrically. Therefore, this research aims to explore the hypothetical learning trajectory in order to develop students' understanding of limit concepts by means of GeoGebra and an approach based on RME. The result shows that students are able to solve limit problems and at the same time they try to make sense of the problem by providing the geometrical representation of it. Therefore, the use of geometric representations by GeoGebra and RME approach can provide a more complete understanding of the concepts of limits.

Keywords: *calculus, design research, GeoGebra, limits, RME.*

Introduction

Calculus has been one of the most fundamental concepts in various areas in life, such as in physics, science, engineering, finance, etc. Moreover, calculus is a compulsary subject not only for college students but also for high school students. Most of the teaching-learning of calculus in classroom is still oriented to the technical procedures in solving problems without having a sufficient understanding of the underlying concepts. Ahuja, et.al (1998), Gravemeijer and Doorman (1999), Lee (2006), and Weigand (2014) suggest the importance of geometrical representations in the teacing-learning of calculus to have a meaningful conceptual understanding.

The concept of limits plays a central role in understanding the concept of derivative and integral in the calculus course. Based on Weigand (2014), a conceptual understanding of the formal limit concept is challenging for students and requires explanations and visualizations using different representations (beyond the symbolic representation). Furthermore, the understanding of the process of the construction or calculation of limits in the sense of step-by-step processes on numerical and graphical levels is essential for the understanding of the limit concept beyond a formal definition of limits and this can be supported by computer visualizations.

Over the last few decades, the use of innovative technology in calculus teaching and learning has been an interest in mathematics education (Gravemeijer&Doorman, 1999; Hohenwarter, et all, 2008; Özmantar, 2009; Diković, 2009; Haciomeroglu& Andreasen, 2013). By means of technology, the teaching and learning of calculus has shifted into more active and dynamic where students can explore various mathematical concepts with multi representations. In this research, a dynamic mathematics software GeoGebra is used in the teaching-learning of limits concept. GeoGebra is an open-source software for mathematics teaching and learning that offers geometry, algebra and calculus features in a fully connected and easy-to-use software environment. By using GeoGebra, students are able to construct, manipulate, and give arguments about mathematical objects (Hohenwarter, et al, 2009). Therefore, it is believed that GeoGebra can support the development of students' thinking in understanding limit concepts by providing its visualization or geometrical representations of the formal definition of limits.

The need of contextual situation in developing an instructional design for the teaching-learning of limit concepts is as important as providing the means. In other words, a specific domain theory Realistic Mathematics Education (RME) is used in this research in designing the instructional design or the so-called the hypothetical learning trajectory. RME was first developed in 1971 in response to a need to improve the quality of mathematics teaching and learning in the Netherlands. The main features of RME are the use of contexts and models, students' own constructions and productions, interactivity, and intertwinement (Treffers, 1987).

Although considerable studies have been done in the teaching-learning of calculus, it is noteworthy that the research in calculus learning and teaching has not capitalized on advances in design research to further link theories of learning with theories of instructional design (Rasmussen, et al. 2014). Moreover, design research provides a productive perspective for theory development, and results of design research which encompass design of activities, materials, and systems are the most useful results regarding the improvement of the education system as the ultimate goal of educational research (Edelson, 2002). Therefore, by design research methodology, this paper aims to explore the hypothetical learning trajectory to develop students' understanding of limit concepts by means of a dynamic software GeoGebra and an approach based on Realistic Mathematics Education (RME) theory.

Methodology

The methodology used in this research is "design research" which was first proposed as "developmental research" by Freudenthal in the Netherlands to develop the so-called domain-specific instruction theory of RME (Gravemeijer & Cobb, 2006; Freudenthal, 1991). The purpose of this design research is to develop a class of theories about both the process of learning and the means designed to support that learning, be it the learning of individual students, of a classroom community, of a professional teaching community, or of a school or school district viewed as an organization (Cobb et al, 2003)

Basically, a design research has three essential phases, which are the design and preparation phase (thought experiment), the teaching experiment phase (instruction experiment), and the retrospective analysis phase (Gravemeijer & Cobb, 2006; Cobb et al., 2003). Each of these forms a cyclic process both on its own and in a whole design research. Therefore, the design experiment consists of cyclic processes of thought experiments and instruction experiments (Freudenthal, 1991).

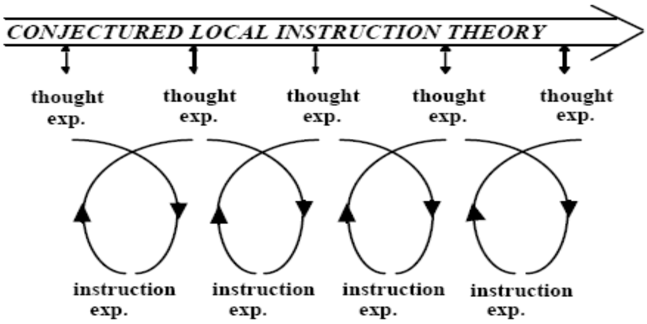


Figure 1. Reflexive relation between theory and experiments (Gravemeijer & Cobb, 2006)

In the first phase of this design research, the instructional design, namely, the hypothetical learning trajectory was developed under the guidance of the domain-specific instruction theory RME, and then put to the test in the teaching experiment phase, and finally the conjectures are either proved or disproved in the analysis phase. In this respect, the conjectured local instruction theory guides the cyclically teaching experiment phase while the experiment contributes to the development of the local instruction theory.

The teaching experiment was conducted in Universitas Negeri Jakarta during the course of Differential Calculus in year 2015/2016 in a class of 49 students enrolled in the course. The data collected during the teaching experiment was video recording of the classroom activities, interviews, students' written test, and field notes in every meeting. The field notes consist of the development of students' thinking in understanding limit concepts as well as students' contribution and interactivity in relation to the GeoGebra as a means to support the learning and the RME approach. During the teaching experiment, the researcher also acted as an observer along with another observer.

The retrospective analysis deals with a set of data collected during the teaching experiment where the HLT was compared with students' actual learning. The HLT functions as guidelines determining what the researcher should focus on in the analysis. The results of the retrospective analysis will form the basis for adjusting the HLT and for answering the research questions.

To ensure the quality of data and instruments as well as to justify the results (the HLT including its instructional activities) of the design research, we refer to the methodological norm of reliability (virtual replicability) and validity. A criterion for virtual replicability is 'trackability' (Gravemeijer & Cobb, 2001). This means that the reader must be able to track the learning process of the researchers and to reconstruct their study: failures and successes, procedures followed, the conceptual framework used, and the reasons for making certain choices must all be reported. Furthermore, internal reliability can be interpreted as inter-subjective agreement among the researchers on the project. Meanwhile, the design research should provide a basis for adaptation to other situations by offering a thick description (results) of what happened in the classroom (Gravemeijer & Cobb, 2006). That is what we mean by the ecological validity of this design research.

Results and Discussions

Results of this paper is described qualitatively where the data were collected from the video recording of the classroom situation, field notes of students' activities and discussion, and students' written test. The development of students' thinking of a number of students was analyzed, not only for the student as individuals, but also for their participation in and contribution to the development of classroom mathematical practice. The role of GeoGebra was highlighted as well in supporting the development of students' understanding of limit concepts. On top of that, the hypothetical learning trajectory was compared to the actual learning process that happened in the teaching experiment. The following section describes the hypothetical learning trajectory along with the results obtained from the teaching experiment.

Before the teaching experiment was conducted, a number of questions were given to the students in order to identify their previous knowledge and understanding about limit concepts. Most of the students' conceptions about limits was that the value of a limit of $f(x)$ when x

approaches c is the same as the value of $f(c)$, that is if $\lim_{x \to c} f(x) = L$ then $f(c) = L$. Students often explain about the procedural techniques in solving limit problems, either by substitution or factorization of the function. However, they failed to explain "What happens to the function $f(x)$ as x gets close to constant c but different from c ?".

Based on the above evidence, students tended to think limits as limit problems that they have to solve algebraically. In other words, students did not have the concept image of limits or the visualization of a function when x approaches some constant. The diagram in figure 2 shows that the concepts of graphs and functions underlie the concept of limits, where limits is essential to the development of the concepts of derivatives and integrals in calculus. Therefore, graphic function was chosen as the context to start the learning trajectory in order to acquire the intuitive meaning of limits. Moreover, a graph of discontinuous function is chosen to manage students' misconceptions about limits. Accordingly, students were expected to be able to think geometrically and algebraically in developing the intuitive meaning of limit.

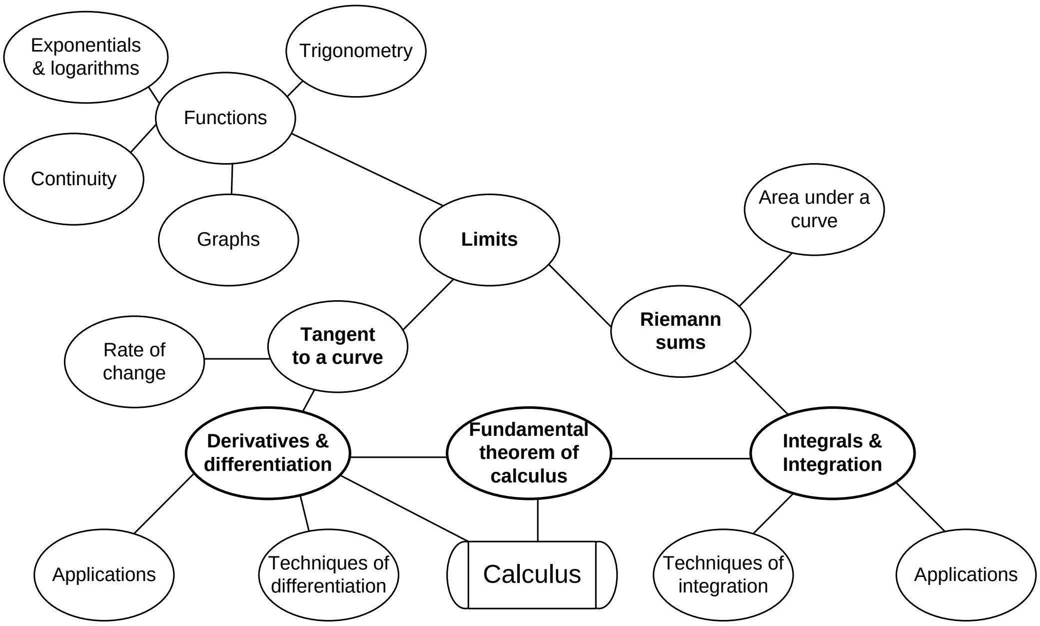


Figure 2. The concepts that underlie calculus concept (source: Lee Peng Yee. 2006. Teaching Secondary School Mathematics.)

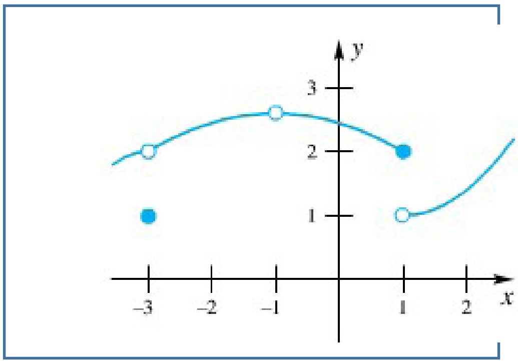


Figure 3. A graph of discontinuous function

By showing the above graph to students and asking “What is happening to *f*(*x*) as *x* approaches 1?”, students were able to reconstruct their understanding about limits, and at the same time, they construct the concepts of right-hand and left-hand limits. Moreover, the graph in figure 4 below is able to strengthen students’ intuitive notion of limits by investigating the behavior of the function at certain points.

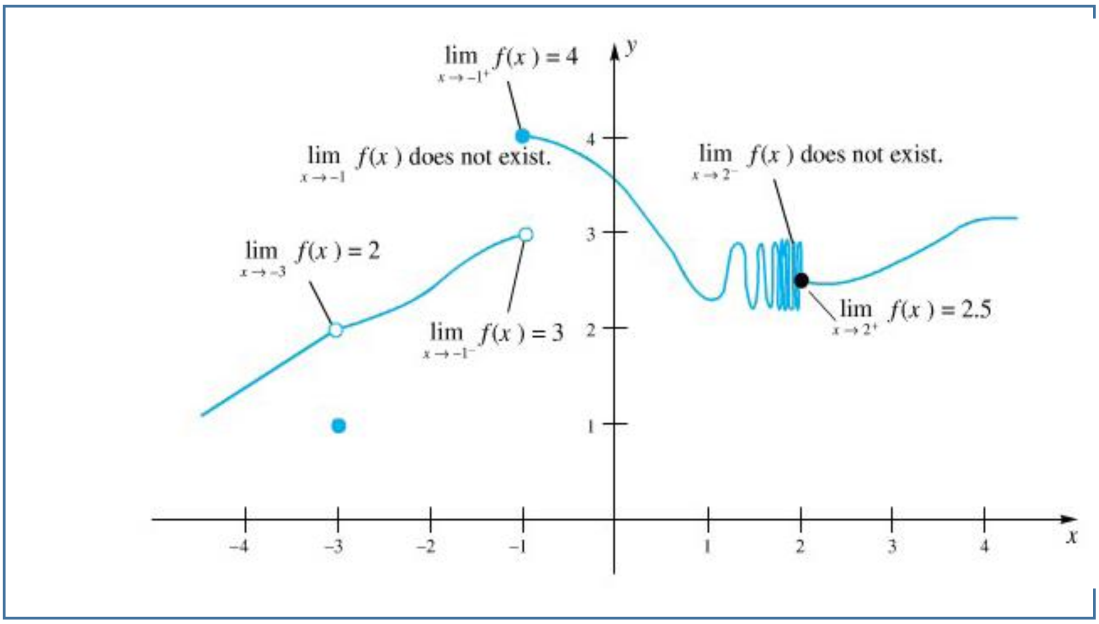
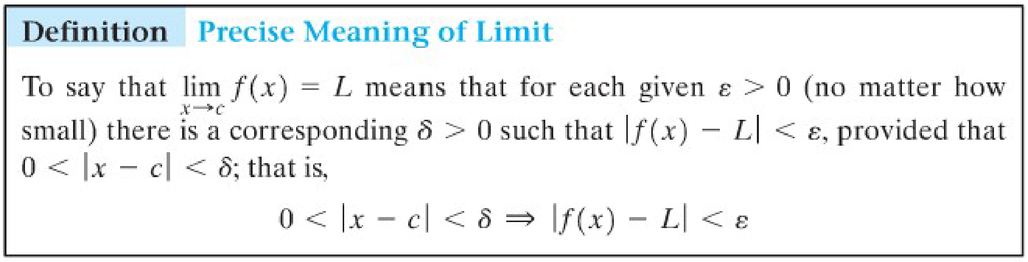


Figure 4. A graph of discontinuous function (source: Varberg, et.al, Calculus 9th Edition, pg.59)

On the next level, the software GeoGebra was applied to construct the formal definition of limit. By providing the dynamic geometrical representation in GeoGebra,, students are able to construct the idea of an arbitrary small positive integer epsilon (ε) in the neighborhood of *L* and the corresponding another small positive integer delta (δ) in the neighborhood of *c*.



Definition Precise Meaning of Limit

To say that $\lim_{x \to c} f(x) = L$ means that for each given $\varepsilon > 0$ (no matter how small) there is a corresponding $\delta > 0$ such that $|f(x) - L| < \varepsilon$, provided that $0 < |x - c| < \delta$; that is,

$$0 < |x - c| < \delta \Rightarrow |f(x) - L| < \varepsilon$$

source: Varberg, Purcell, Rigdon. Calculus 9th Edition (pg: 62)

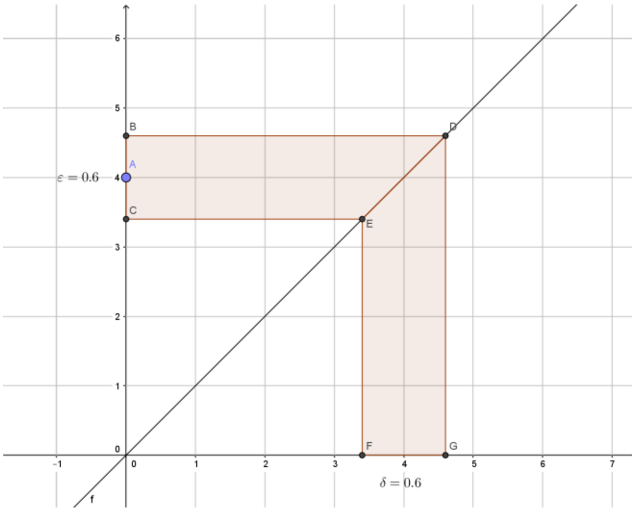


Figure 5. A screenshot of the dynamic visualization of the formal definition of limits.

The advantage of using GeoGebra is its dynamic feature that enable us to see how the changes of the chosen epsilon affect the changes of delta. By using GeoGebra, students can investigate the relation between epsilon and delta geometrically along with the algebraic analysis when giving a formal proof to a limit problem.

There was a huge progress in terms of the classroom culture built in the classroom. During the teaching experiment, students get used to work in group. They had the responsibility to share ideas and contribute to each other. Students were very active in discussing the solution of problems, for instance, when a student wrote down a solution of the given problem on the whiteboard and there was a little error, then another student corrects together and contribute in improving the solution of the problem being discussed.

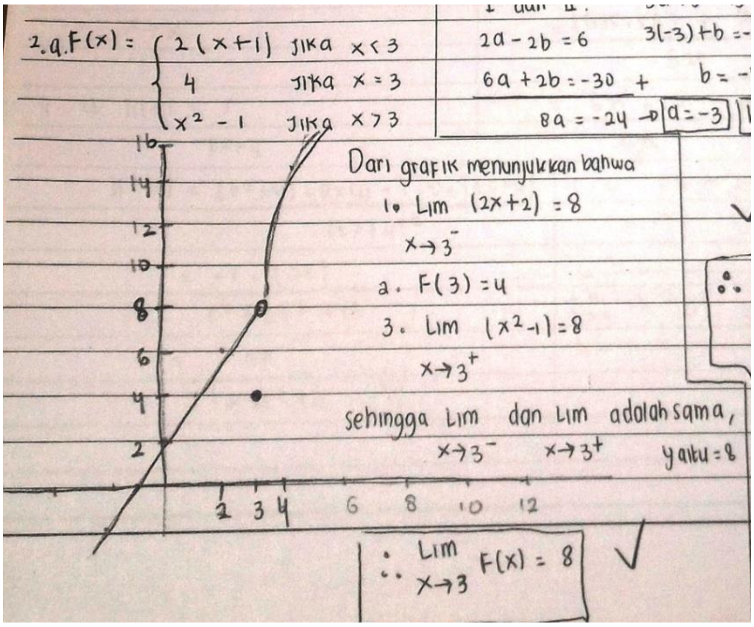
Based on the result of concepts review, most of the student can manage their misconceptions about limits. Figure 6 shows the questions given to students to review their conceptual understanding of limits. The result shows that 20 out of 38 students can respond correctly for numbers 1 and 4, while less than half of the students can answer correctly for numbers 2, 3, and 5, that is 18 students answer correctly for number 2, 17 students for number 5, and 13 students for number 2.

Respond with true or false for each of the following assertions. Be prepared to justify your answer.

1. If $f(c) = L$, then $\lim_{x \to c} f(x) = L$.
2. If $\lim_{x \to c} f(x) = L$, then $f(c) = L$.
3. If $\lim_{x \to c} f(x)$ exist, then $f(c)$ exist.
4. If $\lim_{x \to 0} f(x) = 0$, then for every $\varepsilon > 0$ there exist $\delta > 0$ such that $0 < |x| < \delta$ implies $|f(x)| < \varepsilon$.
5. If $f(c)$ is undefined, then $\lim_{x \to c} f(x)$ doesn't exist.

Figure 6. The questions for concepts review

Students' understanding of the formal definition of limit had developed by means of GeoGebra. Based on the result of students' test, there were 14 out of 38 students who can give a complete formal proof of limit, 7 students made a little error, and 15 students who success in the preliminary analysis but didn't give the formal proof.



$$2.a.\ F(x) = \begin{cases} 2(x+1) & \text{jika } x < 3 \\ 4 & \text{jika } x = 3 \\ x^2 - 1 & \text{jika } x > 3 \end{cases}$$

$2a - 2b = 6$ $3(-3)+b =$

$6a + 2b = -30$ + $b =$

$8a = -24 \rightarrow \boxed{a = -3}$

Dari grafik menunjukkan bahwa

1. $\lim_{x \to 3^-} (2x+2) = 8$ ✓

2. $F(3) = 4$

3. $\lim_{x \to 3^+} (x^2-1) = 8$

sehingga $\lim_{x \to 3^-}$ dan $\lim_{x \to 3^+}$ adalah sama, yaitu = 8

$\boxed{\therefore \lim_{x \to 3} F(x) = 8}$ ✓

Figure 7. a student's written test

Furthermore, the work of a student above shows that the geometrical representation of a function help students to make sense of the problem. In the question, students were asked to find $\lim_{x \to 3} f(x)$ if:

$$f(x) = \begin{cases} 2(x+1) \text{ jika } x < 3 \\ 4 \qquad\quad\ \text{jika } x = 3 \\ x^2 - 1 \quad \text{jika } x > 3 \end{cases}$$

The problem does not provide the graph of the function, so students have to evaluate the limit of f(x) from both sides, right and left-hand limits. Figure 7 is an evidence that by chosing a meaningful context, that is the graph of functions, students can solve problems algebraically, and at the same time they can make the problem meaningful to them by giving the geometrical representation of the problem.

Conclusion

In conclusion, evidence shows that GeoGebra supports the development of students' thinking in understanding limit concepts by its dynamic feature, where the chosen ε (epsilon) in the neighborhood of *L* determines the value of δ (delta) in the neighborhood of *c* in the formal definition of limit. Furthermore, the use of context as one of the characteristics of RME (Realistic Mathematics Education) helps students to make sense of what they are doing. For instance, drawing the graph of a function as an attempt to make sense of the problem in determining the limit of the function at a certain point, and showing in the graph that the right- and left- hand limits are equal.

In addition to this, students' contribution and interactivity in the classroom show a significant influence in the development of students' thinking. By sharing ideas and strategies, giving arguments and justifications, both in whole class discussion and group-discussion, students are able to move to a higher level of thinking in understanding limit concepts.

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SQ3R Model To Develop Students' Mathematical Literacy

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Abstract

The literacy skill of Indonesian students is very low compared to students from other countries in Asia. National library data showed that the average citizen of Indonesian citizens read 0-1 books per year. PISA study showed the ability of Indonesian students' mathematical literacy is ranked 64th out of 65 countries. While the literacy skills needed in the mathematical logical thinking, using mathematical concepts and solve problems. One alternative that teachers can do is apply the learning model SQ3R comprising Survey, Question, Read, Recite, and Review. The advantages applying Sq3r model in developing mathematical literacy are (1) can make students more confident in understanding and solving problems, (2) Assist the concentration of students, (3) Helps focus the attention of students and identify the difficulties in getting an answer, (4) Train provide answers in questions about the material, and (5) Help prepared the records in question and answer form. applying SQ3R model also to support the 'Gerakan Literasi Sekolah (GLS) and in accordance with the prescribed curriculum in 2013 the ministry of education and culture programs.

Keywords: *Learning Model, Literacy, SQ3R*

Introduction

The development of science and technology would not have happened if the perpetrator does not have the thinking and reasoning ability. The highlight of the results of thinking and reasoning is creativity. The creativity enables humans to create the infrastructure to facilitate the participation of culture of human life in the modern society.

Thinking and reasoning ability must be preceded by the ability to read. Reading ability does not a pronounced activity, spell or sound, but the activities are undertaken to draw conclusions intended by the authors using print media or words. Crawley & Mountain (Rahim, 2008) stating that the nature of reading is a complex activity that involves a lot of things, not just recite the writing, but also involves a visual activity, thinking, metacognitive and psycholinguistics. Failure to read lead to misunderstandings. (Putra & Suryana, 2016).

Indonesian Society does not have the habit of reading. This is reflected in the results of research suggested by UNESCO, other United Nations (UN) in the field of education, science and culture in 2012 which states that the index reading interest in Indonesia only reached 0,001 (Http: //www.paud-dikmas .kemdikbud.go.id/news/8459.html). This means that in a thousand people, only one person has the habit of reading. Correspondingly, the National Library of the Republic of Indonesia have data that Indonesian citizens are on average read 0-1 books a year, it lags behind the ASEAN countries who read 2-3 books per year. Indonesia will be left far behind when compared to the Japanese people who read 10-20 books a year.

Data from the Central Statistics Agency (BPS) said only about 23.5% of Indonesia's population interested in reading, behind the interested in watching television 85.59%. Naturally, if the results of the mapping PISA (Programme for International Student Assessment) showed that 76% of Indonesian students how to reading level 1 (out of 6 levels).

The data above shows the low literacy in Indonesia. Though Indonesia has a special day on reading, namely the National Book Day which is celebrated annually on May 17th since 2010 ago.

Discussion

There are several factors that cause low reading of the Indonesian population, including:

1. Reading is not yet a culture. Indonesian society is not yet used to read it because people are more accustomed to narrate. First, as a child, we used to hear a variety of fairy tales, stories customs verbally from parents, grandmother and community leaders.
2. The lack of a means of obtaining readings. When parents want their children to buy quality books, he is difficult to obtain. Not many bookstores are in each district. The quality and ethics book there are quite expensive. The collection of books in the library or libraries are very less.
3. Television and the Internet. Television shows and channels on the Internet very easily seized the attention of children and adults in reading. Although surfing the Internet can be considered the reading but the actual feel of the entertainment is more dominant compared to increase knowledge.

Three things above is describing the cause of the lack of reading culture in Indonesia. If you rarely read, let alone to have the ability and other competencies. Because reading is the door to gain knowledge. By reading someone will be able to try to understand and draw conclusions from the literature sources. By reading, we learn to compose logic, follow the story line even hone our critical power of reason, even the activity of reading is always done to the problem-solving process.

In order to improve the reading culture and develop the school as a learning organisation, ministry of education and culture developed a "*Gerakan Literasi Sekolah* (GLS)". GLS aims to make the school as a learning organisation so that the school community to become lifelong learners and be able to fulfil its role in the era of information technology.

Before that, Fuentas (1998) says that teachers need to develop students' literacy skills because during this time the teachers tend to only develop procedural skills so that students have difficulty understanding the terms or reading a text to resolve the issue. Along with the introduction of school literacy movements, in fact, there has been learning a model that can be used to improve the literacy skills of students is SQ3R learning model.SQ3R learning model is structured so that teachers can guide students to understand the material using a learning structure that focused and systematic. The learning model is a common form of learning activities used by teachers describe students' activities (Trianto, 2013).

SQ3R learning model is a learning model that is designed to help students understand the subject matter with the stages Survey, Question, Read, Recite, and Review. SQ3R learning model developed by Francis P. Robinson at the University of Ohio, United States (Shah, 2010). Originally used as a learning model SQ3R is a system for students in college but this model also fits in the secondary school students and even can be used on any subject as easily adapted to simple text (Sagala, 2009).

Reading in the sq3r learning model is a skill active and dynamic process that involves a complex activity involving physical response (sensation and perception), mental (abstract symbols and meanings), intellectual (critical thinking), and emotional (emotional intensity). In this SQ3R learning models, an activity of reading is the process of balancing between the text that is read with the knowledge of the students so that students can to constructed meaning when read, it means there an interaction between the reader and the text read. feature in SQ3R model very well used in intensive reading of reading comprehension (reading literal, critical and creative) and read the rational, so it is very appropriate to be used to facilitate the students to know and understand the ideas that are relevant, concepts, facts as well as an overview of the reading. SQ3R learning model is practical so that it aligns with a variety of learning approaches (Shah, 2010), including the scientific approach used in the curriculum 2013.

The steps of SQ3R learning model are:

1. *Survey*

At this stage, teachers guide students to observe and identify the subject matter there, either text textbooks and learning activities. In textbooks, students observe or identify all of the text in terms of titles, subtitles, words in italic, or words that are considered important. students mark keywords by highlighting, provide colour, or make notes alongside a page. In the learning activities of students write the activities they have seen, heard or sentence explanation of the teacher.

2. Question

At this stage, the students put questions relevant to the text they have read or activities undertaken.

3. Read

This stage into a key stage, because students intensive reading textbooks or other reference books to get the main idea and the answers to the questions that have been made previously.

4. Recite

At this stage, students write the answers obtained every question that has been made using his own language that is easily understood.

5. Review

Activities of students at this stage is to check, to look back over the question and answer briefly. Students read back part of the material to confirm the previous answers. In this review activity, the teacher can give a quiz to test students' understanding of the material being taught.

The process of reading contained in the learning model SQ3R include Activities Visual, ie reading is the process of translating symbols into spoken and written language, and the process of thinking, ie activities include recognising new words, the literal understanding, understand interpretations, critical reading and creative comprehension (Rahim 2008).

According to Soemarmo (2006), the activity of students in the SQ3R learning model among others:

1. Reading a given reading material, reading texts identified in terms of the title, subtitle, symbols, graphs, charts, tables or terms that a text reading.
2. Make inquiries from observations made in step survey.
3. Reading actively while understanding the concept that exists in the literature for answers that have been prepared and discussed concepts in reading material
4. Reveal the answers that have been prepared with the loud and hard in their own language without taking notes.
5. Checking back and answer questions they have interchanges and make conclusions from reading materials that have been studied.

Student activity above can be observed directly, but the teacher should always guide the students because there is a possibility that students can not find answers to questions that have been drawn. SQ3R learning model application can be combined with a cooperative approach. In order for students' activity in a more controlled group of teachers can divide two or four students into one group.

SQ3R learning model in accordance with learning theory proposed by Bruner, Vygotsky, and Piaget. Bruner argued about discovery learning theory. there are four main ideas in the theory of discovery learning. First, the most appropriate way for students to understand the concepts and principles is to construct its own concept and principles. Students have to understand the new concepts if he had been able to formulate their own ideas and material studies. Second, students will better understand the material presented if it is delivered by the new increased cognitively simple to complex shapes and abstract. Third, procedures and ideas presented should be accompanied by examples and not an example, so that the difference is clearly visible. Fourth, in studying the concept, structure and skills should always be associated with the concept, structure, and other skills. Four of the above is very relevant to the cognitive processes required in SQ3R learning model.

In learning theory of Vygotsky, he argued that learning occurs when students learn and accomplish the tasks that have not been studied but the tasks still within the range zone of proximal development (zone of proximal development) that is within the level of development of the real (the ability to find a solution on their own) was slightly above the level of potential development (ability to find a solution with the help of others). Thus, in order that students can reach the zone, then the task of the teacher provides scaffolding (Slavin in Sugeng, 2015) that aid in the form of guidance, encouragement, describes the problem into steps completion, provide an example, or other actions to students at the beginning of the learning, then gradually reduce it so that students can work independently.

Theory of learning Piaget suggests that learning related to the formation and development of the scheme, namely a cognitive mental structure that can be used by individuals to be able to adjust and coordinate its environment (Hartoyo, 2007). Scheme someone will always change and evolving. The process of change and development is known as adaptation. There are two kinds of processes that lead to the emergence of adaptation. First assimilation, namely the cognitive processes used by someone to be able to integrate the stimulus in the form of principles, concepts, procedures, laws and perceptions into the scheme that has been there before. The second property, namely the process of formation of new schemes that fit the scheme that already exists in the mind or the process of modifying the existing scheme to match the stimulus obtained. In learning, there exist always a balance between assimilation and accommodation.

SQ3R learning model in line with the scientific approach the main approach in 2013. Learning curriculum with scientific approach is a learning process that is designed so that students can actively constructing concepts, laws or principles found in the subject matter. The general steps in the scientific approach to learning that includes observation, questioning, experimentation, process data and information, presenting data or information, followed by analysing, reasoning and then concludes and communicate the results. Having regard to the steps in the learning model and approach Scientific SQ3R seen their suitability. In general suitability of the learning model SQ3R and scientific approach presented in this table:

Table 1. Compliance of Model SQ3R and Scientific Approach

Steps of SQ3R Learning Model	Steps of Scientific Approach	Student Activity
Survey	Mengamati/Viewing	Reading materials provided. Identify reading texts in terms of the title, subtitle, symbols, graphics, or terms that a text reading
Question	Menanya/ Asking	Makes ask questions of the observations made in step survey.
Read	Mengumpulkan informasi/ Collecting information/	Read actively while understanding the concept that exists in the literature for answers that have been prepared. Discuss the concept of the reading material
Recite	Mengasosiasi/ mengolah informasi/ menalar, Menarik Kesimpulan (associate / processing information / reasoning)	Conclusion Reveals Interesting answers have been compiled with loud and hard without taking notes.
Review	Mengomunikasikan/ communicating	Review communicate Checking back and answer questions they have collated. Make inferences from reading material that has been studied.

Having regard to the steps SQ3R learning models, learning models SQ3R seen that does not make students memorise the subject matter but to develop students' thinking skills to think and seek to understand the meaning of the information being studied. To gain an understanding of the information learned, the students must be skilled at understanding the material presented teachers.

According to Sumarmo (2006), the learning model SQ3R can develop literacy skills of students as SQ3R provide the opportunity for students to understand the reading text and develop reading skills even in the field of mathematics, good reading skills math low level (low order mathematical doing) as well as reading skills math high-level (high order mathematical doing). Such skills such as reading a text with a simple operation, the use of mathematical formulas directly, the application flow algorithm standard, read mathematics that includes the ability to understand the idea of mathematics in depth, observe the data and dig text between the lines, compiling a conjecture, analogy and generalization, reasoning logically, solve problems, communicate mathematically and associate mathematical ideas with other intellectual activities belong to the high level of thinking.

The characteristics of SQ3R learning model are:

a. The student-centered
b. Involving scientific skills in constructing concepts, principles, and procedures
c. Involving potential cognitive processes in developing higher level thinking abilities

Some of the benefits of applying the learning model SQ3R among others:

1. Giving the task by reading the text can make students more confident in understanding and solving problems.
2. Facilitate to improve student concentration.
3. To help focus student attention on the most difficult parts in reading, if there are questions can not be answered or not understood, the students could identify the difficulties and get the answer.
4. Train students to be able to convey ideas, ideas or thoughts scientifically.
5. Train students to think systematically.
6. Coaching provides answers in questions about the material.
7. Create a positive character of students.
8. Help prepare the records in question and answer form.

Conclusion

GLS promoted by the ministry of education and culture should be supported by trying to increase students' interest in reading. If students read increases, will indirectly affect the activity and student achievement. SQ3R learning model is one alternative that can be selected teachers to use in developing students' reading interest and improve student achievement because of the steps in the learning model SQ3R accordance with the curriculum in 2013 that implement the scientific approach.

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The Error Analysis of Algebra Operation on the Form of Exponent at the *SMAN 22 Tangerang*

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Abstract

Field Practices Experience (Indonesian called as *Praktek Pengalaman Lapangan or PPL*) is one of activities that can be used in order to carry on a learning evaluation. One of the material which used to learning in PPL is algebra operation of Exponent on Mathematics. Every year, many students were made mistake in this topic. Because of that, in this research, the evaluation needs in this subject. The evaluation form here is an analysis of the students' error answers on the Algebra Operation of Exponent. Subject in this research was 10th grade students of *SMAN 22 Tangerang* on class X-5. Design of this research used qualitative descriptive. The methods to collected data used an essay test method (for students), in-depth interview method (for students), and video documentation (for evaluate the teacher and students on class). The final results showed tha from the total of fourty (40) students, only six (6) who succeeded in solving the five questions. The rest of twenty eight (28) students were left the paper empty and only rewrote the questions given without solving it. In addition, researcher founded that teachers also made some errors while transferring the topic.

***Keywords**: Algebra Operation, error analysis, problem solution, eksponen, qualitative descriptive*

Introduction

According to the Great Dictionary of Bahasa Indonesia (KBBI) mathematics is knowledge about numbers, connection between numbers, and operating procedure of solving problems related to numbers. Sunarsih (2009) defined mathematics as the knowledge of numbers which used symbols with its structure and logical thought to solve problems through deductive thinking without leaving the inductive thinking behind. In Indonesia mathematics has already been taught since elementary school in other words, mathematics is one of important knowledge to study for students.

Mathematics learning activities has also been taught during the field practice experience (PPL) by the students of education study program, especially mathematics. PPL is a compulsory subject for students of education program in Indonesia. Riwa (2015) has wrote PPL is an education process which giving direct practice and learning experience that allow students to connect with society especially education world, where, they can learn to solve problems regarding education. On the process of PPL, there are steps where the students can do some evaluations on the learning process. Sunarih (2009) wrote that, Indonesians students' mathematics ability is very low. Every mathematic learner in every level of school has different goal. That mathematic learning goal is on the unit level of curriculum (KTSP).

Unit level of education curriculum (KTSP) has its own competence standard (SK) and basic competence (KD) which is the objective of every course. Students are demanded to master every basic concept in every SK and KD in every mathematics learning topic. Besides that, students need to fulfill the criteria of minimal completion (KKM) on every SK and KD. However the fact in the field shows that there are still many students who are unable to fulfill those demands and find difficultness in doing that. One of the Mathematics' standard competence in class X in *SMAN 22 Tangerang* is to solve math's problems linked to root, exponent, and logarithm with one of the standard competence of conducting an algebra manipulation in calculation of root, exponent, and logarithm. Based on the field practice (PPL), the researcher founded that the students of class X at *SMAN 22 Tangerang* difficult to achieved the SK and KD.

The material being taught at that time was exponent. The result showed the lack of competence and understanding in discerning the material and solving the given questions regarding the material. At the beginning students had been reminded about basic material which needed to understand, such as addition, subtraction, division, multiplication, and division of real numbers. But the result of evaluation proved, there was still many students who incorrectly answer the questions. The evaluation was given by questions of exponent description. Furthermore, by the result of their middle semester test (UTS) found there were numbers of students who could not reach the criteria of minimal completion (KKM). Based on UTS result mathematics is the hardest subject to be master especially exponent. Likewise Agustin and Linguistika (2016), wrote that exponent is a difficult material because they already have identified that most students made errors on the process of solving the given questions. Due to that reason, further study is needed to analize the mistakes done by students so the further analysis can also be conducted to the evaluation on exponent material.

The word "*Analysis*" according to KBBI is identification towards something (essay, actions, etc.) in order to know the real condition (causes, reasons/case, etc.) and the word "error" in KBBI means incorrectness or mistakes. Based on the two definitions, error analysis is identification toward errors or incorrectness. Sunarsih (2009), wrote that the error should be identified where the results identification help to increase the quality of teaching and learning process, and finally it can help students to improve their mathematics' understanding.

Other than the result that being analyzed, it is also important to have an empirical data of the students which functioned as consideration for teachers to assessing their performance in order to get the right factors causing students difficulties in solving mathematic questions. In a book entitled "*Evaluasi Pembelajaran Matematika*" Hamzah (2014;14) was written that if an information found regarding the students' difficultness because of teachers teaching style or the way s/he teach, it is the teacher responsibility to find a way to solve the problem. Teacher can try to find a new teaching method or approach which can attract the student understanding to the material. Therefore, it is a need to have some interview with students where will be combined with documentation (by recording the teachers' teaching process in the class). There is a possibility that the way teachers' teaching can be one of the causes. Based on the description above that the analysis of error need to carry out especially about exponent, due to that the researcher decided to do a research study entitle "*The Error Analysis of Algebra Operation on the Form of Exponent at the High Schools 22 Tangerang"*

Research Methodology

This research conducted by using qualitative descriptive approach. The subject of this research was several students in class X at *SMAN 22 Tangerang*. The instrument used in this research is the analysis of student's error such as the essay test method, in-depth interview method, and video documentation. The steps conducted in this research were introduction, learning process, and evaluation. Data analysis of this research based on Miles and Huberman in Sugiyono (2015) mentioned the data collection, data reduction, explanation of data, and conclusion.

Result and Discussion

A. Exponent number in curriculum

The *unit level of education curriculum* 2006 *(KTSP)* has content standard about *square numbers* for students in grade X, namely:

a. Standard Competence (*SK*) : Solve problems are relate to exponent, roots and logarithm.

b. Basic Competence (*KD*) : Use the exponent, root, and logarithm rules.

From the SK and KD above, the indicators and the purpose of the study as follows:

a. Indicator

  1) Simplify the form of an exponent number.
  2) Performing algebraic operation on the form of exponential and basic math (*addition, subtraction, division, multiplication).*

b. Purpose of the study

  1) Students are able to determine the simple form of an exponential numbers.

2) Students are able to determine the solution of algebra operation in the form of exponent by using basic math operation (addition, subtraction, division, multiplication).

3) The characteristic of exponent learning in the class, as follows;

   a. $a^x \times a^y = a^{x+y}$

   b. $\frac{a^x}{a^y} = a^{x-y}$

   c. $(a^x)^y = a^{xy}$

   d. $(a \times b)^x = a^x \times b^x$

   e. $(\frac{a}{b})^x = \frac{a^x}{b^y}$

   f. $\frac{1}{a^y} = a^{-y}$

   g. $a^0 = 1, a \neq 0$

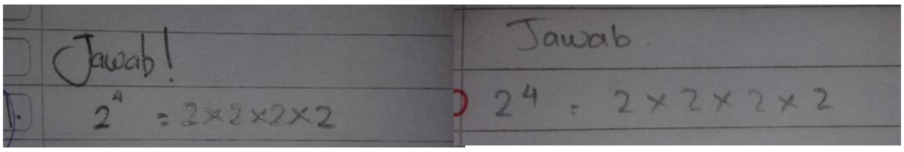
B. Students' error identification.

The test instrument that used in this research is questions which was taken from mathematics book under KTSP 2006 curriculum. Types of errors that analyzed was conceptual errors and procedural errors, based on Basuki in the research of Husain (2013). The conceptual error conducted by students were mistakes do sums, subtraction, multiplication and division, and the procedural errors was wrong in written questions, the process of the test, the answers did not resume until the end, and a mistake because unreadable writing.

1) Question number 1 aimed to know the student's perception of the meaning of exponent. The question was gave: "Please explain what meaning of 2^4".

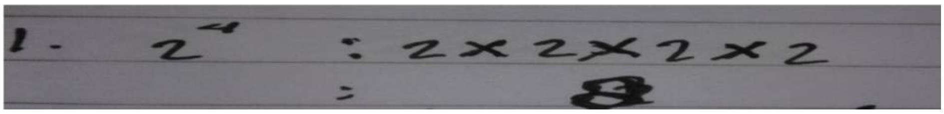
   Student's Errors were founded:

   a. Some of the students did not finish their calculation, example of students worked as Picture 1.



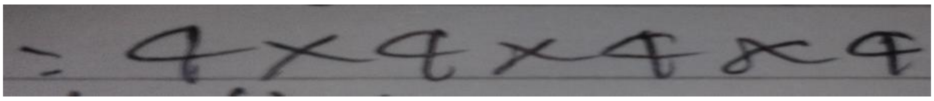
Picture 1a

   b. Students made error during their calculation such as Picture 1b


1. 2^4 = 2 × 2 × 2 × 2
= 8

Picture 1b

c. Students thought 2^4 as 4^4 such as Picture 1c

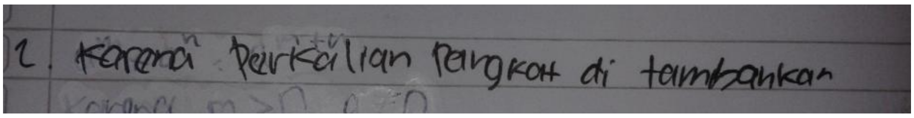

= 4 × 4 × 4 × 4

Picture 1c

2) Question number 2 aimed to know student's perception of the characteristic of multiplication of exponent number with the base number. The question was gave: "Please explain how can $a^m \times a^n = a^{m+n}$"

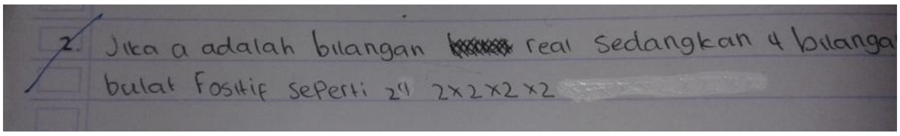
Student's Errors were founded:

a. Students only memorized ($a^m \times a^n = a^{m+n}$) such as Picture 2a


2. Karena perkalian pangkat di tambahkan

Picture 2a

b. Students answered with impropriate example such as Picture 2b.


2. Jika a adalah bilangan real sedangkan 4 bilanga
bulat fositif seperti 2^4 2×2×2×2

Picture 2b

3) Question number 3 aimed to see the student's ability in simplify the exponent. The question was gave :

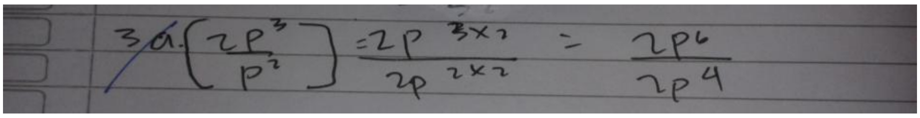
Simplify the form below:

a. $(\frac{2p^3}{p^2})^2 = \cdots$

b. $(-2p^8 \times 4p^3) = \cdots$

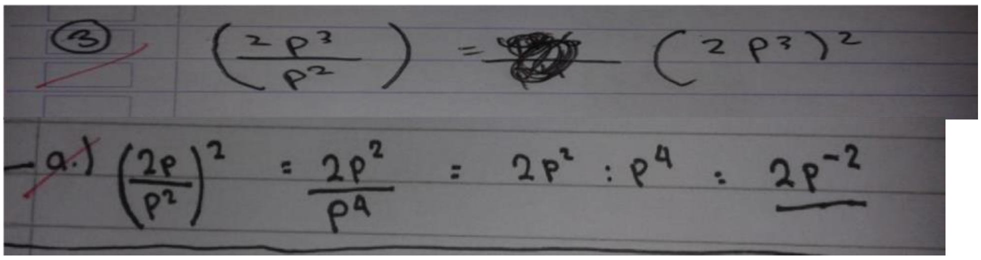
Student's Errors were founded number 3 part a:

a. Students did not know the character of ($\frac{a^x}{a^y} = a^{x-y}$) as the result they could not finish their calculation as Picture 3a.


3a. $\left[\frac{2p^3}{p^2}\right] = \frac{2p^{3\times2}}{2p^{2\times2}} = \frac{2p^6}{2p^4}$

Picture 3a

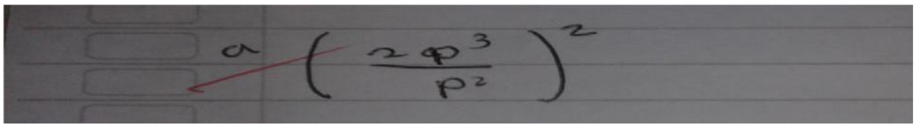
b. Students could not read the question properly as Picture 3b.


③ $\left(\frac{2p^3}{p^2}\right) = \quad (2p^3)^2$

a.) $\left(\frac{2p}{p^2}\right)^2 = \frac{2p^2}{p^4} = 2p^2 : p^4 = 2p^{-2}$

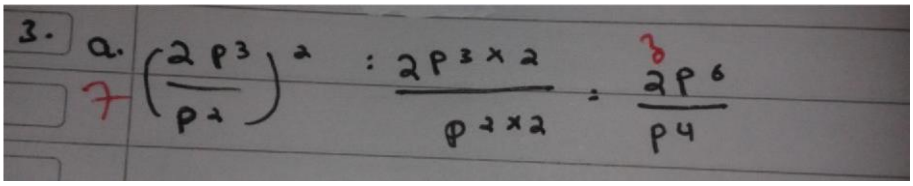
Picture 3b

c. Students only rewrote the question itself as Picture 3c.


a $\left(\frac{2p^3}{p^2}\right)^2$

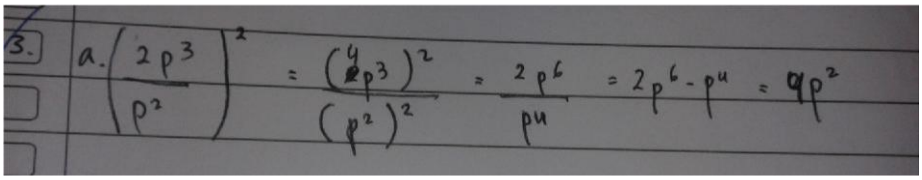
Picture 3c

d. Students forgot to calculate on the constant exponent as Picture 3d.


3. a. 7 $\left(\frac{2p^3}{p^2}\right)^2 : \frac{2p^{3\times2}}{p^{2\times2}} = \frac{2p^6}{p^4}$ 3

Picture 3d

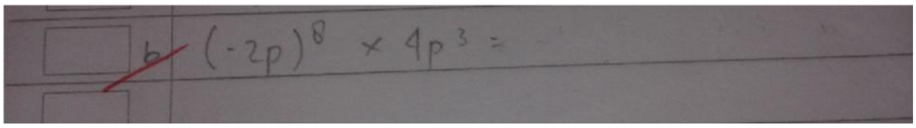
e. Students calculated the constant with the exponent of variable as Picture 3e.


3. a. $\left(\frac{2p^3}{p^2}\right)^2 = \frac{(4p^3)^2}{(p^2)^2} = \frac{2p^6}{p^4} = 2p^6 - p^4 = 4p^2$

Picture 3e

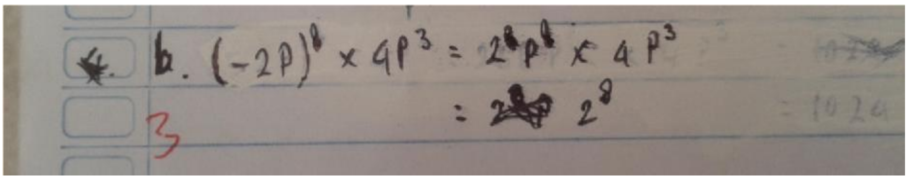
Next, Errors found on question number 3. Part b:

a. Students only rewrote the given question


b $(-2p)^8 \times 4p^3 =$

Picture 3f

b. Students missed some calculation steps which caused error as Picture 3g.

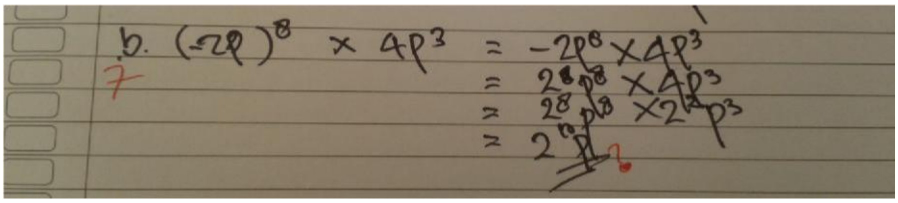

4. b. $(-2p)^8 \times 4p^3 = 2^8p^8 \times 4p^3$

$= 2^8$

3

Picture 3g

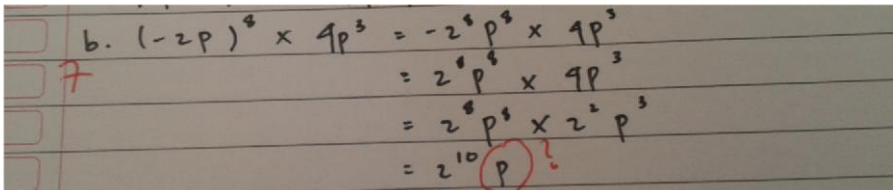
c. Less comprehension of exponent's multiplication with the same base number which caused error during the calculation as Picture 3h.



b. $(-2p)^8 \times 4p^3 = -2p^8 \times 4p^3$
7
$= 2^8 p^8 \times 4p^3$
$= 2^8 p^8 \times 2^2 p^3$
$= 2^{10} p$?

Picture 3h

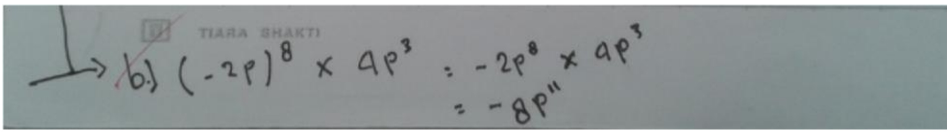
d. Less comprehension of the characteristic of zero exponent and thought $p^0 = p$ as Picture 3i.



b. $(-2p)^8 \times 4p^3 = -2^8 p^8 \times 4p^3$
7
$= 2^8 p^8 \times 4p^3$
$= 2^8 p^8 \times 2^2 p^3$
$= 2^{10} p$?

Picture 3i

e. Students forgot to calculate the exponent on constant number but multiplied constant number without noticing the exponent as Picture 3j.



TIARA SHAKTI
b.) $(-2p)^8 \times 4p^3 = -2p^8 \times 4p^3$
$= -8p^{11}$

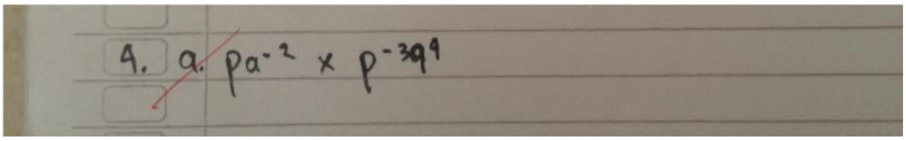
Picture 3j

4) Question number 4 aimed to see the student's ability is exchanging an exponent number into a positive exponent. The question was gave :

a.	$pq^{-2} \times p^{-3}q^4 = \ldots$
b.	$\frac{4a^3b^4c^{-8}}{7a^{-2}b^3c^3} = \cdots$

Student's Errors were founded in number 4. Part a:

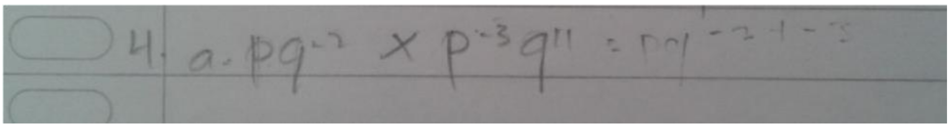
a. Students misunderstand the information of the question as Picture 4.a.



4. a. $pa^{-2} \times p^{-3}q^4$

Picture 4a

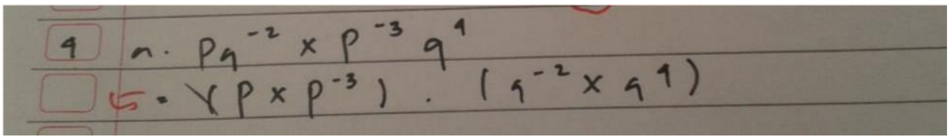
b. Students calculated the exponent from different prime number as Picture 4b.



4. a. $pq^{-2} \times p^{-3}q^{11} = pq^{-2\ 1-3}$

Picture 4b

c. Lack of understanding of the multiplication exponent with the same base number's characteristic that caused students could not finish their answer as Picture 4c.

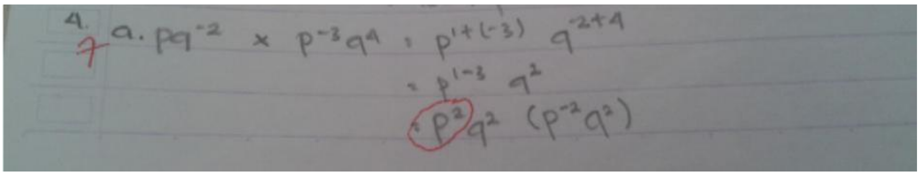


4 a. $pq^{-2} \times p^{-3}\ q^{4}$

5. $(p \times p^{-3})\ .\ (q^{-2} \times q^{4})$

Picture 4c

d. Students made a few errors during the operation of exponent as Picture 4d.



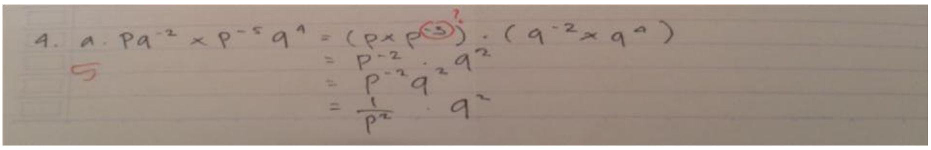
4\. 7 a. $pq^{-2} \times p^{-3}q^{4} = p^{1+(-3)}\ q^{-2+4}$

$= p^{1-3}\ q^{2}$

$= p^{2}q^{2}\ (p^{-2}q^{2})$

Picture 4d

e. Students made error in rewrote the exponent as Picture 4e.



4\. a. $pq^{-2} \times p^{-5}q^{4} = (p \times p^{-3}) \cdot (q^{-2} \times q^{4})$?

5

$= p^{-2} \cdot q^{2}$

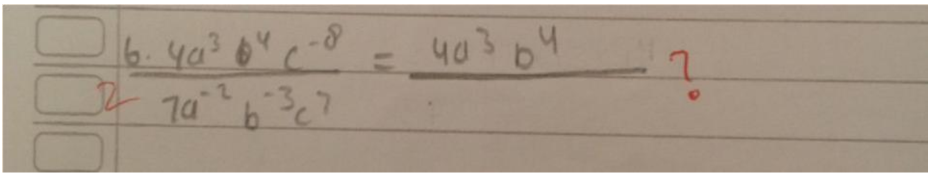
$= p^{-2}q^{2}$

$= \frac{1}{p^{2}} \cdot q^{2}$

Picture 4e

Student's Errors were founded in number 4. Part b:

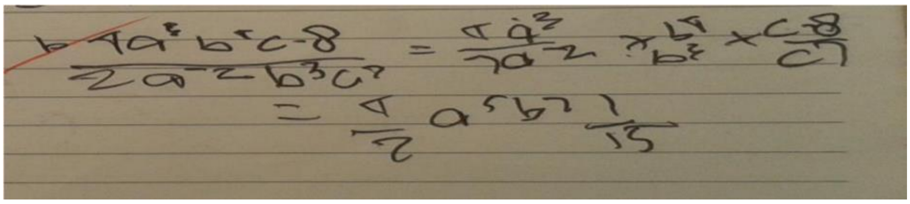
a. Students did not comprehend the concept which leads them to make error in their calculation as Picture 4f.



2 b. $\frac{4a^{3}b^{4}c^{-8}}{7a^{-2}b^{-3}c^{7}} = \frac{4a^{3}b^{4}}{\quad}$?

Picture 4e

b. Students did not know the characteristic of negative exponent which caused them made error in their calculation as Picture 4f.

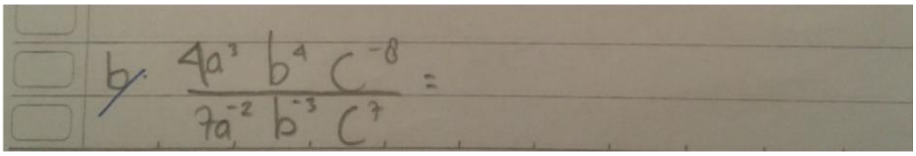


b $\frac{4a^{5}b^{5}c^{-8}}{7a^{-2}b^{3}c^{7}} = \frac{4a^{3}}{7a^{-2}} \times \frac{b^{4}}{b^{3}} \times \frac{c^{-8}}{c^{7}}$

$= \frac{4}{7}\ a^{5}b^{7}\ \frac{1}{15}$

Picture 4f

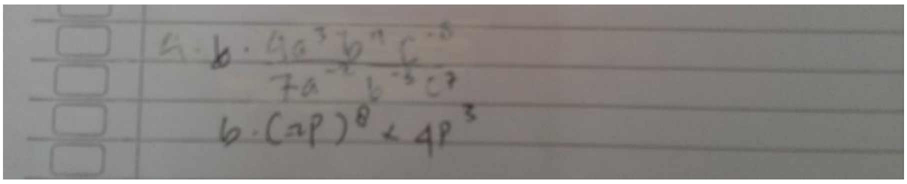
c. Students were just rewrote the given question as Picture 4g.



b. $\frac{4a^3 b^4 c^{-8}}{7a^{-2} b^{-3} c^7} =$

Picture 4g

d. Students rewrote the information of question incorrectly as Picture 4h.



4. b. $\frac{4a^3 b^4 c^{-8}}{7a^{-2} b^{-3} c^7}$

6. $(2p)^8 \times 4p^3$

Picture 4h

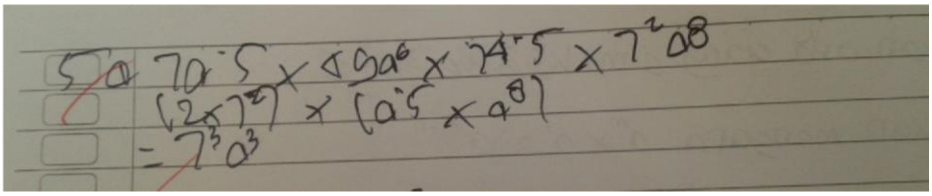
5) Question number 5 aimed to see student's ability in algebra operation on exponent number. The question was gave:

> Determine the value of:
>
> a. $7a^{-5} \times 49a^8 = \dots$
>
> b. $\frac{10b^6}{10b^{-2} \times 10b^8} = \cdots$

For question number 5 half of the students emptied their answer.

Student's Errors were founded in number 5. Part a:

a. Students made some error on deciding the exponent as Picture 5a.



5 a. $7a^{-5} \times 49a^6 \times 7^{-5} \times 7^2 a^8$

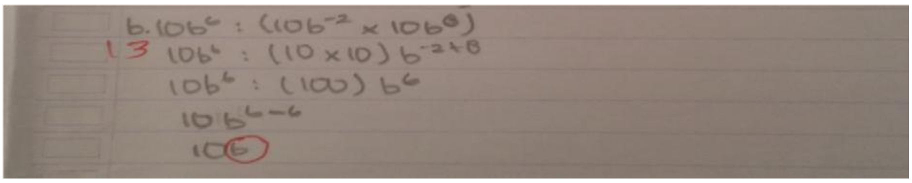
$(2 \times 7^2) \times (a^{-5} \times a^8)$

$= 7^3 a^3$

Picture 5a

Student's Errors were founded in number 5. Part b:

a. Students were lack of comprehension of the characteristic of zero exponent and thought $p^0 = p$ as Picture 5b.



b. $10b^6 : (10b^{-2} \times 10b^8)$

13 $10b^6 : (10 \times 10) b^{-2+8}$

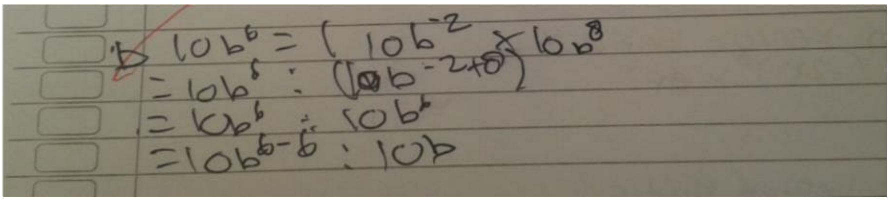
$10b^6 : (100) b^6$

$10 b^{6-6}$

$10\boxed{b}$

Picture 5b

b. Students wrote the fractional exponent form incorrectly without following the procedure. As the result they made some mistake on their calculation as Picture 5c.



b) $10b^6 = (10b^{-2} \times 10b^8$

$= 10b^6 : (10b^{-2+8})$

$= 10b^6 : 10b^6$

$= 10b^{6-6} : 10b$

Picture 5c

Conclusion and Suggestion

From the total of five questions were given, there were found some errors that occasionally did by the students, such as error during the exponent operation, error in multiplying constant number to variable, and error in calculating the exponent number from constant number. Those errors above happened because of the misunderstanding of the concept and mistakes happened during the calculation. As Kelin (2016) explained error that happened occasionally to the students in solving the mathematic questions of exponent number is caused by the misconception and miscalculation that's true. After had interview with some of the research subject which mostly made error, the misconception founded in this research was caused by the misunderstanding of zero exponent's characteristics, miscomprehending of two exponent numbers multiplication with the same base number that caused the further error on the calculation. Moreover there was also some errors in reading and understanding the information and what does the questions demanded them to do. From the total of fourty (40) students, only six (6) who succeeded in solving the five questions. The rest of twenty eight (28) students were left the paper empty and only rewrote the questions given without solving it. The interview result showed, according to the students who could not finish up the questions, they felt the time given to do the task is too short. Also after look back at the documentary video that recorded the learning process of exponent material, teacher did not use any of learning method which lead the learning process into the convectional one, where teacher stand as the centre of learning process and most of the students became passive. Due to that, to teacher, the researchers suggest to use an interesting teaching method, approach or technique which can be use to attract the student's attention into the learning process and turn them to be active in the class.

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The impact of gender, parents' education level, and socio-economic status on Turkish students' mathematics performance

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Abstract

This paper describes secondary data analyses that investigates impact of gender, parents' education level, and socio-economic status on Turkish students' mathematics achievement in Program for International Student Assessment (PISA) 2009. The objectives of this research are to explore the impact of gender, parents' education level, and socio-economic status on Turkish students' mathematics performance. The analyses were done quantitatively using *t-test* analysis, Oneway ANOVA, correlation and linear regression. The findings revealed that gender differences and Turkish students' mathematics performance are statistically significant but practically it is not significant. Parents' education level has an impact on Turkish students' mathematics achievements. Index of Economic, Social, and Cultural Status has a positive effect on mathematics achievements and ESCS is a significant predictor of Turkish students' mathematics performance.

The findings seem to suggest that at the school level, school environment are expected to build students' motivation in learning. Additionally, teachers in the classroom setting are expected to provide extra hours and learning consultation services for students who need it. At students level, students are expected to have high motivation in learning and they are suggested to be not relying on learning facilities provided by their parents. However, further research is required prior to making any recommendation at school policy, teaching and learning practice in the classroom setting.

Keywords: *students' mathematics performance, gender, parents' education level, and socio-economic status*

Introduction

Successful of students in learning is affected by several factors including gender, parents' level education, and socio-economic status. There is a widespread acknowledgment of gender influence, parents' educational level, and socio-economic status on students' mathematics performance. Regarding to gender differences, socio-economic status and mathematics achievement, a study conducted in 2008 in Turkey revealed that there was a statistically significant difference in mathematics performance in favor of cities based on their economics development level and its effect size was quite small indicating in practically its difference was nit significant. In addition, the study also concluded that the development of socio-economic of the areas was not an essential factor for gender differences in mathematics performance (Isiksal & Cakiroghi, 2008, p. 113).

In relation to parents' education level and mathematics achievement, the role of students' parents is paramount for students' future particularly in providing their children with science and knowledge including mathematics. Good parental awareness and good guidance in mathematics is hoped to create and develop children who have good learning outcomes in mathematics so that they are ready for facing the variety of science in the future in which it will always continue to evolve over time. Dalyono (2010) claims that parental awareness and parental guidance are influenced by parents' educational level. He asserts that parents' education level of students plays an important role in educating children. Additionally, he argues that parents with elementary level of education tend to direct their children to basic knowledge and basic skills. Parents with secondary level of education tend to able to direct their children to skills and broader knowledge not only basic skills and knowledge itself. Parents with college level of education, they tend to direct their children to be able to follow recent development of science and technology.

Based on brief explanations above, it is interesting to explore the impact of gender differences, parents' education level, and socio-economics status on students' mathematics performance. This paper will focus on examining these three main issues. The purpose of this study is to explore the influence of gender differences, parents' education level, and socio-economics status on Turkish students' mathematics outcomes.

Research Questions and Hypothesis

There are three research questions will be dealt with in this study as follows.

1) Is there a difference between mathematics achievements by gender?
2) Is there a difference between mathematics achievements by parents' education?
3) What is the influence of Index Economic, Social, and Cultural Status (ESCS) on mathematics achievements?

The hypothesis of this study is as follows.

1) The null hypothesis (H_0) and alternative hypothesis (H_a) of the research question number one as follows.
   H_0: There is no a significant difference between mathematics achievements by gender
   H_a: There is a significant difference between mathematics achievement by gender
2) The null hypothesis (H_0) and alternative hypothesis (H_a) of the research question number two as follows.
   H_0: There are no differences in mathematics achievements based on parents' education
   H_a: There are differences in mathematics achievement between at least one pair of groups
3) The null hypothesis (H_0) and alternative hypothesis (H_a) of the research question number three as follows.
   H_0: There is no influence of Index of Economic, Social, and Cultural Status (ESCS) on mathematics achievements

H_a: There is influence of Index of Economic, Social, and Cultural Status (ESCS) on mathematics achievements

Data and Methods

Data source and participants

The data source of this paper is gained from 2009 administration of Programme International Assessment (PISA), which is a standardized assessment programme developed internationally by engaging OECD countries and conducted to 15-year-olds students in schools (OECD, 2010). Target population of this study is 15-year-olds Turkish students who engaged in PISA 2009. The data included 4996 Turkish students consisting of 2551 male and 2445 female. A brief summary of gender from the sample of this study is presented in Table 1 below.

Table 1 A Summary of Turkish Students who Participated in PISA 2009

Variable	Per cent	Valid per cent
Gender		
Female	48.9	48.9
Male	51.1	51.1
Total		
Percentage	100	100
N	4996	4996

Methods

There are three methods used to investigate the research question. In dealing with research number one, an independent *t-test* analysis will be used to investigate the difference between mathematics achievements by gender. Research question number two will be dealt with using Oneway ANOVA to examine the difference between mathematics achievements by parents' education. Correlation and regression are used to examine relationship between Index of Economic, Social, and Cultural Status (ESCS) and mathematics achievement.

Results

T-test

Female students *(N=2445)* was associated with mathematics achievement *M=441.21 (SD=90.39)*. By comparison, male students *(N=2551)* was associated with mathematics a numerically larger mathematics achievement *M=452.36 (SD=93.16)*. To examine research question number one, an independent *t-test* was undertaken to assess if a difference existed between the mean of mathematics achievement of male and female. As can be seen clearly in

Table 2 below, female and male distributions were normally distributed for the purposes of performing *t-test* (i.e., skewness < |2.0| and kurtosis <|2.0|; (Schmider, Ziegler, Danay, Beyer, & Bühner, 2010see Table 2).

Variable	Sex						95% CI for Mean Difference		
	Male			Female					
	M	SD	SE	M	SD	SE		t	df
Mathematics achievements	452.36	93.16	1.84	441.21	90.39	1.83	-16.24, -6.05	-4.29*	4993.25

Table 2 Descriptive Statistics associated with Mathematics Achievement

	N	*M*	*SD*	Skewness	Kurtosis
Female	2445	441.21	90.39	.318	-.02
Male	2551	452.36	93.16	.349	-.19

In Table 3 below demonstrates presentation of *t-test* results with unequal variances for mathematics achievement by sex.

Table 3 Results of an Independent *t-test* for Mathematics Achievement s by Sex

**p < .01*

The independent sample *t*-test was associated with a statistically significant effect, *t(4993.25)= -4.29, p< .01*. There is a statistically significant difference between mathematics achievements by gender. Results indicate that male is associated with a statistically significant larger mean of mathematics than female. Cohen's *d* was estimated at .12 that is small effect based on Cohen's guidelines (Sullivan, 2012). The 95% confidence interval was -16.24 to -6.05. As a result, the null hypothesis is rejected.

One-Way ANOVA

The descriptive statistics related to mathematics achievements across the seven group of the highest educational level of parents are presented in Table 4. It can be seen clearly that students with none highest educational level is associated with the numerically smallest mean of the mathematics achievements *M=388.64 (SD=79.22)* and students with ISCED 5A, 6 as the highest educational level of parents is associated with numerically the highest mean of mathematics achievements *M=517.62 (SD=92.28)*. In the highest educational level of parents, accounting for 6 (.1%) of respondents answered N/A and 93 (1.9%) was missing data. The percentage of missing data is less that 5% so it is acceptable and it would not impact on the estimation of the whole population (Dong & Peng, 2013). In order to examine the hypothesis the difference between mathematics achievements by parents' education, ANOVA was undertaken. Prior to performing ANOVA, assumption of normality was tested and concluded to be met as distributions of seven groups were associated with skewness and kurtosis less that |2.0| and |2.0| respectively (Schmider et al., 2010;see Table 4).

Table 4 Descriptive Statistics for Parents' Education Level across Mathematics Achievements

Parents' education level	*N*	*M*	*SD*	*Skewness*	*Kurtosis*
None	145	388.64	79.22	.22	.58
ISCED 1	1568	424.28	81.43	.32	.18
ISCED 2	1209	423.88	81.07	.40	.24
ISCED 3B,C	92	417.15	83.49	.77	.55
ISCED 3A,4	971	471.63	85.79	.30	-.70
ISCED 5B	198	479.64	93.40	.22	-.39
ISCED 5A,6	714	517.62	92.28	-.14	-.43

**Notes: N/A: 6 (.1%); Missing: 93 (1.9%)*

In order to investigate possible differences in mathematics between students based on their highest parents' educational level (HISCED), a one-way ANOVA was conducted. We find evidence that there is a difference in mathematics achievements based on HISCED status (F=149.51 (6, 4890), p< .0001). Thus, the null hypothesis of no differences in mathematics achievements based on parents' education was rejected. The ω^2 = .154 indicated that roughly 15% of the variation in the mean of mathematics achievements is attributable to differences between the seven groups of parents' education level.

Table 5 Results of ANOVA

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	6383981.41	6	1063996.90	149.513	.000
Within Groups	34799356.5	4890	7116.43		
Total	41183337.9	4896			

To evaluate the nature of discrepancies between the seven groups means further, the statistically significant ANOVA was followed-up with Tukey's post-hoc tests(Field, 2009). These results are presented in Table 6 and indicate that students' with parents' highest educational level is ISCED 5A *(M=128.98)* scored significantly higher that group 1(none), group 2 (ISCED 1), group 3 (ISCED2), group 4 (ISCED3B, C), group 5 (ISCED 3A, 4), and group 6 (ISCED 5B). The effect size of these significant pairs difference is estimated at 1.53, 1.10, 1.11, 1.19, .54, and .45 respectively. The mean differences of students with parents' none highest educational level *(M=388.64)* and students with parents' educational level is ISCED 3B, C *(M=417.15)* are not statistically significant differences. The effect size of this pairwise is approximately at .34.

Table 6 Tukey Post Hoc Results and Effect Size of Mathematics Achievements by the Highest Parents' Educational Level

		Mean Differences ($\overline{X_i}$ - $\overline{X_j}$) (Effect Size is indicated in parentheses)						
Group	Mean	1.	2.	3.	4.	5.	6.	7.
1. None	388.64	0.00						
2. ISCED 1	424.28	35.64*	0.00					
3. ISCED 2	423.88	35.24*	.40	0.00				
4. ISCED 3B, C	417.15	28.51	7.13	6.73	0.00			
5. ISCED 3A, 4	471.63	82.98*	47.34*	47.74*	54.47*	0.00		
6. ISCED 5B	479.64	91.00*	55.35*	55.76*	62.49*	8.01	0.00	
7. ISCED 5A, 6	517.62	128.98*	93.33*	93.74*	100.47*	45.99*	37.98*	0.00
		(1.53)	(1.10)	(1.11)	(1.19)	(0.54)	(0.45)	

**.The mean difference is significant at the .05 level.*

Correlation

To examine the relationship between Index of Economic, Social, and Cultural Status (ESCS) and mathematics achievements, Pearson's correlation was computed. The results indicated that there was a positive and significant correlation between ESCS and mathematics achievements, $r= .45, n=4967, p< .001$. It suggests that there was a medium effect of ESCS on mathematics achievements (Cohen, 1992; Field, 2009). R-squared was .20, which implies that 20% of variance of mathematics achievements is associated with the variance of ESCS. In this case, missing data is 29 (.58%), which is very small and less that 5%. This missing data would not affect on the estimation of the whole population.

Linear Regression

To investigate the hypothesis of the research question number three, a simple linear regression further was conducted to assess if ESCS predicted mathematics achievements. Prior to analysis, the assumption of normality of independent variable (ESCS) was examined and concluded to be met as its distribution was associated with skewness and kurtosis less than |2.0| and |2.0| respectively (Schmider et al., 2010; see Table 7). The assumption of linearity was met by generating scatter plots of the outcome variable against the explanatory variable (see Figure 1).

Table 7 Descriptive Statistics ESCS and Mathematics Achievements

	M	*SD*	Skewness	Kurtosis
ESCS	-1.16	1.20	.32	-.51
Mathematics achievements	446.91	91.98	.34	-.11

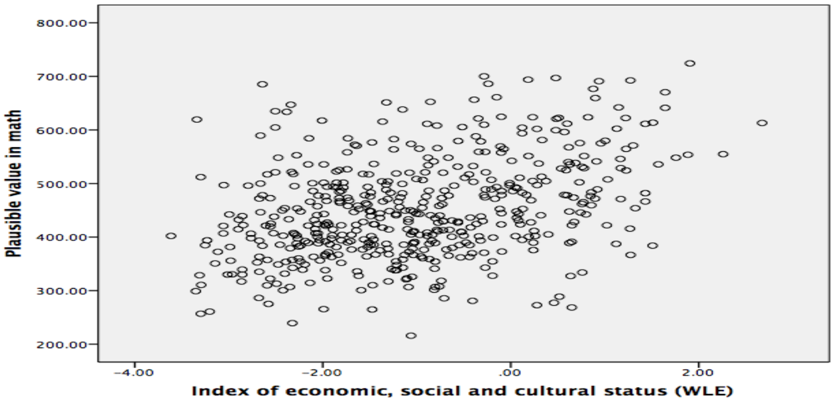

Figure 1 Scatter Plots of ESCS against Plausible Value in Math

The results of linier regression were significant, *F* (1, 4965) = 1281.46, $p< .001$, $R^2 = .20$, suggesting that approximately 20% of the variation in the mathematics achievements score was predicted by ESCS score. ESCS was a significant predictor of mathematics achievements, $B=34.56$, $p< .001$ suggesting that for every one unit increase in ESCS then mathematics achievements increased by 34.56 units. Therefore, we rejected the null hypothesis. Results of linier regression are presented in Table 8 below.

Table 8 Results of Linier Regression with ESCS Predicting Mathematics Achievements

	B	*SE*	*β*	*t*	*p*
Constant	487.63	1.62		301.64	.00
ESCS	34.56	.97	.45	35.80	.00

Discussion

In this study, analysis of data showed that gender differences and mathematics achievement is statistically significant. Findings of the data analysis revealed that male is associated with a statistically significant larger mean of mathematics achievement than female. Although, the mean difference of mathematics achievement between male and female is statistically significant, the effect size was quite small suggesting that the mean difference of mathematics achievement by gender was not significant practically. The findings of this research are similar to the previous study conducted by Isiksal and Cakiroghi in Turkey in 2008. They found that gender differences regarding to mathematics achievements of Turkish middle school students based on level of the economic development city was statistically significant but it was not significant practically (Isiksal & Cakiroghi, 2008).

In relation to mathematics achievements by parents' educational level, statistical findings in this study revealed that there was a significant difference in mathematics achievements by parents' education. Students with their parents' educational level ISCED 5A, 6 showed significant mean difference compare to others group. The effect size between ISCED 5A, 6 and four groups, which are none, ISCED 1, ISCED 2, and ISCED 3B, C was quite large indicating that it practically was significant and the effect size between ISCED 5A, 6 and ISCED 3A, 4 and ISCED 5B was quite small suggesting that it practically was not significant (Cohen, 1992).

Regarding to the influence of Index of Economic, Social, and Cultural Status (ESCS) on mathematics achievements, correlation analysis revealed that there was positive and significant relationship between these two variables. ESCS has a medium influence on mathematics achievement. Linear regression showed an increase in one unit in ESCS would increase mathematics achievements roughly 34.56. It means that there are positive linear relationship between ESCS and mathematics achievement. Additionally, the results of regression linear also indicated that ESCS score predicted approximately 20% variation of mathematics achievement. Therefore, it could be concluded that ESCS is significant predictor of mathematics achievement. And indeed, ESCS score has a positive influence on mathematics achievement. Based on correlation and linear regression analysis, it could be said generally that if Index of Economic, Social, and Cultural Status (ESCS) of students in Turkey were good then students' performance in mathematics would increase.

In conclusion, there are three main issues of the findings in this study. First of all, the finding of this study has revealed that gender differences and mathematics achievement in Turkish students who participated in PISA 2009 are statistically significant but practically is not significant. Secondly, parents' education level of Turkish students in this research has an impact on Turkish students' mathematics achievements. The most significant difference is found between group ISCED 5A, 6 and none highest education level of parents and practically this difference is significant. The last main point of the result in this study is Index of Economic, Social, and Cultural Status (ESCS) has a positive impact on mathematics achievement. ESCS and mathematics achievement show a positive and linear relationship. ESCS is a significant predictor of mathematics achievements.

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INCREASING MOTIVATION AND LEARNING OUTCOMES BY LEARNING METHODE *DIFFERENTIATED INSTRUCTION BY GROUP INVESTIGATION (DIGI)*

Case Study in Topic Geometry Transformation in Senior High School of International Islamic Boarding School Republic of Indonesia (SMA IIBS RI) Class XI Science Girls

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Abstract

This article describes the impact of the use of DIGI methods (Differentiated Instruction by Grouping Investigation) in XI Grade of Senior High School IIBS RI especially Science Girls Class. The objectives of the research focus on the students' achievement and motivation in transformation geometry chapter. DIGI method is DI methods that focus on grouping students to learn and produce mathematics formula as well as the use of the mathematics applications, especially in the transformation geometry chapter. The analysis was done descriptively by using the result of student test score in learning process and student motivation questionnaire. The article report uses processing data and t-test statistical with a degree of freedom 90% to examine the students' motivation and achievement. The findings are students' achievement increased as well as students' motivation in learning process. By using DIGI methods show that students become active learners, and students learn to define their own learning experience. DIGI methods that used are a new experiment in certain grade that is XI grade science girls to achieve results and students' motivation, this study will be better if the methods apply to all grades.

***Keywords**: Differentiated Instruction by Grouping Investigation, learning motivation, learning outcomes, students achievement*

INTRODUCTION

Mathematics, which is considered as the *queen of science*, should be able to be excellent in protecting other science. That has been outlined by mathematicians in the first era. However, the obstacle faced at this time is much different from the previous one. Mathematics is only be used as a routine subject in curriculum charged to students.

A side effect of this, students feel bored in mathematics, especially when teacher use ordinary methods. Students are completely suffered in learning mathematics. Boredom is reflected in students activity, there is no interest to try new things on their mathematics learning.

Senior High School of International Islamic Boarding School Republic of Indonesia (SMA IIBS RI) applies accelerated learning during two years of high school period. This makes teachers should be skilled in presenting teaching methods. Accordingly, an interesting methods makes students interested in learning since it is a must that they spend learning activity in short and fast period.

In fact, some teacher still use the old way of learning strategies in the classroom. Teachers are positioned as the center of attention (teacher centered). Ideal activity demands students to become the center of attention (student centered).

Besides, the students have different intelligence or known by multiple intelligence. By this condition, some teachers still use the old means of learning for method for all students (one size fit all). Using the levelling strategies and methods are not effective to the students. Moreover, in topic Geometry Transformation, there should be different in treating students with various intelligent characteristics

Therefore, appropriate methods and learning strategies are needed to accommodate accelerated students learning styles, for improving their achievement at school. In relation to this matter, I, the author tried to do research of the method by Differentiated Instruction by Group Investigation influencing to students motivation and learning outcomes in mathematics lesson, especially in topic Geometry Transformation.

Learning of Differentiated Instruction

Differentiated Instruction is a method in which students are expected to be more active in learning activity. Differentiated Instruction is intended to facilitate students with special intelligence at maximizing their individu potential. This learning system is used for students with accelerated system, or at this moment using credit semester system.

Differentiated Instruction is developed by education practitioners to anticipate enrichment program or full acceleration that often occurs to students with special intelligence and talents. This system serves special students so that they can learn in regular classes. On Differentiated Instruction, students are offered a series of learning options to dig and direct teaching method on the level of students readiness, interest and various learning profiles. On the other side, teachers use:

1. Various ways used by students to explore content of curriculum,
2. Variety of logical activities or processes so that students not only can fully understand but also have information and ideas,
3. A different choices that students can demonstrate what they have learned. (Tomlinson, 1995)

Tomlinson (1999) state Differentiated Instruction is a learning approach and steps. Differentiation is a flexible arrangement step in teaching and active learning for learners in all conditions and help them to maximize students achievements as learners.

Tomlinson (2015) distinguish Differentiated Instruction is a systematic way to plan curriculum and instruction to learners with diverse intelligence.

From the above explanation, it can be interpreted that the Differentiated Instruction used in regular classroom does not mean that it gives the same task on all the students and make adjustments for special students by distinguishing the question difficulty level, giving more difficult task to them, or allowing them to complete regular program then they are free to do the game as enrichment. Differentiated Instruction is also not to say that students are given more tasks, such as mathematics questions are given more to the special students. There are four characteristics of Differentiated Instruction :

1) Learning focuses on concepts and principles.

   Differentiated Instruction is based on concepts and principles to encourage teachers in providing a wide selection of learning. In this case, all students can explore key concepts of teaching materials. The teaching process emphasizes on students understanding the subject matter rather than memorizing partial information.

2) Evaluation of readiness and learning progress of students are accommodated in the curriculum.

   This provides students challenging learning experience by exploring the students individually. Students do not need to do the same activity or learning process. The teachers should be able to continuosly evaluate students readiness and interest with their support. This is needed by students to require interaction and additional guidance to explore their challenge of learning experience.

3) There is flexibly grouping students.

   On Learning Differentiated Instruction, talented students often study with many patterns; it can be in pairs or be independently individuals. Teachers need to design tasks based on students readiness, interests, and learning styles. To transfer new ideas, teachers also need to learn how linear and classic teaching method.

4) Students become active explorer.

   Talented students own ability to hunt their learning style. Therefore, the teacher role as mentors and facilitators for students activities in order to be directed according to the demands of the curiculum.

The Strategy of Learning Diferentiated Instruction

Teachers can modify teaching activities including five elements, namely:

1) Substance of Learning

   Teacher have to implement curriculum completely and fully give to learners. Teachers provide treatment to students in learning by jumping, as long as they are in the corridor of curriculum that must be accepted by learners.

2) Process

Teachers can develop students thinking skills, in and cross-disciplinary subject, or independent study.

3) Product

The products provide that every student knows the steps and demonstrates knowledge learned by independent effort.

4) Environment

It is a step of conditioning a class, not just about physical condition of the class but also on classroom management from time provided that can be effective.

5) Evaluation

Teachers do understand their students' needs in relation to given learning material (students needs).

Group Investigation

Group Investigation is a learning method that provides students' activities to be more active in classroom. Group Investigation method directs the students in the group to express their understanding on learning. This method takes model of social mechanism in which community members believe to be able to produce better decisions together. This method is used by students so that they can discuss in the group with their appropriate personal understanding to solve problems. (Winataputra, 2001 p. 34)

Group Investigation provides an opportunity for students not only to develop themselves but also their skills in learning experience. Motivation in this study can be formed with students capability and learning experience. Group Investigation has main focus on material investigation of group each member.

Group Investigation Steps

Sharan Supandi (2005 p. 6) reveals that there are steps in learning group investigation as follows:

1. The teacher divides the class into heterogeneous group.
2. Teacher explains the purpose of learning and gives group work.
3. The teacher called the chairmen to take the group task.
4. Each group discusses the task cooperatively.
5. When finished, each group, represented by its chairman or one member delivers the discussion results.
6. Other groups can provide feedback.

7. The teacher gives brief clarification when occuring an error concept and finàlly provides conclusions.
8. The teacher evaluate learning outcomes based on the topic discussed and presented by the students.

Stages in Group Investigation

In connection with the cooperative method, Group Investigation has stages in its implementation, namely:

1. Stage troubleshooting phase.

   At this stage, students solve problems assigned by the teacher, investigate what lies behind the problem and take conclusion.
2. Classroom management phase.

   This stage focuses on how teachers manage the class in guiding students in discussion to solve the problem and demonstrate its answer.
3. Phase of Individual Understanding.

   This stage is the last one where students are able to present problem solving. The teacher has essential duty on guiding and directing students to draw conclusions in accordance with the subject matter. (Thelen in Winataputra, 2001, p. 37)

The successful implementation of Group Investigation learning methods can be influenced by complex factors, namely:

1. The student-centered learning.
2. Group Learning creates an atmosphere of mutual cooperation and interaction among students in the group regardless of students background.
3. Students are trained to have good ability in communication.
4. The motivation that encourages students to be active starting from the first to the final learning stage.

Learning Motivation

Motivation is personal impulse arising consciously or unconsciously to perform an action with a particular purpose. Motivation can also be said as an effort series to provide certain conditions, so that a person is willing and wanting to do something, and when he likes, he eliminates dislike feeling of it. (Slameto in Ridwan, 2005, p. 20)

Learning motivation is students' encouragement that leads toward learning activities to achieve students' desired aim. (Prayitno in Ridwan, 2005, p. 25) Dimensions in Learning Student Motivation are as follows:

1. Attendance at school;
2. Activeness in teaching and learning process in the class;
3. Learning at home;
4. The attitude in facing obstacles
5. Effort to overcome difficulties;
6. Interest and concentration in learning;
7. The learning habit
8. The spirit to learn, and others.

The characteristics of learning motivation as follows:

1. Persevering in facing the tasks (not quit before completion);
2. Resilient in facing adversity (not easily discouraged);
3. Showing interest and ability to achieve goals;
4. More happy to work independently;
5. Be sensitive and responsive to problems (capable of providing solutions).

Since students' motivation role in learning is important, the teacher is expected to increase students' learning motivation to achieve optimal learning results.

According to Winkel and Dimyati (in Ridwan, 2005, p. 35) a teacher should consider the following things: 1) Ability to optimize learning principle application 2) Ability to optimize the dynamic learning elements 3) Ability to optimize students experience and abilities.

Learning Outcomes

In Official Indonesian Dictionary (2004, p. 234), the result is interpreted as something that has been done before. Furthermore, as noted above, the results would not be obtained as long as someone does not perform an activity. Tenacity, perseverance, and optimism can help to achieve an optimal result.

In addition, study is a conscious activity to get some experience of the materials that have been studied. Learning result can be clearly seen from personal behavior change based on his experience.

Metodology

Since it is closely related students motivation and learning outcomes in accelerated schools, the writer tries to combine two learning method becomes an inseparable integral part. The writer call this new learning method as Differentiated Instruction by Group Investigation abbreviated as 'DIGI'.

Differentiated Instruction method is usually used for the applying method of PISA, while Indonesia is still in a low category among the neighboring countries of Southeast Asia.

Differentiated Instruction method focuses on applying students independent learning. This method is also more grounded in students learning experiences. Therefore, teachers play an active role in organizing their students in order to run appropriate learning activities designed by the teacher.

Combining these two methods becomes a reason that learning process designed by the researchers put forward the method of learning investigation problems. Group Investigation Method is already in line with Differentiated Instruction Method, which is more directed to student centered in learning activity. Teachers are only as advisors and counselors without a lot of talking in front of the class. The teacher just give conclusion on what was learned by the students.

Designing of Learning

DIGI Model is designed with students centered strategy. The learning design is as follows:

1. Students are set up in groups seat ;
2. The teacher gives an introduction to the concept of geometry in a video showing a robot moves from one place to another in video from youtube;
3. Students discuss the geometry transformation anf its parts by viewing videos;
4. Students present the results in their groups;
5. Teacher lead to conclusion about the geometry transformation;
6. Teacher gives students workout in group to define parts of geometry transformation and tells the steps how to solve it;
7. Students present the results obtained in their group in front of all students;
8. Teacher lead students to the conclusion about geometry transformation;
9. Teacher gives problem solving to students in small groups;
10. Students discuss the problem solving in small groups;
11. Students present the results from their group in front of class;
12. Teacher provide conclusions and final result of the problem solving;
13. Teacher provide a post test to measure the student final learning.

Unit Analysis

In the test of DIGI learning model, the researcher conducted a population analysis of SMA International Islamic Boarding School RI, and for leearning activity sample is held at class XI, Science Girls, learning topic geometry transformation.

In analysis determination, it is carried out two decision-making methods for motivation of learning; a hypothesis with DIGI and without DIGI. The hypothesis in this study, namely:

Null Hypothesis (H_0): There is no increase in students' mathematics learning motivation by using DIGI.

Alternative Hypothesis (Ha): There is increased motivation in students' mathematics learning by using DIGI.

And for growing mathematical learning outcomes before and after using DIGI only from test by mean and standard deviation value.

Technique of Collecting Data

The data collection to analize students learning motivation is on questionnaire data and their active participation in group. While the data collection to see improvement of students' learning outcomes is through students' post test result.

Analysis of Data

The data analysis to observe students learning motivation is by finding out the R value square that is greater than the error level, and the siginificant value which is smaller than the error. By that data, it shows that students' learning motivation is high with DIGI method.

And The data analysis for drawing conclusion that there is learning outcomes increase is if the final result calculation meets following criteria:

1. t value is greater than t table. To calculate the t value, it is based on the following formula:

$$t = (\bar{X} - \bar{Y}) \sqrt{\frac{n\,(n-1)}{\sum_{i=1}^{n}(\hat{X}_i - \hat{Y}_i)^2}}$$

2. The Mean Value (average) after the DIGI method use is greater compared with the prior use,
3. The standard deviation value after the use of DIGI method is smaller compared with the prior use.

RESULT

Before DIGI method is implemented, there is a fact based on the observation that learning process in the classroom still use lecture method. In addition, the result of informal interview to students shows the information of students activity in class including their participation and ability to answer math questions during the class.

Learning design on geometry transformation focused on the effect of Differentiated Instruction (DI) with following learning sequences:

1. Students are given an exercise in determining the concept of translation, reflection, rotation, and dilation,

2. Then the students along with teacher discuss the concept of translation, reflection, rotation, and dilation,
3. Students are trained to know overall concept of geometry transformation.
4. Students do the exercise independently.
5. Teacher gives explanation and problem solving accomplished by students.
6. Evaluation in the form of assessment.

Having carried out the assessment, found that students do not totally understand the problems of geometry transformation. Then the researcher tried a new method of Differentiated Instruction by Group Investigation (DIGI) with the following steps:

1. Students are made in groups, with the merger between the students having deep understanding and those who do not. It is intended to give chance for lower students in being helped explain by the higher one. See figure 1

Figure 1. Students find the concept of Geometry Transformation by DIGI method

2. Students work on the questions in group based on the same ones as in step 4 DI and added some similar questions with the ones in step 4.
3. Representatives of groups explain to the class. Teachers guide if there are errors. See figure 2.

Figure 2. Presentation by Students

4. Evaluation in the form of assessment. See figure 3.

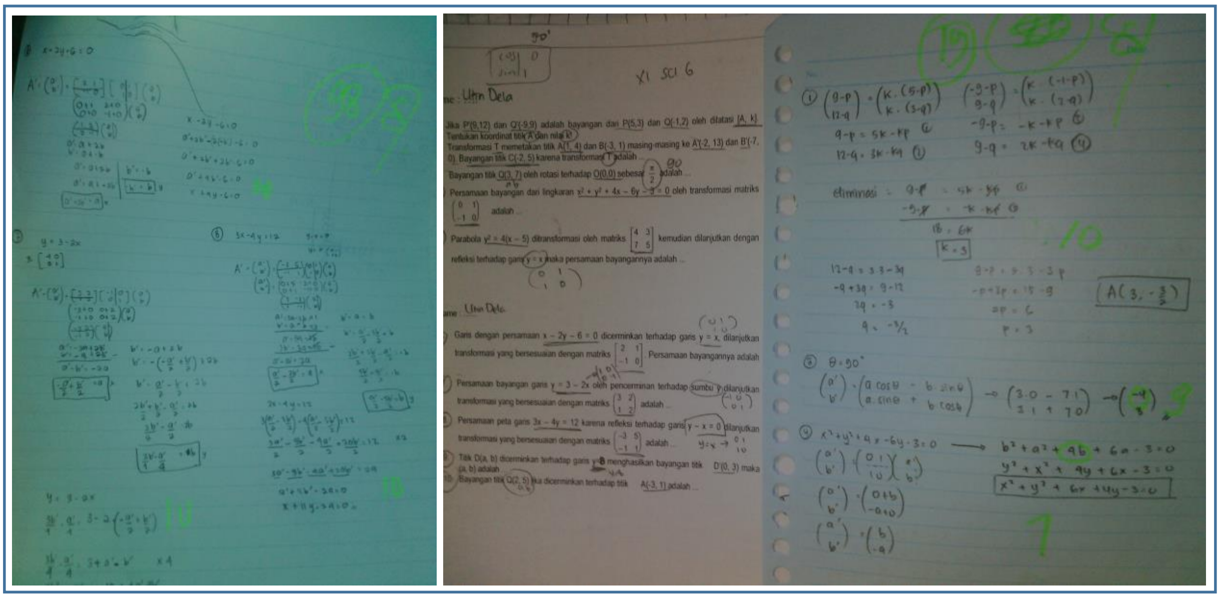


Figure 3. Result of Students Test After DIGI Method Use.

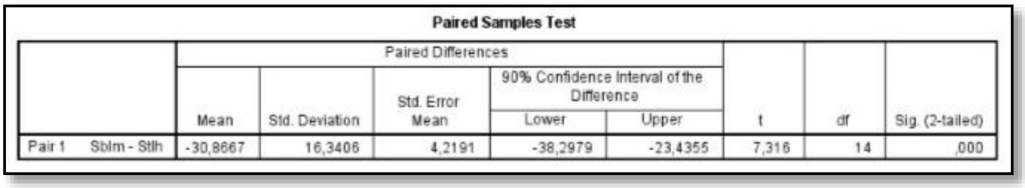
From the final results of DI and DIGI, there is t test of descriptive statistics with the following results (table 1):

Table 1. Scoring of Test Using DI and DIGI Test.

No	Name	DI	DIGI
1	Amalia Salsabila Mumtaz	86	99
2	Aulia Rizky Noer Hikma	43	79
3	Brigitta Nandira Pricilla	58	77
4	Diyanti Pratiwi Br Ginting	39	40
5	Humaira Syahnya Almas	64	76
6	Khonita Maulidiyah	15	67
7	Nabila Syafaqoh Fardam	34	45
8	Nanda Cynthia Huzna	93	98
9	Nisrina Meisya Zahra	83	100
10	Putri Khalilah	40	95,5
11	Rahma Hazfany Hasibuan	14	10
12	Sheila Ayu Kirana Prabani	56	95
13	Siti Dian Meylani	46	30
14	Taqiyatuzzahra Arrawi	40	94,5
15	Utin Dela Nurbani Ramadhan	39	79

From Table 1, it shows that the results (see Figure 4), the average score of DIGI method use is higher than the use of DI, which the average score of DI minus DIGI equals -30.8667. In addition, the standard deviation score of DIGI use is smaller than the standard deviation of

DI, which the DI standard deviation minus DIGI equals 16.3406. There is conclusion that the DIGI method use in learning activity can increase student learning achievement.



Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	90% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Sblm - Stlh	-30,8667	16,3406	4,2191	-38,2979	-23,4355	7,316	14	,000

Figure 4. The Score of Statistical of Student Learning Result Using DIGI

To see students learning motivation is conducted by questionnaire after implementing the learning process with DIGI method (see figure 5).

Table 2. The Qustionare Question of Students Learning Motivation

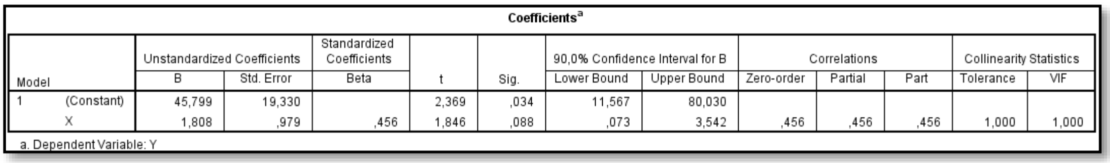
No	Question	SA	A	D	SD
1	DIGI motivate independence in attitutde				
2	DIGI personally motivated to solve problems as a group				
3	DIGI personally motivated to interact to peers together				
4	DIGI personally motivated to solve the HOTS question				
5	DIGI personally motivated to know the lesson				
6	DIGI personally motivated to argue in groups				
7	DIGI personally motivated to unyielding in study				
SA : Strongly Agree , A : Agree , D : Disagree , SD : Strongly Disagree					

The score of questionare question item, SA is 4, A is 3, D is 2 and SD is 1. After collecting the questionnaire score of students (see table 3), analysis the students learn motivation by DIGI methode is held by analysis t test.

Table 3. Questionnaire Result (X) and Post Test Result by DIGI Methode Use (Y).

No	Students Name	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total Score (X)	Y
1	Amalia Salsabila Mumtaz	2	3	3	3	3	3	3	20	99
2	Aulia Rizky Noer Hikma	3	3	3	2	2	3	3	19	79
3	Brigitta Nandira Pricilla	3	3	4	3	3	3	3	22	77
4	Diyanti Pratiwi Br Ginting	2	1	1	2	2	2	2	12	40
5	Humaira Syahnya Almas	3	3	3	2	3	3	3	20	76
6	Khonita Maulidiyah	2	3	2	1	2	2	3	15	67
7	Nabila Syafaqoh Fardam	3	3	3	3	3	3	3	21	45
8	Nanda Cynthia Huzna	3	3	3	3	3	3	3	21	98
9	Nisrina Meisya Zahra	3	2	4	3	2	1	3	18	100
10	Putri Khalilah	2	2	2	3	3	2	3	17	95,5
11	Rahma Hazfany Hasibuan	1	1	1	1	1	1	1	7	10
12	Sheila Ayu Kirana Prabani	3	3	3	3	3	3	3	21	95
13	Siti Dian Meylani	2	4	4	4	3	3	3	23	30
14	Taqiyatuzzahra Arrawi	3	3	3	3	3	3	3	21	94,5
15	Utin Dela Nurbani Ramadhan	3	3	4	3	4	3	4	24	79

From table 3, there is analysis of DIGI method influence on learning student motivation. Using SPSS software, The result in figure 5, It shows that at 90 % degree of freedom, t score is 1.846, while the t table score is 1.75305. Since t score > t table, it can be concluded that the DIGI method use can increase students' learning motivation.



Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	90,0% Confidence Interval for B		Correlations			Collinearity Statistics	
		B	Std. Error	Beta			Lower Bound	Upper Bound	Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	45,799	19,330		2,369	,034	11,567	80,030					
	X	1,808	,979	,456	1,846	,088	,073	3,542	,456	,456	,456	1,000	1,000

a. Dependent Variable: Y

Figure 5. The Score of Statistical t-test of Student Learning Motivation.

CONCLUSION

The use of DIGI method can change the learning mindset from a teacher lecture where students are more passive into active learner method. This method is not only to train students' learning independently, but also to increase students achievement and motivation after the use of DIGI method. The implementation of the DIGI method in SMA IIBS RI, XI class, Science Girls on the topic of Geometry Transformationon, it was found that students motivation and learning achievement has increased.

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Improving Teachers ICT Application Competencies: Case Study at Vocational High School in East Kalimantan Province

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Abstract

Innovation in learning should be done in parallel with ICT literacy development of both students and teachers. One of the problem in learning technology is the unevenly of teachers competency in small city due to the lack of ICT technology development access. This study is qualitative research based on community service activity to enhance the competency of ICT application of mathematics teachers in Vocational High School in Penajam Paser Utara (PPU) district, East Kalimantan Province (Indonesia), especially in utilization of Geogebra and Autograph based on implementation of Lesson Study. This study is a qualitative research that includes 5 phases includes 5 phases ; (1) situation analysis (2) training and workshop of Geogebra and Autograph application (3) implementation of Lesson Study (4) data analysis and presentation, and (5) discussion. In the situation analysis stage, it was found that the ICT literacy of the students and the teachers were adequate, but most of mathematic teachers have not utilized ICT in their classroom. In the training and workshop process, lesson plan utilized Geogebra and Autograph for certain topics was formed. The topics are Quadratic Function, Transformation, Linear Program and Integral Application. As the core of this competency enhancement activity, the teachers have implemented the Lesson Study to see the effectiveness of using Geogebra application in the classroom. The results gained from this study are: (1) by using Geogebra, the students has become more motivated to learn mathematics, (2) the teachers found that Geogebra application can develop student understanding of mathematics concepts more easily. By and large, the competency of ICT utilization in teaching of Vocational High School-mathematics-teachers in PPU, especially in utilization of Geogebra and Autograph, has increased. The recommendation of this study is that the need of the continuity of Lesson Study activity to improve the effectiveness of Geogebra and Autograph utilization in other mathematics topics.

Keywords: *Geogebra, Autograph, lesson study*

Introduction

Utilization of Information and Computer Technology (ICT) especially computer in mathematics learning is getting more relevant considering the characteristics of the mathematics itself. It is known that many topics embedded in mathematics are abstract in nature. There are five important considerations in choosing a computer application so that it can be widely used in learning mathematics, which the application are; (1) using a dynamic view to analyze, (2) expressing personal models, (3) exploring models, (4) storing and processing the real data, (5) sharing and communicating (Pratt, Davies, & Connor, 2011). GeoGebra is a computer application that meets these criterias.

Currently, in Penajam Paser Utara district, there is only 30% of vocational students who have laptops, but 98% of vocational students have gadgets of various brands and types. More than 80% of teachers have laptop facilities of various specifications as a tool in their teaching activities, although its utilization is not optimal as a tool in mathematics learning. The empowerment of ICT (Information and Communication Technology), in terms of computer technology- in learning activities at schools should be supported not only by sufficient infrastructure equipment but also by escalating skill and knowledge of using learning application.

There are various applications that are specifically designed for learning mathematics through various approaches. These applications include: Maple, MuPAD, CABRI, Geogebra, and Autograph. The successful use of computer applications is not located on the sophistication of the application, but rather the strategies of teachers that can collaborate the application in accordance with the character of the material needs and the level of computer literacy of students.

The application that is undergoing fast dissemination among teachers in the world is GeoGebra. Besides as an open source, Geogebra application has now become a network among teachers in the world in discussion communities, so called International Geogebra Institute (IGI). It can create innovative ideas and disseminate the knowledge easily. The growing presence of open-source tools in mathematics classroom on an international scale is calling for in-dept research on the instructional design of Geogebra-based curricular modules and corresponding impact of its dynamic mathematics resource on teaching and learning (Hohenwaer & Lavicza, 2007)

Lewis (2002) defines, “Lesson Study is a cycle in which teachers work together to consider their long-term goals for students, bring those goals to life in actual “Research lessons and collaboratively observe, discuss, and refine the lessons”.

One teacher in the team teaches a lesson, acting as model teacher. The other teachers observe the lesson either in the classroom or from a video of the lesson, focusing on agreed aspects of the teaching and student learning. The lesson study team then meets to reflect on the lesson observations and to review, revise and reframe the lesson prior to the lesson being re-taught by another teacher. The lesson study team engages in cycles of planning, observing, reflecting, reviewing and re-teaching, each time focusing on a pedagogical and content focus. The cycle of instructional development of lesson study has three basic components; Plan, Do, and See. *Plan* is related to the activity of planning and goal-setting. Teaching, observing, and revising the research lesson are covered in *do* component of the cycle. *See* step consists of observing, discussing, and reflecting the Research lesson.

As aforementioned discussion, the problem raised in this research is: "How the competency of ICT application of Vocational High School-Mathematic Teacher in district of Penajam Paser Utara can be enhanced, especially in utilization of Geogebra and Autograph base on Lesson Study activity".

Methodology

The approach taken in this study is a qualitative approach. Frankel & Wallen (2009) stated that qualitative research is research that requires researchers studying the phenomenon that occurs naturally in all its complexity. The subjects were 22 teachers of mathematics and 2 vocational education supervisors in district of Penajam Paser Utara. The phases of the study used a modified Frankel and Wallen (2009) that are summarized in the following table.

Phase 1	SITUATION ANALYSIS It pertains background, teaching experiences level, and computer literacy level of Vocational High School-Mathematic Teacher in district of Penajam Paser Utara
Phase 2	TRAINING AND WORKSHOP The training is a training of Geogebra and Autograph application, while a workshop is a study of writing the Lesson Plan utilizing Geogebra and Autograph
Phase 3	LESSON STUDY IMPLEMENTATION Lesson Plan writing (Plan) Implement and Observe the learning process (Do) Discussion and Reflection (See)
Phase 4	DATA ANALYSIS AND PRESENTATION
Phase 5	DISCUSSION AND CONCLUSION

The first phase, researchers conducted the analysis of the situation through a survey to determine the profile of research subjects, the obstacles encountered in teaching, and the level of ICT literacy of research subjects.

In the second phase, training and workshops on the use of GeoGebra and Autograph were conducted. Training was held for 2 days in a row. The first day was GeoGebra application training, and the second was Autograph. Workshop was held on the second day after Autograph training, and produced several models of mathematics lesson plan utilizited by GeoGebra and Autograph application.

In the third phase, Lesson Study activities were carried out to see the effectiveness of the utilization of GeoGebra and Autograph applications in classroom. Lesson Study were started with made some groups of study, developed lesson plan utilizing Geogebra and Autograph application, and chosed representative teacher of each groups (*plan* step). After careful and mutual discussion among the group member, then they reached their agreement on the selected mathematics topics. The other teachers observe the lesson either in the classroom or from a video of the lesson, focusing on agreed aspects of the teaching and student learning (*do* step) .

The implementation of the Lesson Study was conducted at Public Vocational High School 2 Penajam Paser Utara. The Lesson Study team then meets to reflect on the lesson observations and to review, revise and reframe the lesson prior to the lesson being re-taught by another teacher. The lesson study team engages in cycles of planning, observing, reflecting, reviewing focusing on a pedagogical and the effectiveness of GeoGebra and Autograph utilization in the classromm (*see* step). In the fourth phase, researchers conducted data presentation and analysis of data from activities that have been implemented. The fifth point is a stage for discussion and made conclusion.

Data collection in this study was done by triangulation technique. Researches used different techniques of data collection to get data from the same resource (Sugiyono, 2011). The aforementioned method was done by distributing teachers' and students' activity observation sheets, questionnaires, video recording, and both teachers and students interview to get better understanding of teacher competency improvement in ICT application based on Lesson Study.

The research phases from situation analysis to core activity were done in July-August 2016. The core activities (phase two and three), as a research focus, were held for three days starting in August 1st until August 3rd 2016. The research was conducted in Public Vocational High School 2 Penajam Paser Utara district, East Kalimantan province .

The instruments in this study were 1) ICT literacy questionnaire, 2) model teacher observation sheet for Lesson Study classroom, 3) student observation sheet for Lesson Study classroom, 4) teacher questionnaire to see the teachers' respond of Geogebra utilization, and 5) student questionnaire to get feedback from the students. Moreover, researches also directly involved in research activities ranging from data collection of the situational analysis phase, training and workshops, as well as Lesson Study implementation including *plan*, *do,* and *see.*

Result and Discussion

1. Situation Analysis

In the analysis of the situation process, after having discussion with teachers that belong to MGMP (Lesson Teacher Discussion), and with headmaster of Public Vocational High School 2 Penajam Paser Utara, some points can be concluded bellow.

a. The teachers have difficulty to motivate students to learn mathematics. There are two reasons: first, the mathematics has not been seen as a subject that can be learn visually using computer; secondly, it is still considered as a difficult theory that need high skill of calculation.

b. Most of the Vocational High School-Mathematic Teacher in district of Penajam Paser Utara do not have optimal knowledge about computer application, such as Geogebra, Autograph, and Muped. that can be utilized in learning mathematics. On the other hand, engineering vocational students demand a good visualization in mathematics learning, especially in vector algebra and geometry.

c. The availability of computers and the knowledge of application for mathematics learning are still limited. Some of the teachers has already known the Geogebra application, but it has not been applied in the learning activity.

The survey result of ICT implementation in the learning process of Vocational High School-Mathematic Teacher in district of Penajam Paser Utara is shown on the table 1 bellow.

Table 1. The result of ICT application in the learning process survey

Question/Statement	Answers	
	Yes	No
Do you use student’s laptop in the mathematics learning process?	0%	100%
Do you use laptop and projector for mathematics learning?	57%	43%
Do you use computer facility for mathematics learning?	7%	93%
I utilize a software to visualize the presentation on LCD	29%	71%
I utilize a software to visualize the presentation and support the analytical approach	14%	86%
The students get involve in software application that you use in the learning process	7%	93%

Table 2. The Survey Result of Application Proficiency in Mathematics Learning

Computer Application	**Number of Teachers**	
	Felt competent	**Used in the class**
Power Point	18	18
Geogebra	2	0
Autograph	1	0
Maple	0	0

Mupad	0	0
CABRI	0	0

2. Training and Workshop

In the second stage, training of Geogebra and Autograph application was done for one day. Moreover, workshop to make lesson plan utilized Geogebra and Autograp was done. Lesson plans utilized Geogebra and Autograh was resulted from workshop activities. Selected topic of lesson plans are: 1) Quadatric function with Geogebra, 2) Liner program with Geogebra, 3) Linier program with Autograph 4) Vector with Geogebra, 5) Transformation with Geogebra 6) Integral with Autograph.

3. Lesson Study Activity

Plan. The lesson study participants were divided randomly to two groups to propose a learning plan. Each group agreed in Linear Program as a chosen topic for model teacher's material. The reason was because that the Linear Program was on going topic in one of Public Vocational High School 2 Penajam Paser Utara classroom. The chosen model teachers were Mrs. Lusia Lapik from Public Vocational High School 3, and Mrs. Sriyati from Public Vocational High School 1 represented group one and group two respectively.

Figure 1. Planning Stage of Group 1

Figure 2. Planning Stage of Group 2

Furthermore, in this session, each model teacher presented learning plan that would be implemented. Each group agreed to purpose Geogebra as a learning tool and use problem solving approach for teaching the topic. Both of model teachers implemented cooperative learning method. Each group prepared and developed teaching media including worksheet and slide presentation. Learning class was formed into 5 or 6 groups which each of them consist of 5 or 6 students randomly chosen. Moreover, each student has been given a laptop facilitated by the lesson study group.

Do. The learning implemented by model teachers was conducted in two classes in Public Vocational High School 2 Penajam Paser Utara, which are class 2 TAV (Second Grade Audio Visual Engineering Class) and class 2 AP (Second Grade Office Administration Class). Each class observed by eight teachers, one education supervisor, and one academic facilitator (researcher).

In group one, the implementation of learning constrained by power outage happened after 10 minutes of the activity. So that, the projector could not be used to help the learning process. Teacher model made an improvisation scenario without using projector in the class. However, the students still employed their laptops with the battery. Generally, the enthusiasm of students to use computer applications in mathematics was great. It is seen from the majority of students tried to follow all the instructions of the model teacher even though without LCD projector. But, the instruction and explanation from the model teacher was not effective, as well as it took much effort of the teachers since she had to check each group for each planned activity. In the end of the process, the model teacher summed up the learning activity with a good conclusion. Even though the percentage of the subject delivery was limited due to power outage.

Figure 3. ***Do*** **Stage of Group 1 Model Teacher**

In group two, generally the learning process was implemented in accordance with planned scenario. The power outage also happened in the last 10 minutes, but it did not influence significantly the planned learning process. The students had good enthusiasm in following the process. The model teacher demonstrated the good capability using LCD projector in the learning activities.

Figure 4. Do Stage of Group 2

The learning process of the model teacher in both groups can be concluded in table 3 bellow.

Table 3. Summary of Learning Implementation

Indicator	**Group 1**	**Group 2**
Lesson Study Class	Second Grade of Audio Visual Engineering Class	Second Grade of Office Administration Class
Punctuality	The model teacher was late for 20 minutes	The model teacher was on time
Power constrain	Power outage after 10 minutes of the class started	Power outage in the last 10 minutes of the class
Learning conformity with lesson plan	It was not in accordance with the planning	It was in accordance with the planning
Model teacher satisfaction	Less satisfied	Satisfied

See. After learning process implemented by model teacher, reflection process was sequentially done. The components were moderator (researchers), model teacher, education supervisor, and teachers who participated in Lesson Study. The learning process reflection of each teacher was done alternately. Furthermore, it was done in the same forum, hence both of the groups could observe others reflection discussion. This reflection aimed to see the effectiveness of Geogebra application in the learning process.

Figure 5. Reflection Activity (*see*)

Figure 6. Reflection Activity (*see*)

The result of Group 1 reflection is concluded on table 4 bellow.

Table 4. Reflection of Group 1

Model teacher reflection	Observer Responses
- The teacher was late coming to the class - Power outage made the teacher teaching manually - It was not accordance with the planning due to time limitation - Number of laptop limitation	1. Mrs Heksa - The limitation of laptops availability impacted the activeness of students. - The teacher had good capability to do improvisation 2. Mr. Saiful Ghozi (Researcher) - The scenario, which has been made, changed totally due to the power outage. The learning process needed to be done manually, however the teacher managed it well. - The students was enthusiastic with the new application, however some students did not understand of the teacher explanation, so that, they worked by themselves. - At the beginning of the class, the teacher only sit in front of the class in the first, yet after sometimes she started to supervised the group of the students actively. - There was only 6 students using the stationary to find the answer of the problem, the rest did not. - The appreciation done by the teachers is essential to improve the students' passion. 3. Mr. Sugiono (Education Supervisor) - The teacher was experience teacher, so she was not panic of the power outage. - It is better to name the group of students, so it makes the process of learning more easily. - The explanation of the teacher was not clear, so that, many of the students did not understand of what have been explained. - The teacher gave less attention to all of the groups. She concerned only with one particular group. - A teacher is advisable to check the working result of the students. - The students were less active

	- There was only one group that followed the teacher explanation

Moreover, the result of the reflection of Group 2 is concluded on table 5.

Table 5. Reflection of Group 2

Model teacher reflection	Observer Responses
- The class experienced power outage in the last 10 minutes. - The students could determine the visible area of linier program problem by Geogebra. - All of the groups finished the task using the Geogebra application. - There was a question "Why is this application called Geogebra?".	1. Mrs. Asnidar - The learning process helped by Geogebra application is suitable if each student has laptop. Moreover, the ideal group is that consist of 3 students. - One of the students worked on worksheet when the Geogebra application was utilized. - The students were interested in the new application presented by the teacher. 2. Mrs Risty (Researcher) - The concentration of the students distracted by power failure. - The students gave a good interest. - The process improved the capability of the teacher of IT utilization - The model teacher should not have conversation with the observer teacher. - The model teacher should solved the problem by herself in Lesson Study classroom without asking observer help 3. Mrs Latifah (Education Supervisor) - All of the aspects were monitored, including the teacher and the students. - Classroom activity should directed to student oriented method - Geogreba and Autograph implementation aim to visualize and construct understanding, as well as to check the result between manual and computer calculation.

4. The effectiveness of Geogebra Application in Mathematics Classroom

Indicators to quantify the effectiveness of Geogebra application in this study are: 1) student respond both in the learning process and afterward, 2) teacher respond both in the learning process and afterward.

Table 5. Questionnaire Responses of Students Afterward

Questions	(Group 1)	(Group 2)
1. Was today an interesting learning day?	Yes = 68% No = 32%	Yes = 88% No = 12%
2. Did you understand the mathematics topic that has been explained by the teacher?	Yes = 54% No = 46%	Yes = 84% No = 16%
3. What area that need to be improved in today learning process?	a. Facility = 25% b. Electricity = 2% c. Humor = 12%	a. Facility = 18% b. Humor = 12%

	d. Volume of the voice = 40% e. Not Answer = 33%	c. Volume of the voice = 41% d. Not Answer = 29%
4. Did you find something new in the math class today?	Yes = 100% No = 0%	Yes = 100% No = 0%

From the total of 26 Mathematics teacher of vocational high school in district of Penajam Paser Utara, who were invited into the program of competency improvement, 22 of them attended the first day of training (84.5 % of attendance), and 20 of them attended the second day of training (70 % of attendance)

Furthermore, the respond of the observers is summed up on table 6 bellow.

Table 6 Questionnaire Responses of the Observers

Questions	(Group 1)	(Group 2)
1. Was the learning process in accordance with lesson plan ?	Accordance = 10 % Less Accordance = 84% Not Accordance = 6%	Accordance = 82 % Less Accordance = 18% Not Accordance = 0%
2. Was the class activity effective?	Effective = 45% Less Effective = 40 Not Effective = 5 %	Effective = 85% Less Effective = 12 % Not Effective = 3 %
3. Was there something new in the class today?	Yes = 100% No = 0%	Yes = 100% No = 0%
4. What area that need to be improved in today learning process?	lesson plan, preparation of model teacher, facilities, method.	preparation of model teacher, facilities, method.

The data of capability improvement in ICT implementation are derived from questionnaire after the training and workshop, student and teacher interviews, as well as direct observation from the researchers. Because of that, the data cannot measure quantitatively.

It can be concluded from the data that teachers have motivation and willingness to improve the capability to implement ICT in their classroom, because they have tried Geogebra application from planning, implementation and evaluation through Lesson Study activity. Application that has not been implemented is Autograph due to time limitation in Lesson Study. To sum up, the teachers have improved the basic capability to implement ICT in the learning process.

Conclusion

The conclusions that can be inferred in this research are:

1. The teachers have capability to utilize Autograph 3.3 and Geogebra 5 in mathematics learning.
2. The students have become more enthusiastic to learn mathematics, because they were interested in new application that presented by the teacher.
3. Lesson Study activity was well implemented despite power failure happened in the classroom. However, it affected the effectiveness of learning activities in term of student enthusiasm and understanding.

Recommendations generated from this study are :

1. Classroom action research is required to test the effectiveness of the application implementation in accordance with the defined aim of mathematics learning.
2. Lesson Study activity can be empowered more effectively through MGMP forum to improve the quality of the learning process, in accordance with the potential and obstacle characteristic.

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Learning Module Using Discovery Learning Approach: Assessment of Validity, Practicality, and Effectiveness

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Abstract

The aims of this research were to determine the validity, practicality, and effectiveness of the result of the development of learning module using discovery learning approach in limits algebraically. The development of the learning module was done through the modified Four D, namely define, design, development, and disseminate. Data were analyzed using descriptive analysis. Validity analysis was based on an expert assessment. Practicality analysis was based on implementation of learning module. Effectiveness analysis was based on student response after using learning module. The results concluded that: 1) learning module with discovery learning approach in limits algebraically declared fit for use by experts with the validity score is 3.44 of scale 4 and belonged to very good criteria; 2) learning module showed practical use in learning with the practicality score gained from the sheet of learning implementation is 0.919 of scale 1 and belonged to good criteria; 3) learning module is effective to use in teaching-learning process with the scores of effectiveness obtained from the sheet of students' response to the learning module is 3.39 of scale 4 and belonged to good criteria.

Keywords: *learning module, discovery learning, assessment, limits algebraically*

Introduction

Teaching-learning process is an active process that is resulted through individual active involvement in reflexing experience and action practiced in a certain environment (Huda, 2014, p. 38). Students are expected to get impressive and meaningful experience through active process and individual involvement. Jennifer R. Nichols stated 4 teaching-learning principal in the 21st century, teaching-learning process must be centered on students, education must be conducted collaboratively, teaching-learning process must have contexts and school must be integrated with the society (Nichols, 2013). Teaching-learning approach must be centered on students. Students not only sit, listen, and memorize the materials, but also construct knowledge themselves. Therefore, school needs to facilitate students to be active, provide material that can encourage students to construct knowledge and participate in social environment. Mathematics learning is expected to teach students to study independently in a team, think critically, solve the problems, and communicate well. Critical thinking skill and problem solving of students can be stimulated and sharpened through meaningful teaching-learning process. Teaching-learning process can be meaningful if students can be active during the process. Therefore, students can construct their knowledge through several activities that encourage them to conduct discovery. This teaching-learning process gives impressive experience to the students so that they feel confidence to learn the materials independently using appropriate media

(Chambers, Chambers, & Thiekotter, 2013). The role of teacher is as a facilitator who provides learning experience and media that can stimulate them to think critically, and assesses students' learning result.

Ausubel cited on Orton (1987), "if I had to reduce all of educational psychology to just one principle, I would say this: The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly." Main factor that influences teaching-learning process is what students know, therefore teachers can make sure of it and teach them appropriately. To create meaningful learning, it must be designed to encourage students to conduct discovery. Teaching- learning process is designed and collaborated using discovery learning. Discovery learning takes students to organize knowledge because material is not given in the final form. Students are demanded to conduct various activities such as finding the aim of teaching-learning, collecting information, comparing, categorizing, analyzing, integrating, and organizing materials and making conclusion. Teacher must be able to make students study independently in discovery learning. Bruner stated that learning process will run well and creatively if teachers give chances to students to find concept, theory and understanding through real samples in their life (Budianingsih, 2005, p. 41). Discovery learning has some characteristic, such as stimulation, problem statement, data collection, data processing, verification, and generalization (Syah, 2004). By applying discovery learning, students are expected to participate actively in teaching-learning process, think critically and creatively, learn how to study, obtain optimal skill, memorize in students' thought because students are involved in the process of finding concept.

Based on the interview conducted with mathematics teachers of senior high schools in Sukoharjo, teaching-learning process has not been optimal yet. It is shown when most of students do not pay attention on teachers' explanation, they do not have initiative to read handbook before class is started and they do not finish exercise if there is no command. Students also do not bravery to share their opinion or ask the difficulties.

Based on the result of national examination of senior high school year 2013/2014 (Badan Penelitian dan Pengembangan Kementerian Pendidikan dan Kebudayaan [Balitbang Kemdikbud], 2014), limit of a function is low mastery. In Sukoharjo regency, the mastery is 44.09%. It is the lowest percentage of other regencies, such as Surakarta 54.37%, Klaten 47.06%, Karanganyar 46.90%, Central Java province 51.81%, and national level 51.88%. The result of national examination of senior high school year 2014/2015 (Balitbang Kemdikbud, 2015), percentage of mastery limit of a function in Sukoharjo regency is 43.60%. It is lower than Surakarta 54.49%, Klaten 47.71% and national level 53.41%. The low mastery of material indicates that learning is not optimal. That problem raises because students' handbook is not interesting and difficult to study independently. Therefore, students depend on teachers' explanation.

To facilitate students in conducting independent and meaningful teaching-learning process is by optimizing it through lesson material that encourages them to conduct discovery. Module is a systematic lesson material so that the users can study with or without facilitator (Departemen Pendidikan Nasional, 2008, p. 20). Module is systematic lesson material based on certain curriculum and packed in the smallest learning form which enables to be learned independently in some time (Purwanto, 2007, p. 9). The characteristics of module are having independent direction, content unity, independent, adaptive, and friendly (Daryanto, 2013, p. 9-11). The characteristics of learning module using discovery learning are stimulation, problem statement, data collection, data processing, verification, and generalization. In the introduction, students are faced with problems that take them to learn a certain material. Students are guided to identify problems that relate to lesson material. Students discuss in a group to collect relevant information, process and interpret it. Students examine to prove whether the solution is true or not. Finally, students make conclusion for the same problems.

Method

This research belongs to research and development. Product that developed is learning module using discovery learning approach on limits algebraically. Development model uses modified four D, with steps of define, design, development, and disseminate. Step of define includes early-final analysis, students analysis, task and concept analysis and special instructional aim. Step of design includes arranging module and its lesson plan, and module assessment instrument that includes instrument to measure validity, practicality, and module effectiveness. Step of development includes development to physical result of learning module using discovery learning through validator assessment and limited experiment. The validator is an expert of material, curriculum, media and writing. Aspects validated by material and curriculum expert include content feasibility, display and language component, and discovery learning approach. Aspects validated by media and writing expert include graphic feasibility. After validation process, revision is conducted on the part which needs to be improved. It will result second draft module. Furthermore, second draft module is given to the validator. Second validation process is conducted continuously until there is no revision. Field test is conducted in Insan Cendekia Sukoharjo senior high school to obtain data practicality and effectiveness. Step of disseminate includes printing and distributing module to be ready used in teaching-learning process.

Result

Module development in this research uses modified four D. Step of define results sub-material in limits algebraically that can be conveyed with the characteristic of discovery learning and how to present the material. Material is not presented completely, but students are asked to find material through instructions in module. It is appropriate with the aim of discovery learning in which students think the necessity in studying, how to shape knowledge, and use skill of critical thinking (Cruickshank, Jenskins, & Metcalf, 2009). Material presentation in module uses inductive concluding process. Students are given problem samples to be discussed and concluded generally. Inductive concluding process is justified in mathematics teaching-learning process in senior high school in which concept understanding is often started inductively through system observation or phenomenon, real event experience or intuition (Kementerian Pendidikan dan Kebudayaan [Kemdikbud], 2014, p. 325). Based on its mathematics work way, it is expected to shape critical, creative, honest, and communicative behavior to the students. The use of simple words in module is expected to be understandable for the students of grade 10. Material presentation in module uses instructional sentences that encourage students to conduct something in achieving teaching-learning aims. According to Kemdikbud (2014, p. 364), the role of teachers is stating problems and guiding students to find solution by commands and worksheet use. Students follow the available direction and find its solution. Students are also given discussion room to write step of finishing problems on module.

Step of design includes the making of module, lesson plan and module assessment instrument. Teachers and education practitioners have to present material mastered by students using strategy, method, and appropriate learning media (Kemdikbud, 2014, p. 365). Manipulation of lesson material based on students' cognitive development is conducted to facilitate learning process (Kemdikbud, 2014, p. 376). Cognitive development step of age of 14-18 is iconic step to symbolic. Students understand objects through visualization then start to have abstract idea of language and logic skill. Discovery learning module is conveyed through graphic, table, and instructional sentences that encourage students to solve problems. Students with symbolic cognitive development can solve problems without observing graphic. However, students with iconic cognitive development are helped through observing graphic first. Then, they solve problems by following the instructions given.

Module assessment instrument is an instrument to measure validity, practicality, effectiveness module. Module validity instrument is adapted from guidance of text book assessment of assessment standard of lesson text book by Badan Standar Nasional Pendidikan

(BSNP). It is also added by aspect of discovery learning. Module practicality instrument is based on guidance of teaching-learning implementation of the regulation of the minister of education and culture Indonesia no. 103 year 2014. It states that teaching-learning process on the elementary and middle education consists of some activities such as introduction, content, and closing that is adjusted with module. Module effectiveness instrument is considered from display, material and benefit of module using.

Step of development includes expert assessment and module test. Based on expert assessment, the developed module has been completed with 5 feasibility aspects, such as content feasibility, component of presentation, language, graphic and discovery learning. Validation result of three validators is summarized in Table 1.

Table 1

Experts	Score				The Number of Items	The Number of Score	Mean of Score
	1	2	3	4			
Dr. Sumardyono, M.Pd. (subject matter experts and learning curriculum experts)	0	0	28	26	54	188	3.48
Mujapar, M.Pd. (subject matter experts and learning curriculum experts)	0	0	28	26	54	188	3.48
Dr. Sumardyono, M.Pd. (media experts and scribes)	0	0	34	20	54	102	3.37
Marfuah, S.Si, M.T. (media experts and scribes)	0	0	30	24	54	186	3.44

Based on three validators, module is concluded in a good criteria with score is 3.44 of scale 4.

Furthermore, limited field test is conducted. In this module test, facilitator who guides students is their own mathematics teacher. It is conducted because students have been friendly, comfortable and they do not need to adapt with another facilitator. The function of facilitator is responding students' thinking result. It can be form as question or difficulty in solving problems. Facilitators guide, motivate and encourage students' enthusiasm (Kemdikbud, 2014, p. 384). Before teaching-learning process is begun, facilitators discuss with the writer related to module application. It is conducted to reach vision and mission of module arrangement. In the step of module test is obtained score of practicality and effectiveness module that can be a material of revision before conducting module test. Implementation data for 5 times of teaching-learning process is summarized in Table 2.

Table 2

Meeting	Evaluator	Score		The Number of Items	The Number of Score	Mean of Score
		0	1			
I	Facilitator	3	12	15	12	0.8
	Observer	3	12	15	12	0.8
II	Facilitator	2	13	15	13	0.86
	Observer	3	12	15	12	0.8
III	Facilitator	1	14	15	14	0.93
	Observer	0	15	15	15	1
IV	Facilitator	0	15	15	15	1
	Observer	0	15	15	15	1
V	Facilitator	0	15	15	15	1
	Observer	0	15	15	15	1

Observation result for 5 times of teaching-learning process using module obtains module practicality score of 0.919 of scale 1. It means that module is practically used in teaching-learning process. Result of response sheet questionnaire after using module is 3.39 of scale 4. It means that module is in good criteria. Therefore, module can be stated as effective in use.

Step of disseminate includes module distribution. It is conducted after revision based on suggestion and comments of the students in testing step. Then, module is printed and banded to be ready used in the testing step. Cover and module content is printed in 80 gr HVS paper and cover is press laminated. Module is spirally banded. In the final result, module is printed in size of letter of 216×279 mm. Module is used as learning source in limits algebraically. It also becomes a reference of learning source of the next material of limits algebraically.

Discussion

Based on observation during teaching-learning process and students' comments as module users, there are some weaknesses in using module, such as: 1) Discovery process needs long time (Westwood, 2008). In teaching-learning process, process of students' adaptation on module needs long time especially in characteristic of discovery learning in finding material; 2) Students are encouraged to be involved actively and have experience in conducting experiment (Slavin, 1997); 3) Characteristic of discovery learning depends on students who have adequate language mastery, calculating, independent-self skill and self-management (Westwood, 2008). Students with low cognitive skill will be rather difficult in using module. Students prefer to get intense guidance from teachers; 4) Teachers do not monitor the activity effectively. They cannot also give individual encouragement and guidance (Westwood, 2008). Passive students have no bravery to ask questions. They cannot also request special guidance from teammate or even facilitator. Therefore, they let themselves in a misunderstanding situation.

The result of module development is also compared with another existed module. This module is aimed to make teaching-learning process more meaningful. Characteristic of module development is different from another module in some aspects, such as: 1) Material aspect, it is based on syllabus analysis in the school. Material is presented in a form of discovery. Therefore, it creates independence and appreciation on teaching-learning process. Material is also presented through samples observation. Then, it is concluded inductively; 2) Scenario aspect, module uses discovery learning which creates discovery situation, critical thinking, participation, high interaction and improvement skill of analyzing, synthesizing, and evaluating (Balm, 2009); 3) Students' learning activity, characteristic of development module is: a) Stimulation, students are stimulated by presenting problems through statement or picture, it is expected to create students' encouragement to investigate problems; b) Problem statement, students are asked to identify relevant problem which improve students' skill to identify main problem; c) Data collection, students are asked to collect relevant data to answer problems which improve skill in choosing relevant data to solve problems, and data that is not relevant as a supporting data; d) Data processing, students are asked to process information that has been collected. Then, it is interpreted. Therefore, it can improve students' skill in analyzing and synthesizing; e) Verification, students are asked to prove the true answer. It creates unsatisfied behavior and evaluative thinking skill; f) Generalization, students are asked to conclude problems that have been solved. It improves systematic thinking and skill to relate field data that has been processed with hypothesis.

Conclusion

The conclusion of learning module using discovery learning in limits algebraically can be stated as follows: 1) Valid with score of 3.44 and belongs to very good criteria; 2) Practically used in teaching-learning process with score of 0.919 and belongs to good criteria; 3) Effectively used in teaching-learning process with score of 3.39 and belongs to good criteria.

Suggestions are mentioned as follows: 1) Teachers, module using discovery learning approach can be used as learning source to the next year especially in limits algebraically. By using discovery learning approach, students will be active during teaching-learning process. Students can find idea to solve problems, improve participation and students' interaction. Students who are not accustomed to study independently show that this module is importantly considered to detract teachers' role in explaining the detail material; 2) Students, module can be used to find how meaningful knowledge is formed. Module can also be used to improve skill of analyzing, synthesizing, and evaluating. By using module, students should try to study independently and actively. They should not also depend on teachers' explanation; 3) Researchers, module testing can be used to compare students' achievement that uses or not uses module. It is also to conduct development research to create independent learning, improve activeness, participation, and interaction among the students, and create skill of problem solving

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Teaching Mathematics Holistically: Helping Student to be a Holistic Learner

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Abstract

Education is conducted to develop the potential of students to be faithful and devoted human of God, good morality, mastering of science, and capable to apply its in their life, so it can form their character and human civilization. The paradigm of "integrity" should become educational basic soul, system and practices. Various dimensions of student growth not only intellectually, emotionally, sociality, but also morality must be facilitated in order to develop optimally. Mathematics education, as an integral part of the education system, has to take a significant role to achieve the essence of education. Mathematics education is not only oriented toward mastery of mathematics, as *an sich*, but also it needs to be directed at achieving the essence of learning, namely the integrity of the personal development of students. The teaching of mathematics is implemented holistically, i.e. a mathematics learning which in implementation has integrating the development of the values of humanity, it is expected to help students becoming a holistic learners, the learning which is in addition not only mastering of science but also has values of beneficence.

Keywords: *teaching, mathematics, holistic*

Introduction

The function of education is to develops the ability and character, and the civilization in order to achieve the life of the nation and aims to develop students' potential become a man of faith and fear Allah, the almighty God, noble healthy, knowledgeable, skilled, creative, independent and become citizens of a democratic state and responsible. It shows how high expectations which are based on education. Critics of the current quality of human resources seemed to be a cracked mirror of education which is considered its failure to fulfill duties and responsibilities.

To develop capabilities and to establish character and civilization, its requires education which is able to develop human resources whose has wholeness of knowledge skills and attitudes. Supporting growth and developing of human resources as a whole need for the implementation of a holistic and sustainable education. The paradigm of "integrity" should be the basic spirit system and practice of education. Knowledge, skills and attitudes should be able to be developed integratedly. There is only the education system which is capable of integrating the developent of the various aspects which will be able to drive growth and development of students as a whole.

The above problems requires careful and creative solution. Demands that education should encourage growth and development of students as a whole does not have to be addressed reactive by making subjects from all aspects of life. Curriculum subjects at this time had felt “very much” so need new ways or methods to address the needs of growth and development of students as a whole without adding the burden of student learning. Existing subjects can be empowered in order to contribute more not only in the domain of each field of study but also more open functioned to support the growth and development of students.

Mathematics courses as basic subjects in school should be able to address the above challenges. Learning math should be empowered to support the personal development of students. Learning mathematics should not be oriented a matter of mathematics in *an sich* but it needs to be changed more open touching a wider dimension so as to contribute more. Teaching mathematics holistically is expected to make mathematics education becomes more meaningful.

Teaching Mathematics Holisticaly

Holistic learning terminolgy has actually been much discussed and implemented. But the meaning over holistic learning to date is still diverse. The most common meanings from a holistic learning are learning to integrate various aspect in a learning package. Various aspects are taught in a single unit so expect received and understood by students in wholeness. In contrast to these concepts, holistic learning here is learning to develop various aspects of learning students’ good knowledge, skills and attitudes simultaneous and continuous so that learning becomes more meaningful. Thus, teaching mathematics holisticaly can be interpreted as teaching of mathematics which is in the process not just only teaching mathematical knowledge but also rather expanded by developing skills and attitudes to support the personal development of students in their entirety.

So far success in learning mathematics more stressed the ability to master the material and achieve a high score. This leads to learning often get stuck the teaching materials mathematics as such to the exclusion of the various aspects of student growth. Score is used as the main reference in perceiving the success rate. Parents can be sad if his childres get low math scores. Various measures imposed the student in order to increase the score. Students with all the helplessness finally resigned and follow the will of the parents. At school the same phenomenon are also common. Perceive school success in learning mathematics based on students’ grades. To improve performance the school through various approaches to further raises the score of the student.

Mathematics courses taught since elementary school to higer education cannot be separated from consciousness that mathematics has a great potential supporting the personal development of students. The significance of this has been accepted as a real almost by all parties even mathematics occupies a vital position in the curriculum. In terms of quantity time allocation math lessons all levels always big enough. Its should not simply to pursue value alone but must be able to be utilized to explore and empower potential math the benefits for students. In this context the implementation of holistic learning by teaching mathematics holistically to be relevant and urgent.

Curriculum

Teaching mathematics holistically need curriculum support. The curriculum is the spirit as well as the content and frameworks in education. The curriculum directs all forms educational activities. Therefore mathematics education which is expected to encourage the development of students as a whole requires a curriculum which clearly orient holistic education developing human resources education as a whole both the knowledge skills and attitudes.

The curriculum is a set of plans and setting the objectives, content and learning materials as well as the ways in which as a guideline implementation learning activities to achieve certain educational goals (Act No. 20 of 2003). Teeaching mathemaics holistically will only be successful if the curriculum of mathematics containing formulation which clearly the idea of a holistic education.

Objectives

Mathematics education must consciously and intentionally be intended to encourage the development of student competence as a whole not just knowledge but also skills and attitude, both attitude to God and attitude towards fellow human beings. The development of attitudes is expected hard work if the attitude is only expected to grow as a side effect of development knowledge or skills. Developing of skills also will be difficult optimally if it is not designed clearly from the beginning.

Contents

Fill in mathematics education must be determined which is comprehensive covering a wide range of components which are able to support simultaneously the development of self-esteem intact. Mathematics education should not have just oriented to master the knowledge of mathematics but the content of mathematics education also needs to load the skills and attitudes. Mathematics education which has been more focused the mathematical material it had proved unsuccessful to optimize its potential to support the personal development of students as a whole.

Lessons Material

Teaching mathematics holistically require material which is able to support the implementation the vision of holistic education in learning mathematics. Supporting materials the implementation of learning mathematics e.g. textbooks facilities or supplies other learning to be able to support the implementation learning mathematics which held in a holistic manner. Without the support of appropriate lesson materials the objectives and content of mathematics learning which has been established to support the teaching mathematics holistically will be difficult to be realized in practice.

Used Way

The means used to achieve the holistic learning of mathematics eg how to teach or how to assess should be clearly defined in accordance with the needs of achievement the goal. Successful teaching mathematics holistically is highly dependent on the way of teaching and assessing. Therefore the policy of the process and assessment must support the practice of holistic learning in mathematics.

Currently in Indonesia entered a new phase in the field of education where this time began to apply the new curriculum which is oriented to the development of students become the whole person including knowledge, skills and attitudes namely Curriculum 2013. Curriculum 2013 formulated explicitly four (4) kinds of core competencies namely:

a. Core Competence - 1 (CC-1) for core competencies of spiritual attitude;
b. Core Competence - 2 (CC-2) for the core competencies of social attitudes;
c. Core Competence - 3 (CC-3) for the core competencies of knowledge; and
d. Core Competence - 4 (CC-4) for the core competencies of skills.

At each CC described in some basic competence which supports the achievement of CC. This formula is applied to all levels of schooling from ellementary to secondary school on all subjects including mathematics. Thus competence in mathematics in curriculum 2013 is explicitly formulated into four focuses namely the basic competencies appropriate to support achievement CC-1, CC-2, CC-3 and CC-4. With the curriculum the development of aspects of knowledge, skills and attitudes in mathematics is no longer just a hope but a mandate which has been defined clearly on the curriculum.

Teaching Mathematics holistically in the Class

Willingness and ability of teachers

The commitment and willingness of teachers is a key factor for the success of teaching mathematics holistically. The commitment of teachers will be directional as well as a source of energy in achieving the desired goals. Teaching load as well as all the complexity of the problem can override the intention of teaching mathematics holistically. Without a strong will very likely the teacher will re-stuck the learning the pursuit of material only and sheer score.

Teaching mathematics holistically also requires the ability of teachers. The development of various aspects of mathematics learning students' holistically requires creativity in managing the class. Teachers need to have an understanding of and ability to apply a variety of models, techniques, methods, approaches and strategies of teaching in order to pack the better class. Potions lesson by optimizing the various learning methodologies largely determines how far the development of self-esteem can be succeed. No extinguished candle illuminating the environment nor is there a blind person becomes the guide. There is only a competent teacher who can develop various aspects of learning holistically.

Setting goals

One important initial step the success of learning is the selection of learning objectives (Mercer 1989). The goal would be the director during the learning takes place. Therefore teaching mathematics holistically will be realized if its wanted to be done. Teachers should begin with the understanding that developing various aspects of learning simultaneous and continuous is important for students and can be implemented in learning.

The purpose of developing the various aspects of student holisticaly needs to be communicated to students. Understanding teachers and students is imperative for the achievement of learning objectives (DePorter, 2000). Students need a clear overview learning objectives and what they can do and they earn. Knowing the purpose and usefulness learned will bring students more active and excited. Therefore if a teacher is willing to teach mathematics holisticaly, teachers should communicate the purpose so that students have a direction which is parallel to the teacher during the learning takes place.

Learning Planning

Instructional planning is one of the important factor to reach the successful learning (Burden & Byrd, 1999). The success of teaching mathematics holistically is determined when planning lessons prepared by the teacher. Objectives, procedures and processes of learning needs to be planned clearly so that the learning activity will serve the chance to the student to develop their knowledge, skills and attitude. If teachers really want to teach mathematics holistically the teacher should begin at this stage.

Planning to teach mathematics holistically among others include: syllabus, assessment design and Learning Implementation Plan. On the development of the syllabus the teacher should be able to outline the curriculum becomes more detailed description with attention to the development of a holistic aspect. The syllabus will provide a strong foothold for planning future learning. The draft assessment is also an important aspect to be observed because by designing of assessment the teacher can plan the targets of thr assessment, assessment instruments to be used and how the assessment will be carried out. At the drafting stage of assessment teachers must plan how the feedback to the development of various aspects of student learning will be done. Preparation of lesson plans should also be carried out by teachers by taking into account the development aspects of iterated. At the time of preparing lesson plans teachers should be able to design that encourages learning and ensuring mathematics learning holistically will be implemented and the results as expected.

Learning Process

The success of teaching mathematics holistically depends on how far teachers are able to encourage and monitor student’s progress during the learning takes place. Teachers play a key role in the implementation of learning but by no means should the teacher dominate the class. Attention and teacher feedback influence success or failure the students. Teachers should help students stay on track heading the development all aspects of learning. Understanding in the beginning that the purpose of learning not just the acquisition of knowledge but also develops attitudes and skills should be maintained and translated through the cooperation of teachers and students during the learning takes place.

Creating the conducive situation for teaching mathematics holistically have to be cared absolutely by teachers. To encourage holistic learning, social environment needs to be built positively among members of a learning community. Built up a harmonious relationship among students or between students and teachers will support better learning outcomes (Meier, 1999). Mathematics lessons which tend to be perceived by the load, difficult, boring no excitement, stress and whether negative feelings moreover need to be changed. Teachers must be able to manage learning while maintaining an interest motivation and optimism students. Teachers need to be more creative to change the class more encouraging positive and inspiring students to learn.

Assessment

Assessment can affect the behavior of learning because students tend to direct their learning activities towards the end of the assessment which is conducted by the teacher (Mercer, 1989). Assessment can also provide constructive feedback for students and teachers. Teaching mathematics holistically should be completed by appropriate feedback in learning so that students have high motivation to learn in all aspects. Feedback can be given directly in the learning proccess. Feedback can also be given through quantitative assessment with a range of appropriate instruments. Tests no longer be relied upon to be the only assessment techniques. Teachers should use various assessment techniques assessment in accordance with the domain extension.

Assessment in mathematics learning holistic manner does not have to always use way and formalistic approach. Teachers can consider how to use and style assessment of parents who conducted assessment to their children informally, simultaneously and continuously. Parents do not use a formal approach but in a way which is a direct and spontaneous. In fact the method and approach proven to affect how children react and adapt in life. During this manner and style of the assessment shadowed for various reasons. However, real evidence effectiveness in support growth and development of children should be a concern and inspiration to the implementation of teaching mathematics holistically. Assesment which is done informally, simultaneously and continuously, completed formal approach in assesing the student, will give more chance to support the meaningfull learning.

Conclusion

Teaching mathematics holistically is a new paradigm in learning mathematics which enabled to support the development of student competence as a whole both the knowledge skills and attitudes. The integration of these three aspects is a prerequisite for the realization of superior human resources which is expected to support the creation of life and better civilization. The teaching of mathematics is implemented holistically, i.e. a mathematics learning which in implementation has integrating the development of the values of humanity, it is expected to help students becoming a holistic learners, the learning which is in addition not only mastering of science but also has values of beneficence. Holistic learner is the main pillar to realize more dignified of national life.

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The Effectiveness of Interactive Module Based on Lectora to Improve Student's Spatial Ability

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Abstract

Geometry polyhedral was a material that requires student's spatial ablity. It was the reason to develop the interactive module based on lectora. The study was aimed at knowing the effectiveness of the implementation of interactive module based on lectora for the students. Kualitatif method was adopted in this study. Action research was adopted in this study. This research consists of two cycles. Each cycle followed the steps were plan, implementation, observation, and reflection. The subject of this study was class VIII of SMP Taman Siswa. In general, learning is said to be effective if at least there 85% of students who pass the study. The findings showed that the students reached learning completeness as much as 7**5**% in cycle I, and 87**,5**% in cycle II
.

Keywords:*lectora, interactive module, geometry polyhedral, learning media.*

Introduction

Learning process has three essential component of the national education system that students, teachers, and curriculum. These three components had links that cannot be separated to improve the education quality. Improving the quality of education could be done through a breakthrough in curriculum development, innovation learning, and working facilities. To support improving the quality of education should be channeled renewal to make innovative learning that could lead to students to can learn optimal good independent study and learn in the classroom. It can be done through innovative creativity built by teachers the learning. Teachers the learning must consider and planned to learn the process of interest to students that students are interested and eager learn so teaching be effective and maximum.

But to achieve effective results or an obstacle to the learning mathematics especially for teaching geometry. So far learning mathematics only focused on skill count, still neglected to skill demanding logic and reasoning. While in learning geometry contains many concept and application in life. Learning geometry is learning that provides in the abstract from experience visual and spatial. Because learning geometry which presents in the abstract make students hard to understand well. It is associated with many students who could not visualize two-dimensional picture or three dimensions so students still not counting broad understanding the concept, volume, and the surface area of a particular field. Besides many students consider drawing up space as up flat, students have not been able to line opposite sides or satisfying. Empirical evidence in the field shows that there are many students had difficulty in studies geometry. Sinaga (2001) notes that for junior high school students are still not sure in resolving problems online parallel and still many said rhombus as the parallelogram.

But learning system used by textbook which contains material and training. Textbook is one book important in teaching and learning concepts geometry for now. But textbook is learning one direction not provide feedback of students.This makes students hard to apply the concept of correctly. Conditions seen make learning become less fun so students more often feel saturation in learning geometry. With the advent of technology can support a medium learning interactive can make up difficulties in learning geometry where through the learning interactive visualize and students can count two-dimensional picture and three dimensions that the visual ability and increased spatial students. One of the media interactive learning able to facilitate the difficulty students in study geometric is an interactive module. Module interactive material is compiled as required by students need for learning served in a whole (self-contained), can be used to independent study (self-instructional), give opportunity to students to practice, students can test own, and accommodating students to provide feedback with Through software of text images, sound, or video equipped with interactive buttons and an interactive evaluation.

Interactive module made integrated into a software that benefits for students which students can learn anytime and wherever and students do not need to carry textbook to learn having been stored in a hardware. Software that able to facilitate this interactive module is lectora inspire. Lectora is electronics learning (e-learning) development, also known as software that developed by trivantis corporation. Lectora used to make a course of training online, assessment and presentation. Lectora can also be used for conversion of presentation from microsoft power point into the content of e-learning. Content developed by the software lectora can be published to various output as html, executable single file (*exe) and cd rom. Content lectora furnished with industry standards e-learning as scrom (sharable content object reference model) and aicc (aviation industry computer-based training committee). Content made in lectora can also adequate compliance. In addition, lectora supported by standard based management learning. Lectora also allows users to use multimedia file to create content more interesting.

Based on the issue and student needs, we needs to be made media learning to help students in study geometric by interactive module based on lectora in geometric the polyhedrals for students junior high school grade VIII. Through interactive module based on Lectora in geometric the polyhedrals for students junior high school grade VIII, students can interact directly with modules with matter, colored pictures, animation, simulated, and video. On interactive module happened multimedia integration into a digital module is an interactive suitable used by junior high school students. The object is shown in the form still picture can be displayed in the animated, simulated, and video. Animation and simulation can also be used in the discussion of sample problem so the students could witness a problem that is shown. Interactive module also provides interactive evaluation for the students that can be used students to measure the aptitude cognitive students of the matter have been studied. In the form of evaluation interactive wholes about multiple choice that can be accessed directly by students and results could be known so that spatial ability of junior high school students grade VIII can increase. This study looks at the use of interactive modules based lectora in SMP Taman Siswa as a source of optimal learning to improve student’s achievement.

Method

Kualitatif method was adopted in this study. This study was an action research class (PTK). This study consisted of two cycles of the first cycle and the second cycle. Each cycle following the steps of classroom action research as follows:

$$\text{Plan} \Rightarrow \text{Implementation} \Rightarrow \text{Observation} \Rightarrow \text{Reflection}$$

The subjects of this study were students of class VIII-3 SMP Taman Siswa as many as 24 students. In general, learning is said to be effective if at least there 85% of students who pass the study.

Result

The presentation of material began discussion of material cube, unsure, nets, a surface area and the volume has overall followed by the form of geometric others are beam, prism, and pyramid with the structure discussion the same matter. Display early module presenting home, guidance use, the authors, and menus containing a list of choice matter who want to be studied by students. At the beginning of subjects sub-basic begins with the presentation of competence base and learning experience. Followed by presentation of matter and exercise. At the end given interactive module evaluation for students who compilation of exercise of all sub subjects. Based on systematic and structure interactive module designed produced a prototype. The first prototype of the start of the title interactive module a button navigator menu “start” and menus the authors. The navigator menu “star” useful to lead students start opening module interactive. A title module and menus the team is presented in figure 1.

Figure 1. The Title Interactive Module

The navigator "start" lead students to display next menu the main menu. The main course is presented in figure 2.

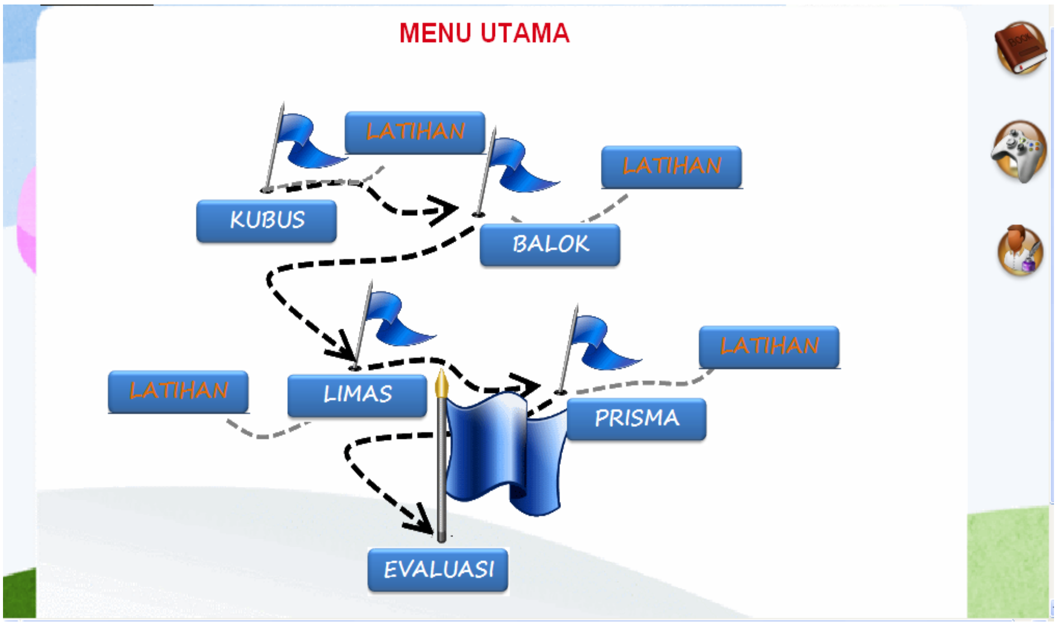
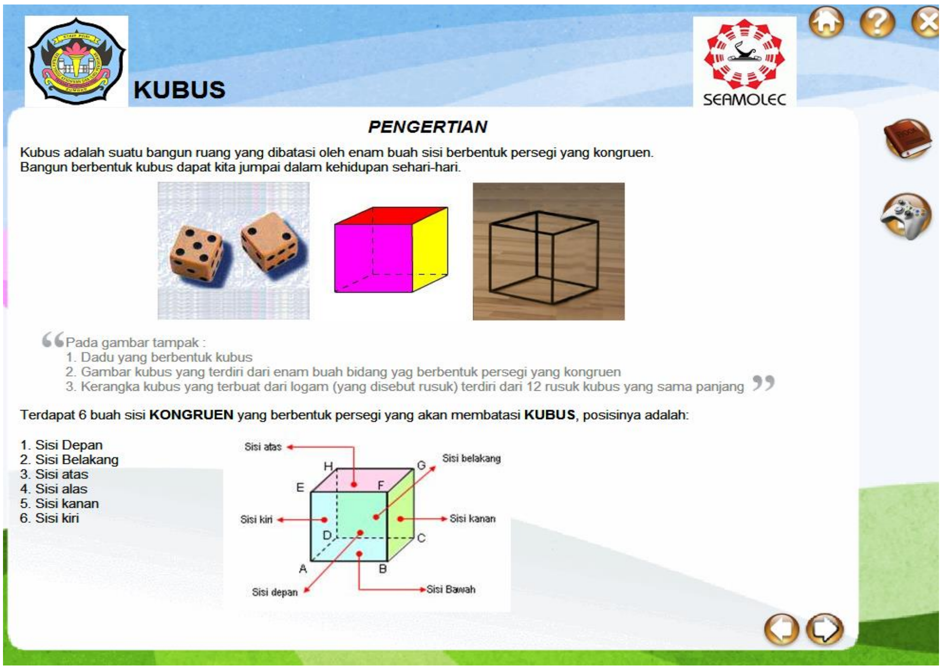


Figure 2. Main Menu

On the primary menu tab having 6 that contains material, exercise, and evaluation related to the title of chapter. Each tab not intertwined and so students can go back to the menu home. On the primary menu buttons there are guidelines use interactive module. Tab 1,2,3,4 contains polyhedral. polyhedral are cube, beam, prism, and pyramid. Matter at interactive module with animation to visualize up the polyhedrals based on every other component the owned. Tab matter is presented in figure 3.



PENGERTIAN

Kubus adalah suatu bangun ruang yang dibatasi oleh enam buah sisi berbentuk persegi yang kongruen.
Bangun berbentuk kubus dapat kita jumpai dalam kehidupan sehari-hari.

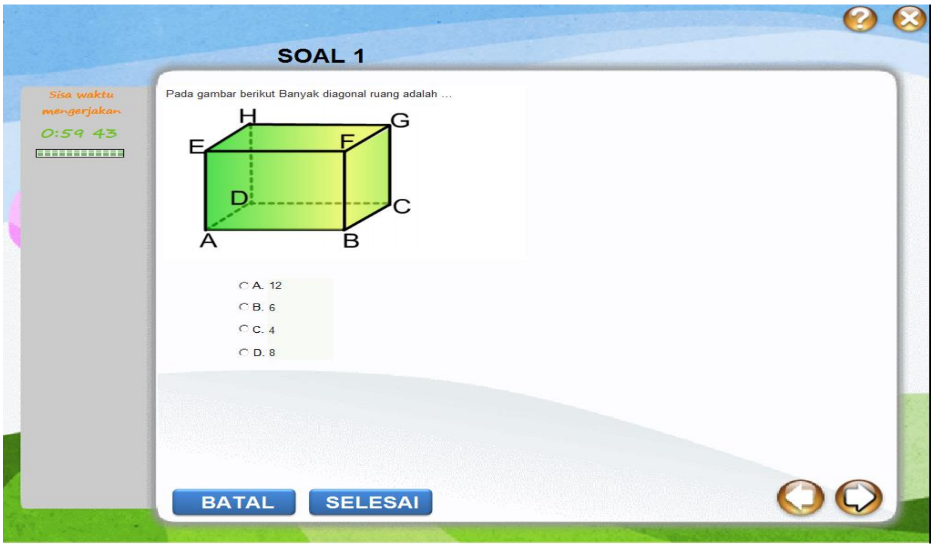
> Pada gambar tampak :
> 1. Dadu yang berbentuk kubus
> 2. Gambar kubus yang terdiri dari enam buah bidang yag berbentuk persegi yang kongruen
> 3. Kerangka kubus yang terbuat dari logam (yang disebut rusuk) terdiri dari 12 rusuk kubus yang sama panjang

Terdapat 6 buah sisi **KONGRUEN** yang berbentuk persegi yang akan membatasi **KUBUS**, posisinya adalah:

1. Sisi Depan
2. Sisi Belakang
3. Sisi atas
4. Sisi alas
5. Sisi kanan
6. Sisi kiri

Figure 3. Tab of Material Menu

Tab 5 is the exercise contains multiple choises problems. Any rehearsal accompanied judgment so students can see that directly without waiting for an explanation of teachers and gain of a feedback of the actual exercises. Tab exercise is presented in figure 4.

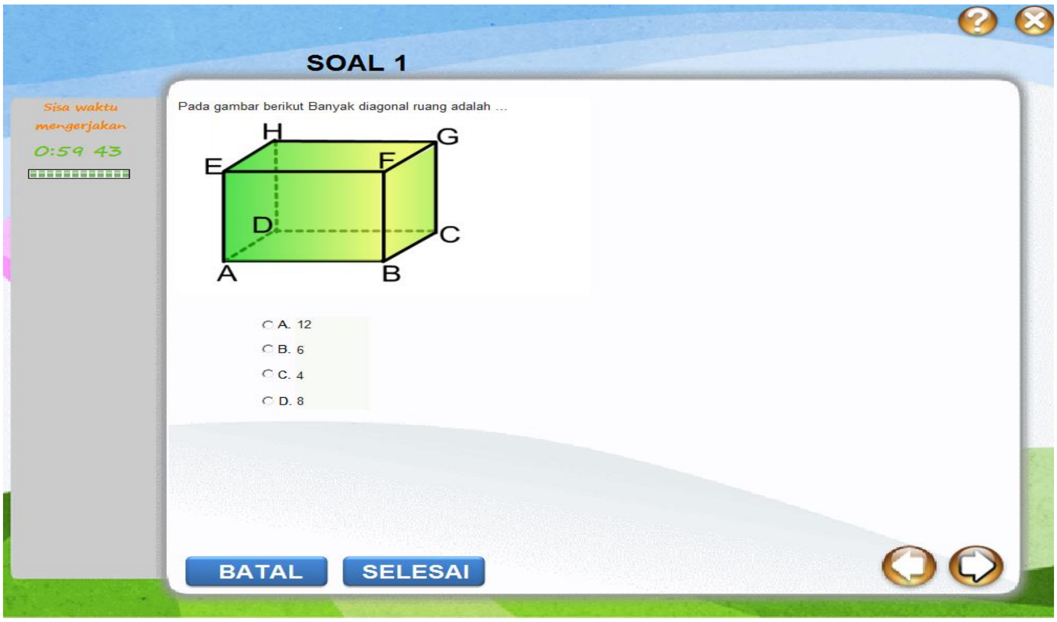


Pada gambar berikut Banyak diagonal ruang adalah ...

○ A. 12
○ B. 6
○ C. 4
○ D. 8

Figure 4. Tab of Exercise Menu

Tab 6 is evaluation contains evaluation overall matter shown in figure 5.



Pada gambar berikut Banyak diagonal ruang adalah ...

○ A. 12
○ B. 6
○ C. 4
○ D. 8

Figure 5. Tab of Evaluation Problems

During the implementation of the learning process, the teacher monitored the student to see the student understanding of the material provided through a process of discussion with each student shown in Figure 6**.**

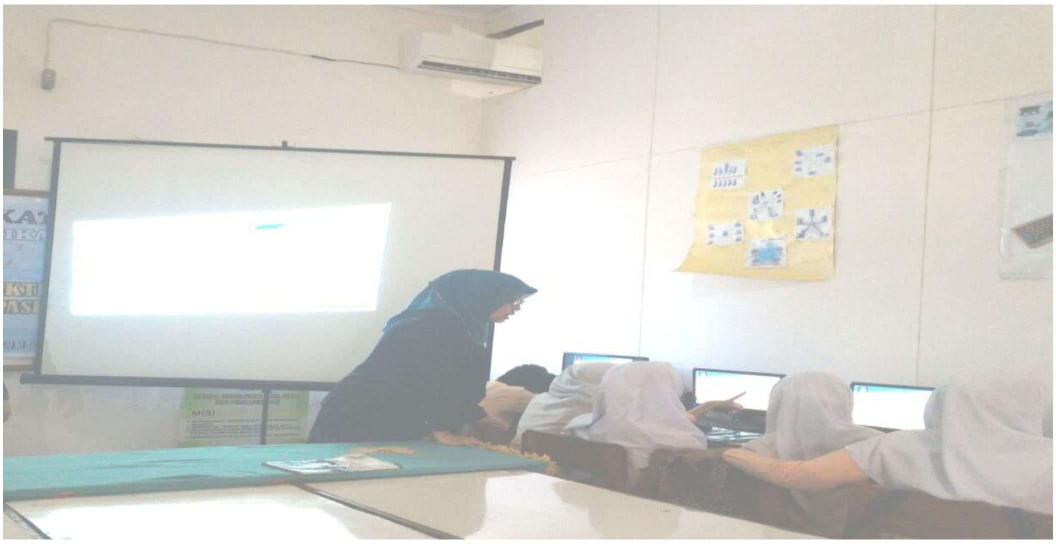
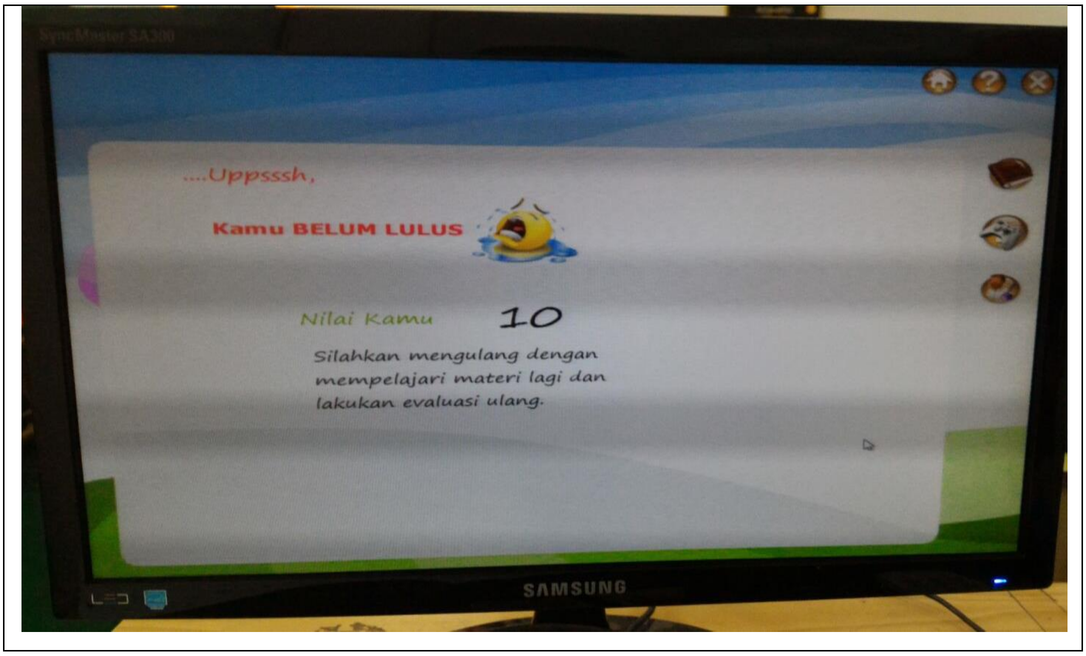


Figure 6. Monitoring Students through Discussion

At first the students' difficult in answering the question because students didn’t understand very well the concept of spatial material cube so that students get unsatisfactory grades are shown in Figure 7.



....Uppsssh,

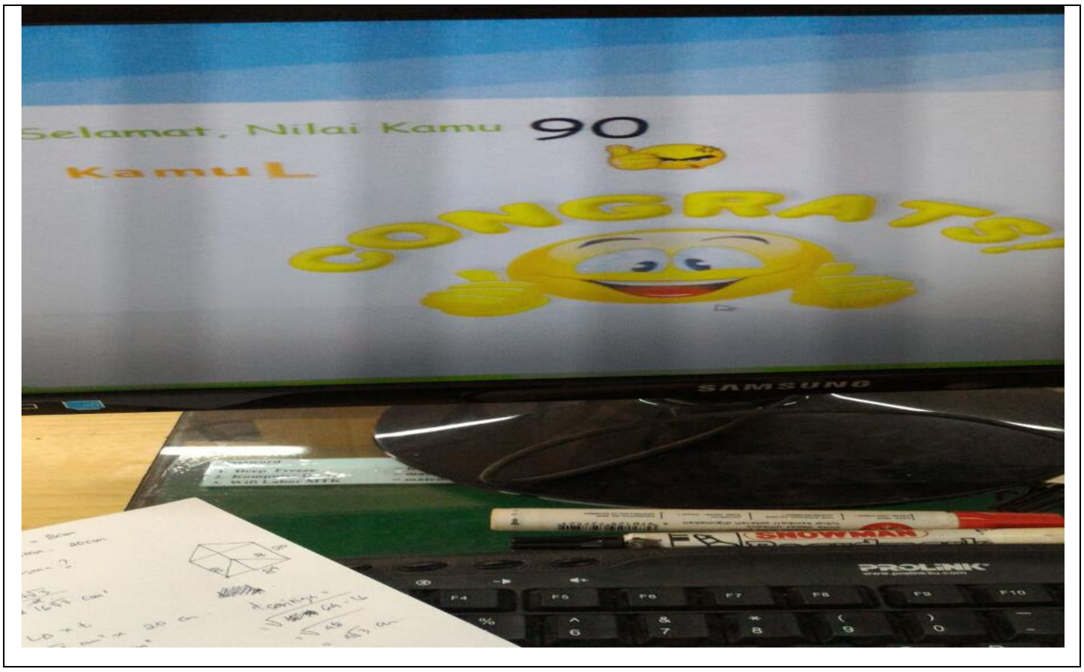
Kamu BELUM LULUS

Nilai Kamu 10

Silahkan mengulang dengan mempelajari materi lagi dan lakukan evaluasi ulang.

Figure 7. The Fisrt Student Result

However, the results didn't make students discouraged, students enthusiastically back to solved the problems so that students get a satisfactory score in Figure 8.



Selamat, Nilai Kamu 90

Kamu L

CONGRATS!

Figure 8. The Final Student Result

To determine the effectiveness of learning using interactive modules lectora, then held a written test. The written test consists of questions containing the three above-mentioned aspects. Written test conducted at the end of the learning material straight line equation. Below is shown a summary of the value obtained by the student in cycle I and II.

Table 1. Student Achievement in Cycle I and II

Keterangan	Nilai Siklus I	Nilai Siklus II
Maximum Score	100	75
Minimum Score	60	40
Average	79,30	82,70
Completeness student	18	21
Uncompleteness student	6	3
Percentage of completeness	75 %	87,5 %

Conclusion

In accordance with the criteria described previously that the effectiveness of learning here must meet the criteria that the test results of students describe at least 85% of students scored at least 75 and increased student interest. From the table values first cycle and second cycle students in mind that the percentage of completeness of 75% so that the learning cycle I have not yet reached the predetermined criteria researchers. It is necessary for the second cycle. After the implementation of learning in the second cycle of 87,5% of students pass the study. So that the learning of Interactive Module Based on Lectora to Improve Student's Spatial Ability was declared effective.

Suggestion

This study takes quite a lot. Further research needs to be done as an effort to improve students' abilities in understanding and applying interactive modules based Lectora.

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ON INEQUALITIES BETWEEN MEANS AND THEIR APPLICATIONS

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Abstract

Inequality is one material that is always examined in senior high school mathematics olympiad, from the simplest to the most complex problems with many variations. Many teachers and students still do not know how to solve inequality problems. This paper discusses basic inequalities between means theory, from arithmetic mean, geometric mean, quadratic mean, harmonic mean, and their applications. Mastering basic algebra and basic problem solving skills is adequate to understand inequalities theory and how to solve their problems.

Keywords: *inequality, quadratic mean, arithmetic mean, geometric mean, harmonic mean*

Introduction

Inequality is one material that is always examined in senior high school mathematics olympiad, from the simplest to the most complex problems with many variations. Inequality between means is the basic inequality theory that must be mastered before we explore more advanced inequality. Inequality between means consist of quadratic mean, arithmetic mean, geometric mean, and harmonic mean. This paper discusses inequalities between means, starting from definition, theorem for two and three variables, extend to n variables, and their applications.

Inequalities between Means with Two and Three Variables

Before exploring inequalities between means, we have to define what is the meaning of quadratic mean, arithmetic mean, geometric mean, and harmonic mean.

Definition (Arithmetic Mean):

The arithmetic mean (AM) of two nonnegative numbers a and b is

$$\frac{a+b}{2}$$

The arithmetic mean of two nonnegative numbers is sometimes referred to as average of the numbers.

Definition (Geometric Mean):

The geometric mean of two nonnegative numbers a and b is

$$\sqrt{ab}$$

Definition (Quadratic Mean):

The quadratic mean (QM) of two real numbers is the square root of the arithmetic mean of the squares of the numbers. In other words, the quadratic mean of the real numbers a and b is

$$\sqrt{\frac{a^2+b^2}{2}}$$

The quadratic mean is also sometimes referred to as the root mean square.

Definition (Harmonic Mean):

The harmonic mean (HM) of two nonzero real numbers is the reciprocal of the average of the reciprocals of the numbers. In other words, the harmonic mean of two nonzero real numbers a and b is

$$\frac{2}{\frac{1}{a}+\frac{1}{b}}$$

Theorem 1. (Cvetkovski, 2012)

Let $a, b \in \mathbb{R}^+$, and let us denote

$$QM = \sqrt{\tfrac{a^2+b^2}{2}}, \quad AM = \tfrac{a+b}{2}, \quad GM = \sqrt{ab}\,, \quad HM = \tfrac{2}{\frac{1}{a}+\frac{1}{b}}$$

then $QM \geq AM \geq GM \geq HM$.

Equalities occur if and only if $a = b$.

Proof:

First of all we will show that $QM \geq AM$.

For $a, b \in \mathbb{R}^+$

$$\begin{array}{rrcl} & (a-b)^2 & \geq & 0 \\ \Leftrightarrow & a^2+b^2 & \geq & 2ab \\ \Leftrightarrow & 2(a^2+b^2) & \geq & a^2+b^2+2ab \\ \Leftrightarrow & 2(a^2+b^2) & \geq & (a+b)^2 \\ \Leftrightarrow & \dfrac{a^2+b^2}{2} & \geq & \left(\dfrac{a+b}{2}\right)^2 \\ \Leftrightarrow & \sqrt{\dfrac{a^2+b^2}{2}} & \geq & \dfrac{a+b}{2} \end{array}$$

Equality holds if and only if $a - b = 0$, i.e. $a = b$.

Furthermore, for $a, b \in \mathbb{R}^+$

$$\begin{array}{rrcl} & \left(\sqrt{a}-\sqrt{b}\right)^2 & \geq & 0 \\ \Leftrightarrow & a+b-2\sqrt{ab} & \geq & 0 \\ \Leftrightarrow & \dfrac{a+b}{2} & \geq & \sqrt{ab} \end{array}$$

So $AM \geq GM$, with equality if and only if $\sqrt{a}-\sqrt{b}=0$, i.e. $a=b$.

Finally we will show that

$$GM \geq HM, \text{ i.e. } \sqrt{ab} \geq \frac{2}{\frac{1}{a}+\frac{1}{b}}$$

We have

$$\begin{array}{rrcl} & \left(\sqrt{a}-\sqrt{b}\right)^2 & \geq & 0 \\ \Leftrightarrow & a+b & \geq & 2\sqrt{ab} \\ \Leftrightarrow & 1 & \geq & \dfrac{2\sqrt{ab}}{a+b} \\ \Leftrightarrow & \sqrt{ab} & \geq & \dfrac{2ab}{a+b} \\ \Leftrightarrow & \sqrt{ab} & \geq & \dfrac{2}{\frac{1}{a}+\frac{1}{b}} \end{array}$$

Equality holds if and only if $\sqrt{a}-\sqrt{b}=0$, i.e. $a=b$. ■

These inequalities usually well be use in the case when $a, b \in \mathbb{R}^+$. Similarly definition of the quadratic, arithmetic, geometric, and harmonic mean for three numbers $a, b,$ and c are:

$$QM = \sqrt{\frac{a^2+b^2+c^2}{3}}$$

$$AM = \frac{a+b+c}{3}$$

$$GM = \sqrt[3]{abc}$$

$$HM = \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$$

With three numbers $a, b,$ and c the following theorem analogous to Theorem 1.

Theorem 2. (Cvetkovski, 2012)

Let $a, b, c \in \mathbb{R}^+$, and let us denote

$$QM = \sqrt{\frac{a^2+b^2+c^2}{3}}, \quad AM = \frac{a+b+c}{3}, \quad GM = \sqrt[3]{abc}\,, \quad HM = \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$$

then $QM \geq AM \geq GM \geq HM$.

Equalities occur if and only if $a=b=c$.

Extension of Inequalities between Means for n numbers

In this section we will develop generalization of inequalities between means for n numbers.

Theorem 3. (Cvetkovski, 2012)

Let $a_1, a_2, \dots, a_n$ be positive real numbers. The numbers

$$QM = \sqrt{\frac{a_1^2 + a_2^2 + \cdots + a_n^2}{n}}$$

$$AM = \frac{a_1 + a_2 + \cdots + a_n}{n}$$

$$GM = \sqrt[n]{a_1 a_2 \cdots a_n}$$

$$HM = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}}$$

are called the quadratic mean, arithmetic mean, geometric mean, and harmonic mean for the numbers $a_1, a_2, \dots, a_n$, respectively, and we have

$$QM \geq AM \geq GM \geq HM$$

Equalities occur if and only if $a_1 = a_2 = \cdots = a_n$.

Proof:

Starting to prove by show that $AM \geq GM$

$$\frac{a_1+a_2+\cdots+a_n}{n} \geq \sqrt[n]{a_1 a_2 \cdots a_n} \qquad \dots\dots\dots.(1)$$

Let

$$x_i = \frac{a_i}{\sqrt[n]{a_1 a_2 \cdots a_n}}\text{ , for } i = 1, 2, \dots, n \qquad \dots\dots\dots.(2)$$

Then $x_i > 0$ for each $i = 1, 2, \dots, n$ and $x_1 x_2 \cdots x_n = 1$

Inequality (1) is equivalent to

$$\frac{a_1}{\sqrt[n]{a_1 a_2 \cdots a_n}} + \frac{a_2}{\sqrt[n]{a_1 a_2 \cdots a_n}} + \cdots + \frac{a_n}{\sqrt[n]{a_1 a_2 \cdots a_n}} \geq n$$

i.e. to

$$x_1 + x_2 + \cdots + x_n \geq n,\ \text{ when } x_1 x_2 \cdots x_n = 1 \qquad \dots\dots\dots.(3)$$

with equality if and only if $x_1 = x_2 = \cdots = x_n = 1$.

We will prove inequality (3) by induction.

For $n = 1$, inequality (3) is true, it becomes equality.

If $n = 2$ then $x_1 x_2 = 1$ and since $x_1 + x_2 \geq 2\sqrt{x_1 x_2}$ we get $x_1 + x_2 \geq 2$.

Hence inequality (3) is true, and equality occurs if and only if $x_1 = x_2 = 1$.

Let us assume that for $n = k$, and arbitrary positive real numbers $x_1, x_2, \ldots, x_k$ such that $x_1 x_2 \cdots x_k = 1$, we have $x_1 + x_2 + \cdots + x_k \geq k$, with equality if and only if $x_1 = x_{2=} \cdots = x_k = 1$.

Let $n = k + 1$ and $x_1, x_2, \ldots, x_{k+1}$ be arbitrary positive real numbers such that

$$x_1 x_2 \cdots x_{k+1} = 1$$

If $x_1 = x_{2=} \cdots = x_{k+1} = 1$ then inequality (3) holds.

Therefore, let us assume that there are numbers smaller then 1. Then clearly, there are also numbers which are greater then 1.

Without loss of generality, assume that $x_1 < 1$ and $x_2 > 1$.

Then, for the sequences $x_1, x_2, x_3, \ldots, x_{k+1}$ which contain k terms we have $(x_1 x_2) x_3 \cdots x_{k+1} = 1$, and according to the induction hypothesis we have that $x_1 x_2 + x_3 + \cdots + x_{k+1} \geq k$. Equality occurs if and only if $x_1 x_2 = x_3 = \cdots = x_{k+1} = 1$.

$$\begin{aligned} x_1 + x_2 + \cdots + x_{k+1} &\geq x_1 x_2 + x_3 + \cdots + x_{k+1} + 1 + (x_2 - 1)(1 - x_2) \\ &\geq k + 1 + (x_2 - 1)(1 - x_2) \\ &\geq k + 1 \end{aligned}$$

with equality occurs if and only if $x_1 x_2 = x_3 = \cdots = x_{k+1} = 1$ and $(x_2 - 1)(1 - x_2) = 0$, i.e. if and only if $x_1 = x_2 = \cdots = x_{k+1} = 1$.

Due to the principle of mathematical induction, we conclude that inequality (3) is proved.

By inequality (2), $\frac{a_1}{\sqrt[n]{a_1 a_2 \cdots a_n}} = \frac{a_2}{\sqrt[n]{a_1 a_2 \cdots a_n}} = \cdots = \frac{a_n}{\sqrt[n]{a_1 a_2 \cdots a_n}}$, i.e. $a_1 = a_2 = \cdots = a_n$. Hence inequality (1) is proved.

Next, we will show that $GM \geq HM$, i.e.

$$\sqrt[n]{a_1 a_2 \cdots a_n} \geq \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}}$$

By $AM \geq GM$ it follows that

$$\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n} \geq n \sqrt[n]{\frac{1}{a_1} \frac{1}{a_2} \cdots \frac{1}{a_n}} = \frac{n}{\sqrt[n]{a_1 a_2 \cdots a_n}}$$

We have

$$\sqrt[n]{a_1 a_2 \cdots a_n} \geq \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}}$$

and equality holds if and only if $\frac{1}{a_1} = \frac{1}{a_2} = \cdots = \frac{1}{a_n}$, i.e. $a_1 = a_2 = \cdots = a_n$.

The proof of $QM \geq AM$ is left because we have to use the Cauchy-Schwarz Inequality that too advanced to discuss in this paper. ■

Application of AM – GM Inequality to Solve Basic Olympiad Problems

In many olympiad problems, especially that include inequalities, inequalities between means can be applied to solve these problems. In this paper, we only focus on how to apply AM – GM Inequality because it can be used as basic thinking skills to understand inequality problem. Start from simple to more advanced problem, let us look at these problems.

Problem 1:

Let x and y be positive real numbers. Show that $\frac{x}{y} + \frac{y}{x} \geq 2$. When does equality occur?

Solution:

Because x and y are positive, so are $\frac{x}{y}$ and $\frac{y}{x}$. Applying AM – GM Inequality to find

$$\frac{\frac{x}{y} + \frac{y}{x}}{2} \geq \sqrt{\frac{x}{y} \cdot \frac{y}{x}} = 1$$

Multiplying the far left side and the far right side by 2, we get

$$\frac{x}{y} + \frac{y}{x} \geq 2$$

Equality occurs if and only if $\frac{x}{y} = \frac{y}{x}$. Cross-multiplying both sides that gives $x^2 = y^2$. So, because x and y are positive numbers equality occurs if and only if $x = y$. ■

In Problem 1, the hint to apply AM – GM Inequality is the fact that $\frac{x}{y}$ and $\frac{y}{x}$ have a constant product. This means that if AM – GM is applied to these quantities, we will have a constant on the lesser side of the inequality. Similarly, if we have nonnegative expressions with a constant sum, AM – GM can be applied to produce an inequality in which the larger side is constant. In other words, to prove an inequality involving nonnegative expressions that have a constant sum or product, we can apply AM – GM Inequality.

Problem 2:

Show that if $x, y > 0$, then $\frac{1}{x^2} + \frac{1}{y^2} \geq \frac{2}{xy}$.

Solution:

In this problem, we have a sum on the greater side and a product on the lesser side. This is a hint to use AM – GM Inequality. Applying AM – GM to $\frac{1}{x^2}$ and $\frac{1}{y^2}$ gives

$$\frac{\frac{1}{x^2}+\frac{1}{y^2}}{2} \geq \sqrt{\left(\frac{1}{x^2}\right)\left(\frac{1}{y^2}\right)}$$

Multiplying both sides by 2 and simplifying the right side gives the result $\frac{1}{x^2}+\frac{1}{y^2} \geq \frac{2}{xy}$.

■

Problem 3

If x, y, and z are nonnegative, show that $xy + yz + zx \geq x\sqrt{yz} + y\sqrt{zx} + z\sqrt{xy}$.

Solution:

In this problem, we are not sure to start to prove. But we can see sums of expressions, so AM – GM can be applied.

The $\sqrt{yz}$ term is a hint to apply AM – GM, because the square root is on the lesser side of the inequality. We produce the square roots by applying AM – GM to each pair of x, y, and z gives

$$\frac{x+y}{2} \geq \sqrt{xy}$$

$$\frac{y+z}{2} \geq \sqrt{yz}$$

$$\frac{z+x}{2} \geq \sqrt{zx}$$

Multiplying these three inequalities by z, x, and y respectively, gives

$$z \cdot \frac{x+y}{2} \geq z\sqrt{xy}$$

$$x \cdot \frac{y+z}{2} \geq x\sqrt{yz}$$

$$y \cdot \frac{z+x}{2} \geq y\sqrt{zx}$$

Adding all of these three inequalities gives

$$xy + yz + zx \geq x\sqrt{yz} + y\sqrt{zx} + z\sqrt{xy}. \ \blacksquare$$

From problem 3 if we want to show that $A \geq B$, then inequalities like $C \geq A$ or $B \geq D$ are not going to help us. We need to work with inequalities in which A is on the greater side or B is on the lesser side.

Problem 4:

Show that $(x+y)(y+z)(z+x) \geq 8xyz$ for all nonnegative numbers x, y, and z.

Solution:

We have a product of sums on the greater side of the inequality, so AM – GM Inequality can be applied. We use AM – GM to produce each sum on the left side, which gives

$$\frac{x+y}{2} \geq \sqrt{xy}$$
$$\frac{y+z}{2} \geq \sqrt{yz}$$
$$\frac{z+x}{2} \geq \sqrt{zx}$$

Multiplying all three inequalities because we want the product of these sums and get

$$\frac{(x+y)(y+z)(z+x)}{8} \geq \sqrt{x^2y^2z^2}$$

We have $\sqrt{x^2y^2z^2} = xyz$ because $x, y, z \geq 0$, so multiplying both side by 8 to give inequality $(x+y)(y+z)(z+x) \geq 8xyz$. ■

Problem 5:

Let x, y, and z be positive real numbers. Show that $(x+y+z)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) \geq 9$.

Solution:

Applying AM – GM to both sides of the inequality because we have two sums on the greater side and get

$$\frac{x+y+z}{3} \geq \sqrt[3]{xyz}$$
$$\frac{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}{3} \geq \sqrt[3]{\frac{1}{x}\cdot\frac{1}{y}\cdot\frac{1}{z}} = \frac{1}{\sqrt[3]{xyz}}$$

Multiplying these inequalities and get

$$\frac{(x+y+z)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)}{9} \geq \frac{\sqrt[3]{xyz}}{\sqrt[3]{xyz}} = 1$$

Finally multiply both sides by 9 and get $(x+y+z)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) \geq 9$. ■

Problem 6:

For all nonnegative real numbers x, y, z, prove that

$$x^2(1+y^2)+y^2(1+z^2)+z^2(1+x^2) \geq 6xyz$$

When does equality occur?

Solution:

We have a sum on the greater side and a product on the lesser side. This is a hint to apply AM – GM Inequality. When applying AM – GM to the three added terms on the left, their geometric means is a very messy expression. We could view xyz as the geometric mean of x^3, y^3, and z^3, but that does not seem to help much, either. If we look at the lesser side, the 6 is a big hint. Expand all the terms on the left side and gives us 6 terms:

$$x^2+x^2y^2+y^2+y^2z^2+z^2+z^2x^2 \geq 6xyz$$

The larger side is now consists of the sum of six terms, and the smaller side is a product. All six terms are nonnegative. Applying AM – GM Inequality and have

$$\begin{aligned} \frac{x^2+x^2y^2+y^2+y^2z^2+z^2+z^2x^2}{6} &\geq \sqrt[6]{x^2 \cdot x^2y^2 \cdot y^2 \cdot y^2z^2 \cdot z^2 \cdot z^2x^2} \\ &= \sqrt[6]{x^6y^6z^6} \\ &= xyz \end{aligned}$$

Multiplying both sides by 6 and gives the desired inequality

$$x^2(1+y^2)+y^2(1+z^2)+z^2(1+x^2) \geq 6xyz$$

Next, we must determine when equality occurs. Equality occurs if and only if $x^2 = x^2y^2 = y^2 = y^2z^2 = z^2 = z^2x^2$. Since x, y, and z are nonnegative and $x^2 = y^2 = z^2$, then $x = y = z$. Hence $x^2 = x^4$, so $x = 0$ or $x = 1$. Therefore, equality occurs only for $(x, y, z) = (0,0,0,)$ and $(x, y, z) = (1,1,1)$. ■

From problem 6 it is often necessary to manipulate a desired inequality (by factor, expand, or regroup terms) into a form for which tools like AM – GM apply.

Problem 7:

Let $x, y, z \in \mathbb{R}^+$ such that $x + y + z = 1$. Prove that

$$\frac{xy}{z}+\frac{yz}{x}+\frac{zx}{y} \geq 1$$

When does equality occur?

Solution:

We will manipulate the greater side and get

$$\frac{xy}{z}+\frac{yz}{x}+\frac{zx}{y} = \frac{1}{2}\left(\frac{xy}{z}+\frac{yz}{x}\right)+\frac{1}{2}\left(\frac{yz}{x}+\frac{zx}{y}\right)+\frac{1}{2}\left(\frac{zx}{y}+\frac{xy}{z}\right)$$

Applying AM – GM Inequality and get

$$\frac{1}{2}\left(\frac{xy}{z}+\frac{yz}{x}\right) \geq \sqrt{\frac{xy}{z}\cdot\frac{yz}{x}} = y$$

$$\frac{1}{2}\left(\frac{yz}{x}+\frac{zx}{y}\right) \geq \sqrt{\frac{yz}{x}\cdot\frac{zx}{y}} = z$$

$$\frac{1}{2}\left(\frac{zx}{y}+\frac{xy}{z}\right) \geq \sqrt{\frac{zx}{y}\cdot\frac{xy}{z}} = x$$

Adding these three inequalities and get

$$\frac{xy}{z}+\frac{yz}{x}+\frac{zx}{y} \geq x+y+z=1$$

It has proved. Equality holds if and only if $\frac{xy}{z}=\frac{yz}{x}=\frac{zx}{y}$, i.e. $x=y=z$. Since $x+y+z=1$, this equality holds if and only if $x=y=z=\frac{1}{3}$. ■

Application of AM – GM Inequality to Solve Maxima and Minima Problems

Many problems about the minimum and maximum values of a function can be solved without using a derivative (calculus), but instead using the inequality between means (Grigorieva, 2015). In this section, AM – GM Inequality can be applied to find the minimum or maximum value of various expressions. The minimization or maximization problems sometimes referred as optimization problems. Minimization and maximization problems are two-part problems. We must show that the maximum or minimum value is attainable, and that no better value (no higher for maximization, no lower for minimization) can be attained.

Problem 9:

Let B be a rectangle with perimeter 36. Find the largest possible area of B.

Solution:

Let the side lengths of the rectangle be x and y. From the information about the perimeter, we have $2x+2y=36$, so $x+y=18$. We wish to maximize the area, which is xy. There is an inequality that relates $x+y$ and xy. Applying AM – GM Inequality and gives

$$\frac{x+y}{2} \geq \sqrt{xy}$$

Since $x+y=18$, then $\sqrt{xy} \leq 9$. Squaring both sides and gives $xy \leq 81$. This shows that the area of the rectangle cannot be greater than 81. We note that the inequality condition of AM –

GM gives us $x = y$ to show that the area of the rectangle can possibly equal 81. It describes the case in which the rectangle is a square with perimeter 36 and area 81. ■

From problem 9 the most common mistake when solving this geometric optimization problem is we only claim that the maximum area occurs when the rectangle is a square. Since the perimeter is 36, each side has length 9, so the area is $9^2 = 81$. The answer is correct, but the most important step is skipped. That is, we have not proved the statement "The maximum area occurs when the rectangle is a square".

Problem 10:

For $x > 0$, let $f(x) = \frac{x^2+16}{3x}$. Find the minimum possible value of $f(x)$.

Solution:

Rewrite the function as a sum of two expressions

$$f(x) = \frac{x}{3} + \frac{16}{3x}$$

We do so because this produces two expressions whose product is a constant. This is a hint to apply AM – GM Inequality to compare $f(x)$ to a constant. AM – GM can be used because $x > 0$, so both $\frac{x}{3}$ and $\frac{16}{3x}$ are positive. Applying AM – GM gives us

$$\frac{\frac{x}{3} + \frac{16}{3x}}{2} \geq \sqrt{\frac{x}{3} \cdot \frac{16}{3x}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

Multiplying both sides by 2 and gives us $f(x) \geq \frac{8}{3}$.

We have equality when $\frac{x}{3} = \frac{16}{3x}$. Solve this equation and get $x = 4$, because $x > 0$. Substitute $x = 4$ to the function and gives

$$f(4) = \frac{4^2 + 16}{3 \cdot 4} = \frac{8}{3}$$

so the minimum value $\frac{8}{3}$ can be achieved. ■

Problem 11:

Bhawara Company is making big rectangular cardboard boxes. They do not want to use more cardboard than they have to. A customer requests boxes that have a volume of 1000 cm^3.

a. Suppose the customer wants each box to have a top, a bottom, and all four sides. Show that the Bhawara Company should make boxes that are cubes in order to minimize the amount of cardboard they have to use.

b. Suppose the customer wants each box to have a bottom and four sides, but no top. What dimensions should Bhawara Company use to minimize the amount of cardboard they use on each box?

Solution:

Let x, y, and z be the dimensions of the box. The volume of the box is $xyz = 1000$.

a. We wish to minimize the surface area of the box, which equals $2(xy + yz + zx)$. This $xy + yz + zx$ on the greater side of an inequality. There is a sum on the greater side of an inequality. This is a hint to apply AM – GM Inequality. Applying AM – GM to the nonnegative quantities xy, yz, and zx gives

$$\frac{xy + yz + zx}{3} \geq \sqrt[3]{(xy)(yz)(zx)} = \sqrt[3]{x^2y^2z^2}$$

We have $xyz = 1000$, so $\sqrt[3]{x^2y^2z^2} = 100$. Multiplying our inequality above by 6 therefore gives $2(xy + yz + zx) \geq 6\sqrt[3]{x^2y^2z^2} = 600$. The equality condition of AM – GM tells us that this surface area occurs when $xy = yz = zx$. From $xy = yz$, we have $x = z$. From $yz = zx$, we have $y = x$. The minimum surface area occurs when $x = y = z$. This means that Bhawara Company should make cubes. If $x = y = z = 10$, then we do indeed have a box with volume 1000 cm^3 and surface area 600 cm^2.

b. Let x be the height of the box, so that y and z are the dimensions of the top and bottom. The surface area of the box is $2xy + 2xz + yz$. We have a sum that we must minimize. Applying AM – GM Inequality and gives

$$\frac{2xy + 2xz + yz}{3} \geq \sqrt[3]{(2xy)(2xz)(yz)} = \sqrt[3]{4x^2y^2z^2} = 100\sqrt[3]{4}$$

Multiplying our inequality above by 3 gives us $2xy + 2xz + yz \geq 300\sqrt[3]{4}$. Since $300\sqrt[3]{4} \approx 476{,}2$, we can make a box with less than 500 cm^2 of cardboard. We use the equality condition of AM – GM to find out how to build the boxes, which gives us $2xy = 2xz = yz$. From $2xy = 2xz$, then $y = z$. From $2xz = yz$, then $y = 2x$. Therefore, we must have $2x = y = z$. So, the dimensions of the bottom must each be twice the height of the box.

To find out what these dimensions are exactly, let $y = 2x$ and $z = 2x$ in $xyz = 1000$ to find $4x^3 = 1000$, so $x^3 = 250$. This gives $x = 5\sqrt[3]{2}$. So $y = z = 10\sqrt[3]{2}$. We have a box without top whose bottom is a square with side length $10\sqrt[3]{2}$ cm and whose sides have height $5\sqrt[3]{2}$ cm does have volume 1000 cm^3 and surface area $300\sqrt[3]{4}$ cm^2. So the minimum surface area is $300\sqrt[3]{4}$ cm^2.

From problem 11 to show that an expression has a specific maximum value, there are two steps, that is, show that the expression cannot be greater than this value and show that the expression can be equal to this value. Similarly, if we wish to show that an expression has a minimum value, show both that the expression cannot be less than this value, and that the expression can equal this value (Rusczyk, 2008).

Conclusion

Inequality is one material that is always examined in senior high school mathematics olympiad, from the simplest to the most complex problems with many variations. Inequality between means consist of quadratic mean, arithmetic mean, geometric mean, and harmonic mean. Inequality between means, especially AM – GM Inequality, can be applied to solve many inequality problems. Many problems about the minimum and maximum values of a function can be solved without using a derivative (calculus), but instead using the inequality between means. Mastering basic algebra and basic problem solving skills is adequate to understand inequalities theory and how to solve their problems.

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Student's Perception on Borobudur Temple as Mathematic Learning Resource

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Abstract

This study was aimed to describe junior high school student's perception on Borobudur Temple as mathematic learning resources. Borobudur Temple is well known having extraordinary architecture built algorithmically. The parts of Borobudur Temple such as stupa, statue, and wall carvings (relief) consist of many geometric models. This study employs ethnomathematics perspective in describing perception about cultural artefact as a mathematical model. The result of this study is used in a basis for developing meaningful mathematic learning in school. The sample of the study was 313 students of junior high school located near Borobudur Temple. The measure of the sampling adequacy with KMO is 0.86 from which elucidate that the number of the sample is sufficient. The data were collected using questionnaire with Likert scale 1 to 4 which mean disagree (1), neutral (2), agree (3) and strongly agree (4). The exploratory factor analysis yielded in three factors of perception of Borobudur Temple as mathematic model, those are: 1) Borobudur Temple is geometry model, 2) Borobudur Temple can be used as mathematic learning source at school, and 3) learning mathematics from Borobudur Temple is helpful for students. Total variance can reach 49,572%. The value of Cronbach alpha was 0,8204 for the 14 items. The data were analyzed using descriptive statistics to attain average items of mean and average standard deviation for each factors. The result of the research shows that: 1) students agree that Borobudur Temple is geometry model, 2) students agree that Borobudur Temple can be used as mathematic learning source at school, 3) students agree that learning mathematics from Borobudur Temple is helpful for them.

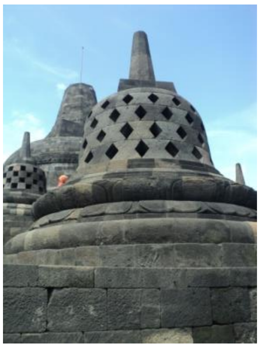
Keywords: *Borobudur Temple, mathematics, perception, students*

Introduction

Borobudur Temple is Indonesian cultural heritage which is recognized worldwide. Many tourists come to visit the Borobudur and admire the beauty of its architecture. Borobudur Temple has become a symbol of world civilization into the laboratory of history and architecture. However, Borobudur as a laboratory of mathematic education is not popular among the people. Only few people know that Borobudur is a very interesting mathematical objects.

Characteristic of Mathematics at Borobudur Temple Building

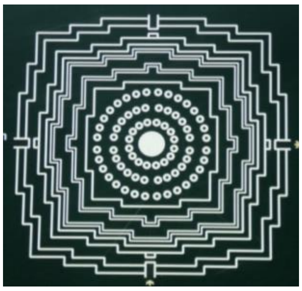
Research by Situngkir, Hokky (2010) found that Candi Borobudur was built algorithmically from which it applied fractal geometry and self-similarity. The essential building of Borobudur is stupa, but not as commonly stupa-shaped dome, but it is punden with six levels of a square (Soetarno, 1988: 90). Three levels of a circular and a main stupa are considered as the peak. All parts belong together, and overallis builds a stupa. The self-similarity of Borobudur Temple can be observed through the dimensionality of stupa (Situngkir, Hokky, 2010:1).



(Stupa of Borobudur, source: private document)

(Self-similarity at Borobudur, source: private document)

In accordance to the presumption that Borobudur was built during the Sailendra dynasty of Buddhist Mahayana, the Borobudur stupa is seen as an imitation of the universe in relation to the philosophy of Buddhism. There are three parts to the Borobudur temple, namely: 1) the foot of the temple called Kamadhatu as natural under the ordinary human being, 2) parts consisting of four levels of a square called Rupadhatu as natural between a man after leaving all worldliness, and 3) the top in the form of three terraced plains including the stupa called Arupadhatu as natural on where the gods (Soetarno, 1988: 91). At the level of Rupadatu are successively from the bottom to the top, there are 32, 24, and 16 or multiples of four pieces stupa containing Buddha statues.



(**Sketch of Borobudur views from the Top, source: private document**)

Research by Parmono Atmadi (1988) on the design principle at Borobudur find their patterns of regularity in the physical building of Borobudur temple, but it has not found a standard unit of measurement for the design principle of Borobudur. Parmono Atmadi filed Borobudur component ratio, although this ratio can not be a reference to an overall ratio of building components.At the temple building in Java, including Borobudur, has a ratio of 4: 6: 9 which states a height comparison base or foundation of the building, the building height of the temple and temple roof heights.

The Ethnomathematic Perspective on The Borobudur

The physical building Borobudur covered a giant geometry model with self-similarity and regularity shapes from its parts. This conclusion came from a rigorous and continuous study of the experts. Free observation might be come to the same conclusion that Borobudur is a mathematics object. Parts of Borobudur are geometry shapes such as cuboid, prism, pyramid, trapezoid, and other interesting shapes that is unknown for its mathematical term.

Borobudur is a cultural artefact which embedded with the philosophical values. Based on the research above, the physical building of the temple is also a mathematic model. Study of mathematics that embedded in culture was done with ethnomathematics. Ethnomathematics is derived from three terms *ethno*, *mathema*, and *tics*. Ethnomathematics is not only ethnic mathematics or mathematics that applies to a particular ethnic, but *ethno* includes all the cultural groups that can be identified with the jargon, codes, symbols, myths and even specific ways in thinking and concluded, then *mathema* interpreted as knowledge and attitudes about space, time, measurement, grouping, comparison, quantity and conclusions, as well as *tic* covers methods or techniques and codes are received, distributed, transmitted, and spread by certain individuals or groups (Francois & Van Kerkhove, 2010: 127).

Mathematics in everyday situation could not be separated from its culture. The idea and practice of mathematics was influenced by the values, norms, customs, and traditions prevailing in the society. For instance, ancient javanese people used the metric system in the development process Borobudur Temple, namely *tala*. Length measurement was defined as the length of a human face from the forehead's hairline to the tip of the chin or the distance from the tip of the thumb to the tip of the middle finger when both fingers are stretched at their maximun distance (Situngkir, Hokky, 2010:2). It is not universal, even varied among people. For that time, it is the most feasible strategy to measure which influenced by their tradition using part of their body.

These ideas or thoughts that then arranged as a representation of concepts and manifest as artifacts, such as images and the arts and techniques. (D'Ambrosio, 2006:6). Finding the characteristics of mathematics inside an artefact was similar to looking for something hiding behind it. According to Gerdes (2014:55), we have to try to reconstruct or unfreeze the frozen mathematics inside the artefact.

Borobudur as Mathematic Learning Resources at School

The process of reconstruct or rediscover the hidden mathematics inside Borobudur could be the starting points for doing and elaborating mathematics in the calssroom. That is the pedagogical values from using cultural artefact as mathematics learning resources. The activity of learning mathematics could be developed more interesting. Beside of that, the most important part is the engagement of students with the object.

Students enable to think and do mathematics by observing, investigating and exploring the physical building of Borobudur. For junior high school, mathematical topics which might be the geometry of two-dimensional such as square, rectangular, trapezoidal studied on its properties. Students can investigate the pattern tiling in the walls and floors of Borobudur. The geometry of three-dimensional such as cubes, cuboid, prism or trapezoid are common geometric shapes for students. At Borobudur, they can find uncommon three-dimensional shapes and study for it. Students can practice to measure length, width, area, or volume those object and then to compare the size. Not only about geometry topic, but the students can also be challenged to find weather the number of stupa has a pattern of numbers.

The usage of cultural artefact as mathematic learning resources at school is to implement a culturally relevant pedagogy. It will helps students to develop their intellectual, social, emotional, and political learning because it using their own cultural to deliver their previous competence (Rosa, M & Orey, D.C, 2013:74). Using object from their culture to study led them to a meaningful learning.

Students' Perception and their Intuition of Mathematics

According to Tall, D (2013:2), mathematics begins from perceptions of and actions on the natural world around us. Begin with practical mathematics in real world, students built their perception about shape and space. Action in counting and measuring lead them to arithmetic operation and then algebra.

Practical mathematics in everyday situation may not applying the formal mathematics. For example, students can distinguish simple or complex shape by using their intuition. When students deal with mathematics at school, they will turn the intuition of mathematics from everyday experiences with a mathematical concept, sometimes later changes or substitutions, but sometimes also occur between everyday experience and mathematical concepts are connected and activated for a particular situation context (Prediger, S, 2004:384).

The connection between their intuition with formal mathematics will emerge if the subject matter related to something they have already know. Mathematical knowledge is built on the knowledge of students from outside the school (Masingila, J.O, 1993:19). It means that they had experience with the matter. The process of building a formal mathematics of intuitive mathematics is an effort to construct knowledge that occurs within the students themselves supported by the socio-cultural environment. Furthermore, formal knowledge will rise an understanding for the students that mathematics is related to everyday life and culture.

Student’s perception of how mathematics is used as a function of their views on mathematics (Masingila, J. O, 2002:37). Mostly students think that mathematics is identic with school mathematics. Students generally also believes that learning mathematics is an isolated and individual activities, mathematics learned in school have little relevance or are not related at all to the real world (Schoenfeld, 1992: 359-360). The student’s belief may be influenced by their perception about mathematics in real world. They did not recognize the connection between school mathematics and mathematics in real world.

Mathematics education in school emphasizes the procedural knowledge or “how’ than emphasizes on meaning, and less focused on “why” so it is less encouraging student to use intuition in learning. Mathematical intuition are similar to sensory perception in some respects and different in other respects (Chudnoff, E. 2016:7). The similarity among others is that they both need interaction with socio-cultural environment. From this interaction, students develop their perception about space and time in line with their intuision of mathematics. The teacher can not disregard this issue and have to use it in the teaching and learning of mathematics to be more helpful for them.

There are still many schools in Indonesia that teach mathematics as a discipline, separated from the real world perspective. Junior high school students have studied formal mathematics at least since they have begun the primary school, so how about their perception about mathematics in real world? Borobudur Temple have been known by students, but do they have positive perception about Borobudur as a mathematical model? Do they have positive perception about Borobudur Temple as a source of learning for school mathematics?

Aim of the Study

The aim of this study was to describe student's perception on Borobudur Temple as mathematic learning resource. This study focus on the perception of the cultural artefact as mathematics model and source for learning mathematics at school. This study involved junior high school students because they have begun formal education at least for six to seven years. Students were taken from surrounding Borobudur Temple because in order to built perception they need interaction and experience with the object.

Context and Methodology

This study uses a survey to collect students' perceptions about Borobudur Temple as mathematics learning resources. Exploratory descriptive associative approach was used to explore the position of the variables and the relationship between one variable with another.

The Places

The study took place at SMP Negeri 1 Borobudur and SMP Muhammadiyah Borobudur on April 2015. These schools are located near to the Borobudur Temple.

The Participants

A total 313 students were the subject of the study. They were 218 seventh grade students of SMP Negeri 1 Borobudur and 95 seventh grade students of SMP Muhammadiyah Borobudur.

Instrument

The data were collected using questionnaire with Likert scale 1 to 4 which mean disagree (1), neutral (2), agree (3) and strongly agree (4). At first, the questionnaire was developed base on two main focuses, those are: 1) Borobudur Temple is a mathemacal object (11 items), and 2) utilization of Borobudur Temple as learning source of school mathematics (3 items).

In order to obtain valid and reliable instrument, expert validity have done by involving two mathematics teacher of Junior High School and one widyaiswara PPPPTK Matematika. Readability test has been carried out with 5 students of seventh grader. Statistical analysis used exploratory factor analysis to determine the item which is not valid and the tendency of the collected items. These factors will describe of how the student's perception on Borobudur Temple.

Findings and Discussion

The Results of Exploratory Factor Analysis

Students were asked to rate their agreement to each item statement on the instrument "Borobudur Temple as Mathematics Learning Resource". A total 313 students have completed the questionnaire and returned it to researcher.

The results of Exploratory Factor Analysis are as follows.

Table 1. KMO AND Bartlett's Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		0,860
Bartlett's Test of Sphericity	Approx. Chi-Square	1,038E3
	df	91
	Sig.	0,000

KMO is 0,86 which mean the number of the sample is sufficient and Bartlett's test sig <0,05, then the matrix is not an identity matrix.

Table 2. Total Variance Explained

Component	Initial Eigenvalues	
	Total	Cumulative
1	4,459	31,853%
2	1,368	41,623%
3	1,113	49,572%

From 14 component, there are three component with eigenvalue > 1 and these 3 component can explained about 49,572% of students' perception.

Table 3. Rotated Component Matrix

No Items	Component		
	1	2	3
1	,799	,137	-,106
2	,333	,402	,339
3	-,344	,334	,544
4	,492	,473	,107
5	,542	,148	,321
6	,339	,215	,526
7	,087	-,063	,752
8	,196	,362	,377
9	,528	,133	,438
10	,647	,127	,109
11	,610	,368	,142
12	,246	,668	,168
13	,173	,777	,036
14	,073	,701	,076

There are no item with loading factor< 0,3 which means that all items are valid. Three factors were obtained from rotated component matrix, namely:1) Borobudur Temple is geometry model (6 items), 2) Borobudur Temple can be used as mathematic learning source at school (4 items), and 3) learning mathematics from Borobudur Temple is helpful for students (4 items).

The Results of Descriptive Statistics Analysis

Table 4. Cronbach alpha, Average item mean, Average standard deviation scores of each factors (N=313)

Factor	Items	Means	Standard Deviation	Cronbach α
1. Geometry Model	1, 4, 5, 9, 10, 11	3,03	0,482	0, 8204
2. Mathematics Learning Source	2, 12, 13, 14	2,95	0,549	
3. Helpful for Student	3, 6, 7, 8	2,82	0,495	

The average items mean for factor 1). Borobudur Temple is geometry model is 3,03. It means that the average students on the position “agree” for this statement. The average items mean for factor 2). Borobudur Temple can be used as mathematic learning source at school is 2,95 and it is approaching score 3. It means that average students also on the position “agree” for this statement. The average items mean for factor 3). learning mathematics from Borobudur Temple is helpful for students is 2,82 and it is approaching score 3. It means that average students on the position “agree” for this statement.

The highest mean score was attained for factor 1. Students have perception that Borobudur Temple is a geometrical model. They agree that mathematics model such as cube, cuboid, pyramid, prism can be found at Borobudur. They also agree that geometric pattern can be found at the Temple and whole building of Borobudur is a geometrical model.

Student have positive perception because they can see those model as part of the building. These experience was connected with their knowledge about varied geometry models from school. In accordance to geometry topics they have learned since elementary school until the seventh grade now, they were able to identify that some part of Borobudur is cuboid or prism.

Student agreed that mathematical identified in Borobudur should be applied in school activity. They opine school mathematics topics that can be learned from Borobudur including the comparison of the length and width, area and volume and pattern of numbers. This positive perception associated with the perception of student that Borobudur is a geometry model, therefore it can be utilized as a source of learning in school.

The lowest mean score was attained for factor 3. learning mathematics from Borobudur Temple is helpful for students. These results could be affected by the condition of the students who have never experienced using Borobudur in the learning of mathematics at school. Student had positive perception that the application and practice of mathematics at Borobudur temple is important to learn because it can generate a sense of pride in their own cultural riches. Mathematical contained in historical sites such as Borobudur can be lost if it is not taught in schools. Some mathematical topics identified in Borobudur temple should be included in the curriculum of mathematics at junior high school.

Student thought that mathematical knowledge can be reached through investigation in Borobudur. This activity is useful for them to learn mathematics meaningfully. By building a mathematical knowledge based on experience of students in everyday life, teacher can encourage student to make the connection between mathematics outside and inside the school in ways that will help student formalized the informal knowledge of mathematics and to learn mathematics more meaningful (Masingila, J.O, 2002:30).

The practical experience of mathematics in everyday life is useful as a specific context for student, which can be used as a basis to make a connection with the relevant school mathematics. The context of the problems from everyday situation has the advantage of the significance of the issues to be resolved. Student was encouraged to resolve the matter in earnest by using a strategy that may be different with that is learned at school.

It is a challenge for teachers to help student see and find that they had engaged with mathematicsin their daily activities. Mostly student and teacher themselves have the perception that mathematics is present in school only through the display of numbers and formulas. Students need to be given the opportunity to do activities in the classroom which is generally carried out in the culture of the community. Teacher must show them how mathematics is used in everyday life, especially in their culture. Therefore, mathematics must be associated with the real matter for students and mathematics should be viewed as a human activity (Gravemeijer, 1994: 82).

Teachers should be able to understand the disposition of student and determine the scope of the disposition of student that includes three dimensions namely student perspective on the importance of mathematics, the student's motivation to engage in the task of mathematics, and perceptions of the students (self perception) about their own mathematics abilities (Campbell, P. F, et. al 2014: 429). The student had a positive perception of the existence of mathematics in the real world based on their intuition. The student can learn mathematics at intuitive level long before the reflective level (Skemp, 1971: 66). This carries important consequences in the learning of mathematics.

Conclusion and Recommendation

Based on findings of the study, we can conclude that junior high school student have positive perception about Borobudur Temple as learning source for school mathematics. Students agree that Borobudur Temple is a geometrical model, it can be used as a source of learning mathematics school and will be beneficial for them. This positive perception comes from students whose school is located near Borobudur Temple. This means that the interaction and experience is important in learning. These perceptions bring implication for teacher. Teacher should be able to draw on intuition and experience of students when presenting the material in order to make it more meaningful. Teachers can design learning activities that take advantage of Borobudur as mathematical objects.

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