Three-phase algorithms in solving full fuzzy transportation problem by using fuzzy analytical hierarchy process

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Abstract: In the fuzzy transportation problem, the ranking function is widely used to order fuzzy number or convert fuzzy number to crisp number. Its process is easy to understand and implement. However, the ranking fuzzy number still has significant weakness in which there is still subjectivity or do not pay attention to real life such that sometimes the input and output disconnected the fully fuzzy transportation problem (FFTP) such as there is negative fuzzy optimal solution. In some cases, it was found that FFTP had equal values of the fuzzy distribution least costs such that the existing methods will be generated two or more fuzzy initial basic feasible values. The proposed algorithm, i.e., three-phase algorithm-based fuzzy AHP, is capable of obtaining the fuzzy optimal solution of FFTP Based on the numerical example used to evaluate the performance of the three phases algorithm. The computational performances have been compared to the existing methods in the literature and the results shown this algorithm can solve the FFTP with similar values fuzzy optimal solution even better minimal solutions than existing methods.

Keywords: fuzzy number; fully fuzzy transportation problem; FFTP; ranking function; fuzzy AHP; fuzzy optimal solution.

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1 Introduction

The transportation problem (TP) is one of the well-known linear programming problem (Hitchock, 1941). The modelling of TP had been widely applied in logistics or supply chain management to achieve minimum total distribution costs where the demand for products still satisfies based on the existing supply. In real life, the TP variables (supply, demand, and transportation cost) are imprecise value. Zadeh (1965) introduced TP with the numbers of supply, demand, and transportation cost are represented by fuzzy number that is fully fuzzy transportation problem (FFTP).

The challenge of FFTP is to build an optimisation model that can produce an optimal solution which is represented by uncertain data or fuzzy optimal solution. Therefore, many researchers were studied in this field. In solving of FFTP, various researchers used different approaches, such as Lotfi et al. (2009) proposed the new method to determine the fuzzy optimal solution of balanced fully FLP. This method can only be applied to fully FLP variables which are represented to symmetric fuzzy number. Kumar et al. (2011) updated Lotfi's method by defuzzifying the symmetric fuzzy number to obtain the fuzzy optimal solution. Najafi and Edalatpanah (2013) improved Kumar's method by showing that, there is a non-negative fuzzy optimal solution so that they recalculated to determine the fuzzy optimal solution. Kumar and Singh (2012) proposed the linearity properties of the Yanger's ranking to rank the triangular fuzzy number of fully FLP in determining a fuzzy optimal solution. Hatami-Marbini et al. (2013) presented the gradually of the LP model to solve FLP in which objective function, coefficients of constraint variable from the right and left sides are represented fuzzy numbers. Ezzati et al. (2015) changed full FLP to be multi-objective linear programming (MOLP) problem by using the lexicographic method. Ebrahimnejad (2017) presented a similar method approach in which the variables of FLP are represented by triangular fuzzy numbers.

The approach of determining fuzzy initial basic feasible solution (IBFS) such as Kumar and Murugesan (2012) presented a modified simplex method to find the fuzzy IBFS in which the variables of FFTP are represented by triangular fuzzy numbers. Kaur and Kumar (2011, 2012) showed out the fuzzy classical algorithms, i.e., generalised fuzzy of north-west corner method (GFNWCM), least-cost method (GFLCM), and Vogel's approximation method (GFVAM) to find fuzzy IBFS. They also used a ranking function to rank fuzzy number variables of FFTP and also used a generalised fuzzy modified distribution (MODI) method to obtain fuzzy optimal solution based on fuzzy IBFS. The fuzzy classical algorithm is also proposed by Ebrahimnejad (2014) to find fuzzy IBFS in which integral value functions to rank the fuzzy number variables of FFTP. Rani and Gulati (2014) solved unbalanced FFTP used the ranking function to rank fuzzy number variables and FVAM to obtain fuzzy IBFS of FFTP. Sudhagar and Ganesan (2016) presented a ranking score method to rank the fuzzy number variables of FFTP. They also presented a modified FLCM to find fuzzy IBFS and fuzzy MODI method to determine the fuzzy optimal solution. Ebrahimnejad (2016) proved that there are the non-negative fuzzy optimal solution of a numerical example that was solved by Sudhagar and Ganesan's method. The new multiplication operation of the fuzzy number, ranking function to rank fuzzy number, the fuzzy classical algorithm to find fuzzy IBFS, and fuzzy MODI to obtain fuzzy optimal solution were proposed by Chakraborty et al. (2016). Fuzzy allocation table method (ATM) was introduced by Hunwisai and Kumam (2017). Preference index-based integral value with LR generalised fuzzy number and fuzzy VAM was presented by Rani and Gulati (2017). Integral ranking based generalised triangular-trapezoidal fuzzy numbers that was implemented with minimum row-column method was proposed by Saini et al. (2018). The graded mean of ranking function and harmonic mean to find penalty value of each row was proposed by Balasubramanian and Subramanian (2018). Kaur et al. (2019) presented fuzzy pythagorean to rank a fuzzy number and VAM to find IBFS in the form of a crisp number. The development of Balasubramanian and Subramanian (2018) method by adding the concept of switching even row/column and odd row/column with supply and demand was discussed by Balasubramanian and Subramanian (2019). Classical ranking function and generalised fuzzy min demand supply approach-based generalised trapezoidal fuzzy was presented by Mathur and Srivastava (2020). Segregated scheme method without ranking function with minimum demand-supply method and stepping stone technique was introduced by Srivastava and Bisht (2020). The solving of unbalance FTP with triangular fuzzy numbers without adding a dummy to supply and demand was presented by Muthuperumal et al. (2020).

Furthermore, the direct approach of solving FFTP such as zero point method to obtain fuzzy optimal solution was introduced by Pandian and Natarajan (2010). Robust ranking to rank fuzzy number and zero suffix method to obtain the fuzzy optimal solution were proposed by Fegade et al. (2012). Mohanaselvi and Ganesan (2012) and Samuel and Venkatachalapathy (2012) proposed a fuzzy dual matrix to obtain a fuzzy optimal solution. Particle swarm optimisation algorithm (PSO) with fuzzy constraint and conjugate constraint was proposed by Baykasolu and Subulan (2019)

The using of ranking functions to convert fuzzy variables on FFTP to crisp variables is often used. This is because the ranking function can simplify the fuzzy transportation algorithm to determine fuzzy optimal solution. In practice, the ranking function have a weakness. The ranking function still find failure in converting two fuzzy numbers and the dominant subjectivity value so that sometimes the input and output disconnected the TPs in real life, for example there is a negative fuzzy optimal solution. Therefore, this study uses the fuzzy analytical hierarchy process (AHP) to rank fuzzy variables on FFTP.

The AHP was introduced by Saaty (1980) to select important factors in decision making. This method was developed to solve the problem by separating, grouping, and arranging into hierarchical structure. In order to achieve the priority criteria, the method used the pairwise comparison matrix by predetermined measurement scale (Saaty, 1990). The input of AHP is the perception of experts, such that there is a subjective factor in decision making that is compatible with a real-life problem. This method also takes into account the validity of data by the limit of inconsistency. However, the uncertainties and doubts that are quite a lot in giving an assessment certainly have an impact on the accuracy of the data and the results obtained. Nevertheless, uncertainties and doubts which are a lot of quite in giving an assessment certainly have an impact on the accuracy of data and the results obtained. Therefore, van Laarhoven and Pedrycz (1983) introduced fuzzy AHP. They used a triangular fuzzy number and logarithmic least squares method to produce priority vectors or eigenvectors (fuzzy weight) in fuzzy AHP. Extent analysis which was proposed by Chang (1992) was used by many researchers to solve fuzzy AHP and to rank alternatives. Chang (1996) had been implemented to calculate the extent synthetic value of pairwise comparison matrix. Praščević and Praščević (2016) implemented fuzzy AHP based on the expected value on the construction industry. Therefore, in this paper we implement fuzzy AHP that was proposed by Praščević and Praščević (2016) to rank fuzzy fuzzy number, modified fuzzy transportation algorithm to find fuzzy IBFS and fuzzy MODI to obtain fuzzy optimal solution based on fuzzy IBFS.

2 Shortcoming of ranking function

This section shows out the shortcomings of ranking functions were proposed by Liou and Wang (1992), Kaur and Kumar (2011) and Ebrahimnejad (2014). Example 1 can be seen that the ranking function method were presented by existing ranking methods failed to rank two triangular fuzzy numbers and also did not reasonable properties for ordering of fuzzy numbers.

Example 1: Let two triangular fuzzy numbers $\tilde{a} = (16, 25, 64)$ and $\tilde{b} = (9, 36, 49)$. Clearly, $\tilde{a} \neq \tilde{b}$. By using existing ranking methods to rank fuzzy numbers can be obtained $a = 32.5$ and $b = 32.5$. So, the ranks of fuzzy numbers are equal, i.e., $\tilde{a} = \tilde{b}$.

3 FFTP formulation

The modelling of TP is very important for production planning, transportation routing, and networking. Generally, the TP model is used to distribute products from supply to demand. The amount of supply and demand of course adjusts to existing needs. Several variables are used on the TP model, including distribution cost (c_{ij}), the numbers of demand (d_j), and supply (s_i). Essentially, the value of variables cannot be certainly known at the formulation of the TP model. Several constraints allowed this uncertainly, such as imprecise data as a consequence of company policies and lack of information.

Table 1 The FFTP table

Source	*Destination*										$\tilde{s}_i$
	U_1		U_2		$\dots$	U_q		$\dots$	U_n		
V_1	$\tilde{c}_{11}$	$\tilde{x}_{11}$	$\tilde{c}_{12}$	$\tilde{x}_{12}$	$\dots$	$\tilde{c}_{1q}$	$\tilde{x}_{1q}$	$\dots$	$\tilde{c}_{1n}$	$\tilde{x}_{1n}$	$\tilde{s}_1$
V_2	$\tilde{c}_{21}$	$\tilde{x}_{21}$	$\tilde{c}_{22}$	$\tilde{x}_{22}$	$\dots$	$\tilde{c}_{2q}$	$\tilde{x}_{2q}$	$\dots$	$\tilde{c}_{2n}$	$\tilde{x}_{2n}$	$\tilde{s}_2$
	$\vdots$		$\vdots$		$\dots$	$\vdots$		$\dots$	$\vdots$		$\vdots$
V_p	$\tilde{c}_{p1}$	$\tilde{x}_{p1}$	$\tilde{c}_{p2}$	$\tilde{x}_{p2}$	$\dots$	$\tilde{c}_{pq}$	$\tilde{x}_{pq}$	$\dots$	$\tilde{c}_{pn}$	$\tilde{x}_{pn}$	$\tilde{s}_p$
	$\vdots$		$\vdots$		$\dots$	$\vdots$		$\dots$	$\vdots$		$\vdots$
V_m	$\tilde{c}_{m1}$	$\tilde{x}_{m1}$	$\tilde{c}_{m2}$	$\tilde{x}_{m2}$	$\dots$	$\tilde{c}_{mq}$	$\tilde{x}_{mq}$	$\dots$	$\tilde{c}_{mn}$	$\tilde{x}_{mn}$	$\tilde{s}_m$
$\tilde{d}_j$	$\tilde{d}_1$		$\tilde{d}_2$		$\dots$	$\tilde{d}_q$		$\dots$	$\tilde{d}_n$		

Based on uncertain variables of the TP model, Kaufman and Gupta (1998) represented fuzzy variables of TP, i.e., fuzzy distribution cost ($\tilde{c}_{ij}$), the numbers fuzzy of demand ($\tilde{d}_j$) and supply ($\tilde{s}_i$) so that the FFTP model can be illustrated FFTP table in showed Table 1 and can be formulated as follows:

$$\min \tilde{\mathrm{T}} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij} \tag{1}$$

subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij} \leq \tilde{s}_i \tag{2}$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \leq \tilde{d}_j \tag{3}$$

and

$$\tilde{x}_{ij} \geq 0 \forall i, j (i = 1, 2, \dots, m; j = 1, 2, \dots, n). \tag{4}$$

where m represents total of supply, n represents total of demand, $\tilde{s}_i = (s_{i,a}, s_{i,b}, s_{i,c})$ is i^{th} fuzzy supply, $\tilde{d}_j = (d_{j,a}, d_{j,b}, d_{j,c})$ is j^{th} fuzzy demand, $\tilde{c}_{ij} = (c_{ij,a}, c_{ij,b}, c_{ij,c})$

is fuzzy distribution cost from i^{th} fuzzy supply to j^{th} fuzzy demand, $\tilde{x}_{ij} = (x_{ij,a}, x_{ij,b}, x_{ij,c})$ is the number of fuzzy approximation unit to assign from i^{th} fuzzy supply to j^{th} fuzzy demand, $\min \tilde{\mathrm{T}}$ is minimised total fuzzy distribution cost and the fuzzy number is represented by the fuzzy triangular number.

If total supply is equal to total demand then the FFTP is called a balance FTTP and given as,

$$\sum_{i=1}^{m} \tilde{s}_i = \sum_{j=1}^{n} \tilde{d}_j \tag{5}$$

4 Fuzzy analytical hierarchy process method

Fuzzy analytical hierarchy process method (AHP) is a combination of the AHP method by the approach of fuzzy concept was developed by Saaty (1980). Fuzzy AHP is used to cover the weaknesses of AHP method, such as the problem of criteria having more subjective properties. In fuzzy AHP, the fuzzy rational scale is used to identify the relative strength of a given criterion. So, a pairwise comparison matrix can be determined and its final value is represented by a fuzzy number.

Fuzzy set theory helps in assessment-related measures subjectively humans use language or linguistic. Linguistic variable is certainly and useful for processing information within the triangular fuzzy number (TFN). The essence of fuzzy AHP lies in the pairwise comparison matrix with a ratio scale related by fuzzy scale value. This research used the intensity value of AHP into TFN on the criteria and alternatives presented by Chan and Wang (1993) in shown Table 2.

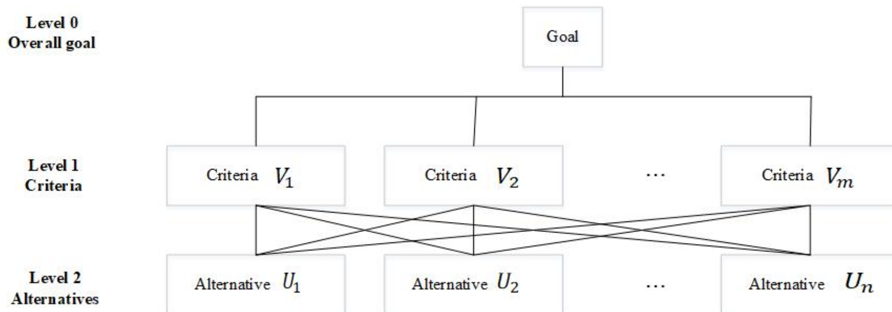
Table 2 TFN rating scale for measuring mutual importance

TFN		*Reciprocal TFN*		*Lingustic value*
AHP scale	*Fuzzy scale*	*AHP scale*	*Fuzzy scale*	
$\tilde{1}$	(1, 1, 1)	$\tilde{1}^{-1}$	$(\frac{1}{1}, \frac{1}{1}, \frac{1}{1})$	equally significant
$\tilde{3}$	(1, 3, 5)	$\tilde{3}^{-1}$	$(\frac{1}{5}, \frac{1}{3}, \frac{1}{1})$	slightly significant
$\tilde{5}$	(3, 5, 7)	$\tilde{5}^{-1}$	$(\frac{1}{7}, \frac{1}{5}, \frac{1}{3})$	very significant
$\tilde{7}$	(5, 7, 9)	$\tilde{7}^{-1}$	$(\frac{1}{9}, \frac{1}{7}, \frac{1}{5})$	greatly significant
$\tilde{9}$	(7, 9, 9)	$\tilde{9}^{-1}$	$(\frac{1}{9}, \frac{1}{9}, \frac{1}{7})$	absolutely significant
$\tilde{2}$, $\tilde{4}$, $\tilde{6}$, $\tilde{8}$	$(x-1, x, x+1)$	$\tilde{2}^{-1}$, $\tilde{4}^{-1}$, $\tilde{6}^{-1}$, $\tilde{8}^{-1}$	$(\frac{1}{x+1}, \frac{1}{x}, \frac{1}{x-1})$	intermediate values

Generally, the fuzzy AHP method describes the problem into several levels, namely the objective is at the highest level, the criteria are followed by sub-criteria (if any) is at level 1 and the alternative is at the lowest level Saaty (1990). In this research, fuzzy AHP is used to rank fuzzy numbers, especially in column destination as an alternative of FFTP such that a hierarchical illustration of FFTP can be seen in Figure 1.

In ranking column destination as an alternative, this research uses the fuzzy AHP based on expected value proposed by Praščević and Praščević (2016). The steps of fuzzy AHP to rank a fuzzy number of FFTP are shown in Algorithm 1.

Figure 1 Hierarchical structure



Algorithm 1 Fuzzy AHP algorithm

Step 1 Construct a hierarchical structure of FFTP. Starting from the highest level, i.e., the objective function, mid-level, i.e., source of row, and lowest level, i.e., destination of column. The hierarchical structure can be illustrated in Figure 1.

Step 2 Input data:

The number of criteria m;

The number of alternative n;

Fuzzy pairwise comparison matrix of criteria $\tilde{V} = (V_a, V_b, V_c)$ by

$$V_a = \begin{pmatrix} 1 & v_{12,a} & \dots & v_{1k,a} \\ v_{12,a}^{-1} & 1 & \dots & v_{2k,a} \\ \vdots & \vdots & \ddots & \vdots \\ v_{1k,a}^{-1} & v_{2k,a}^{-1} & \dots & 1 \end{pmatrix}, \; V_b = \begin{pmatrix} 1 & v_{12,b} & \dots & v_{1k,b} \\ v_{12,b}^{-1} & 1 & \dots & v_{2k,b} \\ \vdots & \vdots & \ddots & \vdots \\ v_{1k,b}^{-1} & v_{2k,b}^{-1} & \dots & 1 \end{pmatrix}$$

$$V_c = \begin{pmatrix} 1 & v_{12,c} & \dots & v_{1k,c} \\ v_{12,c}^{-1} & 1 & \dots & v_{2k,c} \\ \vdots & \vdots & \ddots & \vdots \\ v_{1k,c}^{-1} & v_{2k,c}^{-1} & \dots & 1 \end{pmatrix}$$

Fuzzy pairwise comparison matrix of alternatives still have related of criteria

$$\tilde{U}^{(i)} = \begin{array}{c|cccc} V_i & U_1 & U_2 & \dots & U_n \\ \hline U_1 & 1 & \tilde{u}_{12}^i & \dots & \tilde{u}_{1n}^i \\ U_2 & (\tilde{u}_{12}^i)^{-1} & 1 & \dots & \tilde{u}_{1m}^i \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ U_n & (\tilde{u}_{1n}^i)^{-1} & (\tilde{u}_{2n}^i)^{-1} & \dots & 1 \end{array}$$

Step 3 Test the consistency of ratio (CR) from fuzzy pairwise comparison matrix for $\tilde{V}$ and $\tilde{U}^{(i)}, i = 1, 2, \dots, m$.

For CR of $\tilde{V}$ by solving fuzzy eigenvalues problem equation as follows

$$\tilde{V} \otimes \tilde{\omega} = \tilde{\lambda} \otimes \tilde{\omega} \tag{6}$$

where $\tilde{\omega} = (\omega_a, \omega_b, \omega_c); \tilde{\lambda} = (\lambda_a, \lambda_b, \lambda_c)$ by using expected value was presented by Praščević and Praščević (2016), equation (5) divided to be three equation system

$$\begin{aligned} \bar{V}_a \otimes \omega_a &= \bar{\lambda}_a \otimes \omega_a \\ \bar{V}_b \otimes \omega_b &= \bar{\lambda}_b \otimes \omega_b \\ \bar{V}_c \otimes \omega_c &= \bar{\lambda}_c \otimes \omega_c \end{aligned} \tag{7}$$

where

$$\begin{array}{ll} \bar{V}_a = 2V_a + V_b & \omega_a = [\omega_{1,a}, \omega_{2,a}, \ldots, \omega_{m,a}]^T \\ \bar{V}_b = V_a + 4V_b + V_c & \omega_b = [\omega_{1,b}, \omega_{2,b}, \ldots, \omega_{m,b}]^T \\ \bar{V}_c = V_b + 2V_c & \omega_c = [\omega_{1,c}, \omega_{2,c}, \ldots, \omega_{m,c}]^T \\ \bar{\lambda}_a = 2\lambda_a + \lambda_b & \lambda_a = \frac{(\bar{\lambda}_a - \lambda_b)}{2} \\ \bar{\lambda}_b = \lambda_a + 4\lambda_b + \lambda_c & \lambda_b = \frac{(2\bar{\lambda}_b - \lambda_a - \lambda_b)}{6} \\ \bar{\lambda}_c = \lambda_b + 2\lambda_c & \lambda_c = \frac{(\bar{\lambda}_c - \lambda_b)}{2} \end{array}$$

calculate the fuzzy priority weights of criteria $\tilde{\bar{w}} = (\bar{w}_a, \bar{w}_b, \bar{w}_c)$ as follows

$$\begin{aligned} \bar{w}_a &= \frac{\omega_a \lambda_a}{z_a \lambda_b} \\ \bar{w}_b &= \frac{\omega_b}{z_a} \\ \bar{w}_c &= \frac{\omega_c \lambda_c}{z_c \lambda_b} \end{aligned} \tag{8}$$

where $z_a = \sum_{j=1}^{m} w_{j,a},\ z_b = \sum_{j=1}^{m} w_{j,b},\ z_c = \sum_{j=1}^{m} w_{j,c}$.
Calculate the index of consistency (CI) uses formulation as follows:

$$CI = \frac{\lambda_{\max} - m}{m - 1} \tag{9}$$

$$CR = \frac{CI}{RI} \tag{10}$$

where $\lambda_{max} = \lambda_b$ (Buckley, 1985), RI is random consistency

If CR > 0.1 then repeat step 1 until step 3 such that this condition is satisfied (CR ≤ 0.1).

For CR of $\tilde{U}^{(i)}, i = 1, 2, \ldots, m$ by expressed $\tilde{U}^{(i)}$ be $\tilde{U}_a^{(i)}, \tilde{U}_b^{(i)}, \tilde{U}_a^{(i)}$ based on criteria $\tilde{V}$ to solve eigenvector equation as follows:

$$\tilde{U}^{(i)} \otimes \tilde{\rho}^{(i)} = \tilde{\lambda}^{(i)} \otimes \tilde{\rho}^{(i)} \tag{11}$$

The finding fuzzy principal eigenvalues $\tilde{\lambda}_{max}^{(i)} = (\lambda_a^{(i)}, \lambda_b^{(i)}, \lambda_c^{(i)})$, fuzzy eigenvector $\tilde{\rho}_{max}^{(i)} = (\rho_a^{(i)}, \rho_b^{(i)}, \rho_c^{(i)})$, index of consistency $CI^{(i)}$, and consistency of ratio $CR^{(i)}$ by equaitons (9) and (10).

If $CR^{(i)} > 0.1$ then repeat step 1 until step 3 such that this condition is satisfied ($CR^{(i)} \leq 0.1$).

Step 4 Determine the fuzzy weight vector of alternatives $\bar{\tilde{\rho}}^{(i)} = (\bar{\rho_a}^{(i)}, \bar{\rho_b}^{(i)}, \bar{\rho_c}^{(i)})$ by formulation as follows:

$$\begin{aligned} \bar{\rho}_a^{(i)} &= \frac{\rho_a^{(i)}\lambda_a^{(i)}}{z_a^{(i)}\lambda_b^{(i)}} \\ \bar{\rho}_b^{(i)} &= \frac{\rho_b^{(i)}}{z_a^{(i)}} \\ \bar{\rho}_c^{(i)} &= \frac{\rho_c^{(i)}\lambda_c^{(i)}}{z_c^{(i)}\lambda_b^{(i)}} \end{aligned} \tag{12}$$

where $z_a^{(i)} = \sum_{j=1}^{m} \rho_{j,a}^{(i)},\ z_b^{(i)} = \sum_{j=1}^{m} \rho_{j,b}^{(i)},\ z_c^{(i)} = \sum_{j=1}^{m} \rho_{j,c}^{(i)}$

Step 5 Calculate fuzzy global weight $\tilde{g}_j = (g_{j,a}, g_{j,b}, g_{j,c})$ of alternatives as follows:

$$\begin{aligned} g_{j,a} &= \bar{\rho_a}\bar{\omega}_a = \begin{pmatrix} g_{1,a} & g_{2,a} & \dots & g_{n,a} \end{pmatrix}^T \\ g_{j,b} &= \bar{\rho_b}\bar{\omega}_b = \begin{pmatrix} g_{1,b} & g_{2,b} & \dots & g_{n,b} \end{pmatrix}^T \\ g_{j,c} &= \bar{\rho_c}\bar{\omega}_c = \begin{pmatrix} g_{1,c} & g_{2,c} & \dots & g_{n,c} \end{pmatrix}^T \end{aligned} \tag{13}$$

where

$$\begin{aligned} \bar{\rho_a} &= \begin{pmatrix} \bar{\rho_a}^{(1)} & \bar{\rho_a}^{(2)} & \dots & \bar{\rho_a}^{(m)} \end{pmatrix} \\ \bar{\rho_b} &= \begin{pmatrix} \bar{\rho_b}^{(1)} & \bar{\rho_b}^{(2)} & \dots & \bar{\rho_b}^{(m)} \end{pmatrix} \\ \bar{\rho_c} &= \begin{pmatrix} \bar{\rho_c}^{(1)} & \bar{\rho_c}^{(2)} & \dots & \bar{\rho_c}^{(m)} \end{pmatrix} \end{aligned} \tag{14}$$

and

$$\begin{aligned} \bar{\omega}_a &= \begin{pmatrix} \bar{\omega}_{1,a} & \bar{\omega}_{2,a} & \dots & \bar{\omega}_{m,a} \end{pmatrix}^T \\ \bar{\omega}_b &= \begin{pmatrix} \bar{\omega}_{1,b} & \bar{\omega}_{2,b} & \dots & \bar{\omega}_{m,b} \end{pmatrix}^T \\ \bar{\omega}_c &= \begin{pmatrix} \bar{\omega}_{1,c} & \bar{\omega}_{2,c} & \dots & \bar{\omega}_{m,c} \end{pmatrix}^T \end{aligned} \tag{15}$$

Step 6 Ranked of alternatives $U_j (j = 1, 2, \dots, n)$ based on expected value ($g_{j,e}$), standard deviation (σ_j) and coefficient of variation (CV_j) (Cheng, 1993) as follows:

$$g_{j,e} = \frac{g_{j,a} + 2g_{j,b} + g_{j,c}}{4}, \qquad j = 1, 2, \dots, n \tag{16}$$

$$\sigma_j = \sqrt{\frac{(3g_{j,a}^2 + 4g_{j,b}^2 + 3g_{j,c}^2 - 4g_{j,a}g_{j,b} - 2g_{j,a}g_{j,c} - 4g_{j,m}g_{j,c})}{80}}, \quad j = 1, 2, \dots, n \tag{17}$$

$$CV_j = \frac{\sigma_j}{\tilde{g}_{j,e}}, \qquad j = 1, 2, \dots, n \tag{18}$$

5 Three-phase algorithm

If there are two different fuzzy cost distributions in where the smallest ranking fuzzy numbers are equal, then we will fail in allocating $\tilde{x}_{ij}$ of FFTP table such that will produce more than one fuzzy IBFS. Three phases algorithm is proposed which consist of stage 1 to rank fuzzy number (Algorithm 1), stage 2 to find fuzzy IBFS (Algorithm 2) and stage 3 to obtain the fuzzy optimal solution based on fuzzy IBFS (Algorithm 3).

Algorithm 2 Modified fuzzy transportation algorithm

Step 1 Check balanced FFTP is satisfied equation (5). If it is unsatisfied then it is an unbalanced problem. Convert an unbalanced FFTP into a balanced FFTP by introducing a dummy fuzzy supply $\tilde{s}_{m+1} = (s_{i+1,a}, s_{i+1,b}, s_{i+1,c})$ and a dummy fuzzy demand $\tilde{d}_{n+1} = (d_{j+1,a}, d_{j+1,b}, d_{j+1,c})$ satisfying following two conditions:

1. $0 \leq s_{i+1,a} \leq s_{i+1,b} \leq s_{i+1,c}$
2. $0 \leq d_{j+1,a} \leq d_{j+1,b} \leq d_{j+1,c}$

Step 2 Using Algorithm 1 to rank alternatives or column destinations. Ranking of column destinations are denoted $CV_j, j = 1, 2, \ldots, n$. Selected smallest ranking of CV_j (SR) denoted by $SR = \min(CV_j)$

Step 3 Select the smallest $c_{ij,b}$ of SR. Allocating $\tilde{x}_{ij} = (x_{ij,a}, x_{ij,b}, x_{ij,c})$ of $c_{ij,b}$ is satisfied all conditions as follows:

$$\begin{aligned} &0 \leq s_{i,a} - x_{ij,a} \leq s_{i,b} - x_{ij,b} \leq s_{i,c} - x_{ij,c} \\ &0 \leq d_{j,a} - x_{ij,a} \leq d_{j,b} - x_{ij,b} \leq d_{j,c} - x_{ij,c} \\ &0 \leq x_{ij,a} \leq x_{ij,b} \leq x_{ij,c} \end{aligned} \tag{19}$$

If the smallest $c_{ij,b}$ is more than one SR then select $c_{ij,b}$ by smallest $s_{i,b}$ to allocate $\tilde{x}_{ij}$ which satisfy the conditions equation (19).

Step 4 Calculate the remaining unsatisfied demand and supply at all nodes. Computation of unsatisfied supply and demand are as follows:

$$\tilde{s}_i^1 = \left(s_{i,a} - x_{ij,a}, \quad s_{i,b} - x_{ij,b}, \quad s_{i,c} - x_{ij,c}\right)$$

and

$$\tilde{d}_j^1 = \left(d_{j,a} - x_{ij,a}, \quad d_{j,b} - x_{ij,b}, \quad d_{j,c} - x_{ij,c}\right)$$

If $\tilde{s}_i^1 = \tilde{d}_j^1 = (0, 0, 0, 0)\ \forall i, j$ then iteration is over. Then, fuzzy IBFS is $\tilde{x}_{ij} = (x_{ij,a}, x_{ij,b}, x_{ij,c})$. Otherwise, repeat Step 3 until Step 4 such that $\sum_{i=1}^{m} \tilde{s}_i = \sum_{j=1}^{n} \tilde{d}_j = (0, 0, 0)$

Step 5 Calculate minimal total fuzzy distribution cost by equation (1)

Algorithm 3 Fuzzy modified-distribution method

Step 1	The IBFS obtained of FFTP by using Algorithm 2;
Step 2	Introduce $\tilde{u}_{i,b}$ and $\tilde{v}_{j,b}$ as variable convenient for every i^{th} and j^{th}, respectively. In front of i^{th} write $\tilde{u}_{i,b}$ in row and at $\tilde{v}_{j,b}$ the under ofj^{th} in column. Let $\tilde{u}_{i,b} = 0$ is maximum number of allocations row;
Step 3	Determine $\tilde{\lambda}_{i,j,b}$ and $\tilde{v}_{j,b}$ by using $\tilde{c}_{ij,b} = \tilde{u}_{i,b} + \tilde{v}_{j,b}$ for base of cell, then determine $\tilde{\lambda}_{i,j,b} = \tilde{c}_{ij,b} - (\tilde{u}_{i,b} + \tilde{v}_{j,b})$ of non-base of cells. Next, two possibilities as follows: 1 If $\tilde{\lambda}_{i,j,b} \geq 0, \forall i, j$, then the resulted of fuzzy IBFS is done. In other words, fuzzy optimal solution has been satisfied; 2 Otherwise, $\exists \tilde{\lambda}_{i,j,b}$, then the resulted of fuzzy IBFS do not finished yet. In other words, fuzzy optimal solution is not optimal. Therefore, fuzzy optimal solution is chosen a cell of $(i, j)^{\text{th}}$ in which $\tilde{\lambda}_{i,j,b}$ is smallest negative. Next, make a horizontal and vertical closed path that starts from unchosen base of cell of $(i, j)^{\text{th}}$. The path can only replace to angle on base of cell $(i, j)^{\text{th}}$ and the path is chosen must pass through base and non-base cell of $(i, j)^{\text{th}}$:
Step 4	Give sign $(+)$ and $(-)$ for closed loop started with $(+)$ for chosen non-base cells. After that, determine fuzzy quantity on cells with signs $(+)$ and $(-)$. Consequently, new FTTP table is obtained.
Step 5	Repeat of steps 2, 3 and 4 for FTTP table until $\tilde{\lambda}_{i,j,b} \geq 0, \forall i, j$
Step 6	Obtain a new improved solution by allocating units to the unfilled cell according step 5 and calculate the new FFTP.
Step 7	Determine the value of fuzzy optimal solution or objective function by using equation (1).

6 Numerical example

In this section, we illustrate the proposed method by using numerical example in real life and the resulted fuzzy optimal solution is compared with the existing algorithms (Kaur and Kumar, 2011; Kumar et al., 2011; Kumar and Singh, 2012; Ezzati et al., 2015; Chakraborty et al., 2016; Ebrahimnejad, 2017).

Example 2 (Liang et al., 2005): A Dali company of Taiwan supply a soft drink from three sources, i.e., Changhua (V_1), Touliu (V_2), and Hsinchu (V_3) of Taiwan to four destinations situated at Taichung(U_1), Chiayi (U_2), Kaohsiung (U_3), and Taipei (U_4). The main goal is to minimise the distribution cost. According to the preliminary information. Table 3 summarised supply available from three-point, the demand from the four destination centers and the unit distribution cost for each route used the soft drink company. The environmental coefficients and related parameters are normally are imprecise in real-life due to the incomplete or unavailable information.

Table 3 Data of Example 2 (in US dollar)

Source	*Destination*				*Supply*
	Taichung (U_1)	*Chiayi* (U_2)	*Kaohsiung* (U_3)	*Taipei* (U_4)	
Changhua (V_1)	(\$8, \$10, \$10.8)	(\$20.4, \$22, \$24)	(\$8, \$10, \$10.6)	(\$18.8, \$20, \$22)	(7.2, 8, 8.8)
Touliu (V_2)	(\$14, \$15, \$16)	(\$18.2, \$20, \$22)	(\$10, \$12, \$13)	(\$6, \$8, \$8.8)	(12, 14, 16)
Hsinchu (V_3)	(\$18.4, \$20, \$21)	(\$9.6, \$12, \$13)	(\$7.8, \$10, \$10.8)	(\$14, \$15, \$16)	(10.2, 12, 13.8)
Demand	(6.2, 7, 7.8)	(8.9, 10, 11.1)	(6.5, 8, 9.5)	(7.8, 9, 10.2)	

The solution of the FFTP Example 2 is solved by using Algorithms 1, 2 and 3 as follows:

Step 1 Obvious that based on equation (5), if $\sum_{i=1}^{3} \tilde{s}_3 = \sum_{j=1}^{4} \tilde{d}_j = (29.4, 34, 38.6)$ then FFTP is balanced.

Step 2 Using the Algorithm 1 to rank $\tilde{U}_j, j = 1, 2, 3, 4$. The steps of Algorithm 1 as follows:

1 Obvious that $m = 3$ and $n = 4$. For fuzzy pairwise comparison matrix of criteria on sources as follows:

$$\tilde{V} = (V_a, V_b, V_c) = \begin{pmatrix} \tilde{1} & \tilde{3} & \tilde{5} \\ \tilde{3}^{-1} & \tilde{1} & \tilde{3} \\ \tilde{5}^{-1} & \tilde{3}^{-1} & \tilde{1} \end{pmatrix}.$$

Obvious that

$$V_a = \begin{pmatrix} 1 & 1 & 3 \\ \frac{1}{5} & 1 & 1 \\ \frac{1}{7} & \frac{1}{5} & 1 \end{pmatrix}, V_b = \begin{pmatrix} 1 & 3 & 5 \\ \frac{1}{3} & 1 & 3 \\ \frac{1}{5} & \frac{1}{3} & 1 \end{pmatrix} \text{ and } V_c = \begin{pmatrix} 1 & 5 & 7 \\ 1 & 1 & 5 \\ \frac{1}{3} & 1 & 1 \end{pmatrix}.$$

In order to fuzzy pairwise comparison matrix of alternatives which are related by criteria as follows:

$$\tilde{U}^1 = \begin{pmatrix} \tilde{1} = (1,1,1) & \tilde{2} = (1,2,3) & \tilde{3} = (1,3,5) & \tilde{5} = (3,5,7) \\ \tilde{2}^{-1} = (\frac{1}{3}, \frac{1}{2}, 1) & \tilde{1} = (1,1,1) & \tilde{2} = (1,2,3) & \tilde{3} = (1,3,5) \\ \tilde{3}^{-1} = (\frac{1}{5}, \frac{1}{3}, \frac{1}{1}) & \tilde{2}^{-1} = (\frac{1}{3}, \frac{1}{2}, 1) & \tilde{1} = (1,1,1) & \tilde{5} = (3,5,7) \\ \tilde{5}^{-1} = (\frac{1}{7}, \frac{1}{5}, \frac{1}{3}) & \tilde{3}^{-1} = (\frac{1}{5}, \frac{1}{3}, \frac{1}{1}) & \tilde{5}^{-1} = (\frac{1}{7}, \frac{1}{5}, \frac{1}{3}) & \tilde{1} = (1,1,1) \end{pmatrix}$$

$$\tilde{U}^2 = \begin{pmatrix} \tilde{1} = (1,1,1) & \tilde{1} = (1,1,1) & \tilde{2} = (1,2,3) & \tilde{3} = (1,3,5) \\ \tilde{1} = (1,1,1) & \tilde{1} = (1,1,1) & \tilde{1} = (1,1,1) & \tilde{2} = (1,2,3) \\ \tilde{2}^{-1} = (\frac{1}{3}, \frac{1}{2}, 1) & \tilde{1} = (1,1,1) & \tilde{1} = (1,1,1) & \tilde{5} = (3,5,7) \\ \tilde{3}^{-1} = (\frac{1}{5}, \frac{1}{3}, \frac{1}{1}) & \tilde{2}^{-1} = (\frac{1}{3}, \frac{1}{2}, 1) & \tilde{5}^{-1} = (\frac{1}{7}, \frac{1}{5}, \frac{1}{3}) & \tilde{1} = (1,1,1) \end{pmatrix}$$

$$\tilde{U}^3 = \begin{pmatrix} \tilde{1} = (1,1,1) & \tilde{1} = (1,1,1) & \tilde{3} = (1,3,5) & \tilde{5} = (3,5,7) \\ \tilde{1} = (1,1,1) & \tilde{1} = (1,1,1) & \tilde{1} = (1,1,1) & \tilde{3} = (1,3,5) \\ \tilde{3}^{-1} = (\frac{1}{5}, \frac{1}{3}, \frac{1}{1}) & \tilde{1} = (1,1,1) & \tilde{1} = (1,1,1) & \tilde{2} = (1,2,3) \\ \tilde{5}^{-1} = (\frac{1}{7}, \frac{1}{5}, \frac{1}{3}) & \tilde{3}^{-1} = (\frac{1}{5}, \frac{1}{3}, \frac{1}{1}) & \tilde{2}^{-1} = (\frac{1}{3}, \frac{1}{2}, 1) & \tilde{1} = (1,1,1) \end{pmatrix}$$

2 CR test from $\tilde{V}$ by solving equation (6) and (7) are obtained

$$\bar{V}_a = \begin{pmatrix} 3 & 5 & 11 \\ 0.733 & 3 & 5 \\ 0.485 & 0.733 & 3 \end{pmatrix} \quad \omega_a = \begin{pmatrix} 0.62 & 0.26 & 0.85 \end{pmatrix}^T \quad \lambda_a = 2.045$$

$$\bar{V}_b = \begin{pmatrix} 6 & 18 & 30 \\ 2.533 & 6 & 18 \\ 1.276 & 2.533 & 6 \end{pmatrix} \quad \omega_b = \begin{pmatrix} 0.61 & 0.27 & 0.11 \end{pmatrix}^T \quad \lambda_b = 3.034$$

$$\bar{V}_c = \begin{pmatrix} 3 & 13 & 19 \\ 2.333 & 3 & 13 \\ 0.867 & 2.333 & 3 \end{pmatrix} \quad \omega_c = \begin{pmatrix} 0.58 & 0.30 & 0.12 \end{pmatrix}^T \quad \lambda_c = 5.058$$

Calculate fuzzy weight priority of criteria $\tilde{\bar{w}} = (\bar{w}_a, \bar{w}_b, \bar{w}_c)$ by equation (8), are obtained

$$\bar{w}_a = \begin{pmatrix} 0.418 & 0.175 & 0.080 \end{pmatrix}^T$$
$$\bar{w}_b = \begin{pmatrix} 0.615 & 0.272 & 0.112 \end{pmatrix}^T$$
$$\bar{w}_c = \begin{pmatrix} 0.964 & 0.504 & 0.196 \end{pmatrix}^T$$

Obvious that by equation (9), $m = 3$ is resulted $CI = 0.017$

Obvious that by equation (10),$RI = 0.58$ is resulted $CR = 0.029$, Because of $CR < 0.1$ then comparison matrices $\tilde{V}$ are satisfied

CR test from $\tilde{U}^i, i = 1, 2, 3$ = $(\tilde{U}_a^{(i)}, \tilde{U}_b^{(i)}, \tilde{U}_a^{(i)}), i = 1, 2, 3$ by solving equation (11)

Obvious that $i = 1$ then

$$\tilde{\rho}^{(1)} = \begin{pmatrix} 0.401 & 0.464 & 0.421 \\ 0.239 & 0.261 & 0.260 \\ 0.281 & 0.203 & 0.231 \\ 0.080 & 0.072 & 0.088 \end{pmatrix}$$

and

$$\tilde{\lambda}^{(1)} = \begin{pmatrix} 2.943 & 4.182 & 6.769 \end{pmatrix}$$

Obvious that $CI^{(1)} = 0.061, n = 4$ and $CR^{(1)} = 0.067, RI^{(1)} = 0.90$. Because of $CR^{(1)} < 0.1$ then comparison matrix $\tilde{U}^1$ is satisfied

If $i = 2$ then

$$\tilde{\rho}^{(2)} = \begin{pmatrix} 0.295 & 0.347 & 0.344 \\ 0.295 & 0.265 & 0.219 \\ 0.312 & 0.289 & 0.281 \\ 0.098 & 0.100 & 0.156 \end{pmatrix}$$

and

$$\tilde{\lambda}^{(2)} = \begin{pmatrix} 3.261 & 4.189 & 6.113 \end{pmatrix}$$

Obvious that $CI^{(2)} = 0.063, n = 4$ and $CR^{(2)} = 0.070, RI^{(2)} = 0.90$. Because of $CR^{(2)} < 0.1$ then comparison matrix $\tilde{U}^2$ is satisfied

If $i = 3$ then

$$\tilde{\rho}^{(3)} = \begin{pmatrix} 0.385 & 0.424 & 0.403 \\ 0.301 & 0.287 & 0.247 \\ 0.216 & 0.199 & 0.216 \\ 0.098 & 0.090 & 0.134 \end{pmatrix}$$

and

$\tilde{\lambda}^{(3)} = \begin{pmatrix} 3.266 & 4.106 & 6.501 \end{pmatrix}$

Obvious that $CI^{(3)} = 0.035, n = 4$ and $CR^{(3)} = 0.039, RI^{(3)} = 0.90$. Because of $CR^{(3)} < 0.1$ so that comparison matrix $\tilde{U}^3$ is satisfied

3 Determine fuzzy weight vector of alternatives $\bar{\tilde{\rho}}^{(i)} = (\bar{\rho_a}^{(i)}, \bar{\rho_b}^{(i)}, \bar{\rho_c}^{(i)}), i = 1, 2, 3$ by using equation (12) is obtained

If $i = 1$ then $\bar{\tilde{\rho}}^{(1)} = \begin{pmatrix} 0.282 & 0.464 & 0.682 \\ 0.168 & 0.261 & 0.420 \\ 0.198 & 0.203 & 0.374 \\ 0.056 & 0.072 & 0.143 \end{pmatrix}$

If $i = 2$ then $\bar{\tilde{\rho}}^{(2)} = \begin{pmatrix} 0.230 & 0.347 & 0.502 \\ 0.230 & 0.265 & 0.319 \\ 0.243 & 0.289 & 0.410 \\ 0.076 & 0.100 & 0.228 \end{pmatrix}$

If $i = 3$ then $\bar{\tilde{\rho}}^{(3)} = \begin{pmatrix} 0.306 & 0.424 & 0.638 \\ 0.240 & 0.287 & 0.391 \\ 0.172 & 0.199 & 0.341 \\ 0.078 & 0.090 & 0.213 \end{pmatrix}$

4 Calculate fuzzy weigth global $\tilde{g}_j = (g_{j,a}, g_{j,b}, g_{j,c}), j = 1, 2, 3, 4$ by using equations (13), (14) and (15) are obtained

If $\bar{\rho}_a = \begin{pmatrix} 0.282 & 0.230 & 0.306 \\ 0.168 & 0.230 & 0.240 \\ 0.198 & 0.243 & 0.172 \\ 0.056 & 0.076 & 0.078 \end{pmatrix}$ and $\bar{\omega}_a = \begin{pmatrix} 0.419 \\ 0.175 \\ 0.080 \end{pmatrix}$ then

$g_{j,a} = \begin{pmatrix} 0.183 \\ 0.130 \\ 0.139 \\ 0.043 \end{pmatrix}$

If $\bar{\rho}_b = \begin{pmatrix} 0.464 & 0.347 & 0.424 \\ 0.261 & 0.265 & 0.287 \\ 0.203 & 0.289 & 0.199 \\ 0.072 & 0.100 & 0.090 \end{pmatrix}$ and $\bar{\omega}_b = \begin{pmatrix} 0.615 \\ 0.273 \\ 0.122 \end{pmatrix}$ then

$g_{j,b} = \begin{pmatrix} 0.427 \\ 0.265 \\ 0.226 \\ 0.082 \end{pmatrix}$

If $\bar{\rho}_c = \begin{pmatrix} 0.682 & 0.502 & 0.638 \\ 0.420 & 0.319 & 0.391 \\ 0.374 & 0.410 & 0.341 \\ 0.143 & 0.228 & 0.213 \end{pmatrix}$ and $\bar{\omega}_b = \begin{pmatrix} 0.965 \\ 0.507 \\ 0.196 \end{pmatrix}$ then

$$g_{j,c} = \begin{pmatrix} 1.037 \\ 0.644 \\ 0.635 \\ 0.295 \end{pmatrix}$$

Obvious that for $j = 1$ then $\tilde{g}_1 = \begin{pmatrix} 0.183 & 0.427 & 1.037 \end{pmatrix}$

Obvious that if $j = 2$ then $\tilde{g}_2 = \begin{pmatrix} 0.130 & 0.265 & 0.644 \end{pmatrix}$

Obvious that if $j = 3$ then $\tilde{g}_3 = \begin{pmatrix} 0.139 & 0.226 & 0.635 \end{pmatrix}$

Obvious that if $j = 4$ then $\tilde{g}_4 = \begin{pmatrix} 0.043 & 0.082 & 0.295 \end{pmatrix}$

5 Ranking of alternatives $U_j (j = 1, 2, 3, 4)$ by expected value in equation (16), standard deviation in equation (17) and coefficient of variation in equation (18) is obtained

if $j = 1$ then $g_{1,e} = 0.519$, $\sigma_1 = 0.141$ and $CV_1 = 27.21(\%)$

if $j = 2$ then $g_{2,e} = 0.326$, $\sigma_2 = 0.086$ and $CV_2 = 26.30(\%)$

if $j = 3$ then $g_{3,e} = 0.307$, $\sigma_3 = 0.086$ and $CV_3 = 28.15(\%)$

if $j = 4$ then $g_{4,e} = 0.125$, $\sigma_4 = 0.044$ and $CV_4 = 35.36(\%)$

Obvious that

$$\begin{aligned} SR &= \min(CV_1, CV_2, CV_3, CV_4) \\ &= \min(27.21, 26.30, 28.15, 35.36) \\ &= 26.30 \\ &= CV_2 \end{aligned}$$

Step 3 Select $c_{32,b} = 12$ is smallest $c_{ij,b}$ of CV_2. allocate $\tilde{x}_{32} = (x_{32,a}, x_{32,b}, x_{32,c})$ of $c_{32,b}$ which satisfy the conditions equation (19) so that $\tilde{x}_{32} = (8.9, 10, 11.1)$.

Step 4 Calculate remaining $\tilde{s}_3^1$ and $\tilde{d}_2^1$ is obtained

$$\tilde{s}_3^1 = \begin{pmatrix} 10.2 - 8.9, & 12 - 10, & 13.8 - 11.1 \end{pmatrix} = (1.3, 2, 2.7)$$

and

$$\tilde{d}_2^1 = \begin{pmatrix} 8.9 - 8.9, & 10 - 10, & 11.1 - 11.1 \end{pmatrix} = (0, 0, 0)$$

Obviously, $\tilde{s}_3^1 \neq \tilde{d}_2^1$. Cross out of $\tilde{d}_2^1$ because $\tilde{d}_2^1 = (0, 0, 0)$. Repeat Step 3 without considering the cross out of column so that $\sum_{i=1}^{m} \tilde{s}_i = \sum_{j=1}^{n} \tilde{d}_j = (0, 0, 0)$. In detail, fuzzy IBFS of FFTP on Example 2 can be seen in Table 4.

Table 4 Fuzzy initial basic feasible solution of Example 2

Source	*Destination*				*Supply*
	Taichung (U_1)	*Chiayi* (U_2)	*Kaohsiung* (U_3)	*Taipei* (U_4)	
Changhua (V_1)	(6.2, 7, 7.8)	(0.0, 0.0, 0.0)	(1.0, 1.0, 1.0)	(0.0, 0.0, 0.0)	(7.2, 8, 8.8)
Touliu (V_2)	(0.0, 0.0, 0.0)	(0.0, 0.0, 0.0)	(4.2, 5, 5.8)	(7.8, 10, 10.2)	(12, 14, 16)
Hsinchu (V_3)	(0.0, 0.0, 0.0)	(8.9, 10, 11.1)	(1.3, 2, 2.7)	(0.0, 0.0, 0.0)	(10.2, 12, 13.8)
Demand	(6.2, 7, 7.8)	(8.9, 10, 11.1)	(6.5, 8, 9.5)	(7.8, 9, 10.2)	

Step 5 Calculate minimal fuzzy total distribution cost by using equation (1) is obtained $\min \tilde{\mathrm{T}} = (241.98, 352, 433.36)$. It can be represented as follows,

1 the least amount of minimal fuzzy total distribution cost is 241.98 unit

2 the most possible amount of minimal fuzzy total distribution cost is 352 unit

3 the greatest amount of minimal fuzzy distribution cost is 433.36 unit.

Next, we test the optimality of fuzzy IBFS obtained by Algorithm 2 via fuzzy MODI (Algorithm 3).

Step 1 In order to calculate the values of fuzzy variable $\tilde{v}_{i,b}(i = 1, 2, 3)$ and $\tilde{\nu}_{j,b}(j = 1, 2, 3, 4)$. we freely choose $\tilde{\nu}_{4,b} = 0$ to simplify calculations and using successively the relation $\tilde{c}_{ij,b} = \tilde{u}_{i,b} + \tilde{v}_{j,b}$ for filled cells as shown as follows:

$$\tilde{u}_{3,b} + \tilde{v}_{4,b} = 15 \Longrightarrow \tilde{u}_{3,b} = 15$$
$$\tilde{u}_{2,b} + \tilde{v}_{4,b} = 8 \Longrightarrow \tilde{u}_{2,b} = 8$$
$$\tilde{u}_{1,b} + \tilde{v}_{4,b} = 20 \Longrightarrow \tilde{u}_{1,b} = 20$$
$$\tilde{u}_{1,b} + \tilde{v}_{3,b} = 10 \Longrightarrow \tilde{v}_{3,b} = -10$$
$$\tilde{u}_{2,b} + \tilde{v}_{4,b} = 12 \Longrightarrow \tilde{v}_{2,b} = -3$$
$$\tilde{u}_{1,b} + \tilde{v}_{1,b} = 10 \Longrightarrow \tilde{v}_{1,b} = -10$$

Step 2 Compute the value of $\tilde{\lambda}_{i,j,b}$ for each unfilled cell by using the formula $\tilde{\lambda}_{i,j,b} = \tilde{c}_{ij,b} - (\tilde{u}_{i,b} + \tilde{v}_{j,b})$ is shown below:

$$\tilde{\lambda}_{1,2,b} = \tilde{c}_{12,b} - (\tilde{u}_{1,b} + \tilde{v}_{2,b}) = (22 - (20 + (-3)) = 5$$
$$\tilde{\lambda}_{1,4,b} = \tilde{c}_{14,b} - (\tilde{u}_{1,b} + \tilde{v}_{4,b}) = (20 - (20 + 0) = 0$$
$$\tilde{\lambda}_{2,1,b} = \tilde{c}_{21,b} - (\tilde{u}_{2,b} + \tilde{v}_{1,b}) = (15 - ((-10) + 8) = 17$$
$$\tilde{\lambda}_{2,2,b} = \tilde{c}_{22,b} - (\tilde{u}_{2,b} + \tilde{v}_{2,b}) = (20 - (8 + (-3)) = 15$$
$$\tilde{\lambda}_{3,1,b} = \tilde{c}_{31,b} - (\tilde{u}_{3,b} + \tilde{v}_{1,b}) = (20 - ((-10) + 18) = 15$$
$$\tilde{\lambda}_{4,4,b} = \tilde{c}_{44,b} - (\tilde{u}_{4,b} + \tilde{v}_{4,b}) = (15 - (15 + 0) = 0$$

Step 3 Obviously, based on Step 2 produced $\tilde{\lambda}_{i,j,b} \geq 0$. In other words, $\tilde{\lambda}_{i,j,b}$ is a positive fuzzy number, then the current fuzzy IBFS in Table 3 is an optimal solution.

We also solved it by using the existing algorithms (Kaur and Kumar, 2011; Kumar et al., 2011; Kumar and Singh, 2012; Ezzati et al., 2015; Chakraborty et al., 2016; Ebrahimnejad, 2017). The comparison results of minimal fuzzy total distribution cost and fuzzy optimal solution between existing algorithms and new proposed algorithm is shown in Figures 2 and 3, respectively.

Based Figures 2 and 3 show that the existing method proposed by Kaur and Kumar (2011) to produce two fuzzy optimal solutions such that produces two minimal fuzzy total transportation costs. This is caused the existing method uses a ranking function to rank fuzzy number where is the ranking value of triangular fuzzy number, i.e., $\tilde{c}_{1,3}$ = (\$8, \$10, \$10.6) = 9.65 has equal ranking to the ranking value of triangular fuzzy number, i.e., $\tilde{c}_{3,3}$ = (\$7.88, \$10, \$10.8) = 9.65. Moreover, two minimal fuzzy total distribution costs produced by Kaur and Kumar (2011) more than the minimal fuzzy total distribution cost is produced by the proposed algorithm. Meanwhile, the fuzzy optimal solution and minimal fuzzy total distribution cost are produced by Kumar et al. (2011) is equal to the proposed algorithm. However, Kumar et al. (2011) use the FLP technique and ranking function without considers conditions in real life.

Figure 2 The result comparison of fuzzy optimal solution (see online version for colours)

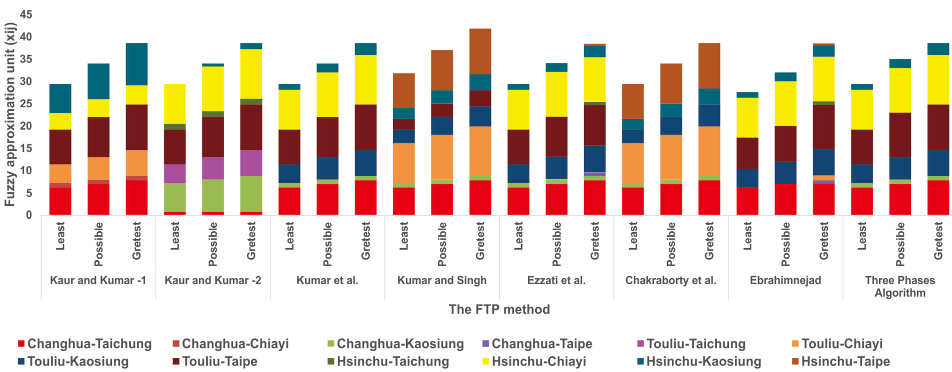
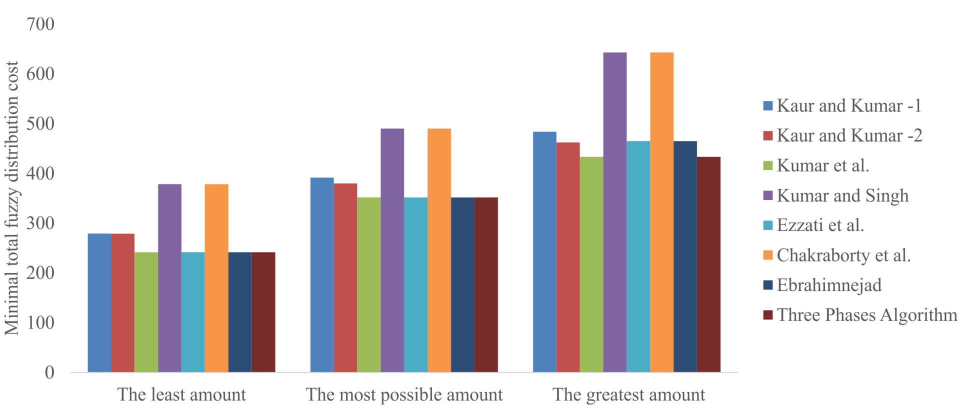


Figure 3 The result comparison of minimum fuzzy distribution cost (see online version for colours)



The least amount, most possible amount and greatest from minimal fuzzy total distribution cost are produced by Kumar and Singh (2012) less than to the new proposed algorithm is shown in Figures 2 and 3. Kumar and Singh (2012) uses the FLP technique and ranking function based on parametric value without considers external factors in solving FFTP such that let's appear equal ranking value. Meanwhile, the fuzzy optimal solution and minimal fuzzy total distribution cost produced by Chakraborty et al. (2016) are equal to Kumar and Singh (2012). The difference is Chakraborty et al. (2016) proposes a modification of the classical fuzzy transportation algorithm on operation fuzzy numbers.

Figures 2 and 3 also show that Ezzati et al. (2015) produces the amounts of least and most possible minimal fuzzy total distribution cost are equal to the proposed algorithm, meanwhile the greatest of minimal fuzzy total distribution cost more than to the proposed algorithm. Ezzati's paper claims that the result of minimal fuzzy total distribution cost more effective than the proposed algorithm. However, the solving FFTP of Example 2, Ezzati's method does not consider an external factors in real life which used the definition of ranking to rank the variables of FFTP and used FLP technique based on lexicography method with Multi objective linear programming. Whereas, Ebrahimnejad (2017) produces the amounts of least, most possible and greatest from minimal fuzzy total distribution cost are equal to Ezzati's paper. The difference is Ebrahimnejad's paper proposes the FLP technique based on the lexicography method without ranking fuzzy number.

7 Conclusions

The determination of fuzzy IBFS is an important part of FFTP to obtain a fuzzy optimal solution which is the minimal fuzzy total distribution cost. Three phases algorithm is proposed algorithm which consist of three stages for solving FFTP. Another unique advancement of the proposed algorithm is the capability to solve FFTP that has equal values of the minimal fuzzy distribution cost. We present capability of the proposed algorithm which produce minimum total distribution cost better than the existing algorithms (Kaur and Kumar, 2011; Kumar et al., 2011; Kumar and Singh, 2012; Ezzati et al., 2015; Chakraborty et al., 2016; Ebrahimnejad, 2017).

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