



International Journal of Management Science and Engineering Management

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tmse20

Improved Segregated Advancement (I-SA): a new method for solving full triangular fuzzy transportation problems

Muhammad Sam'An, Yosza Dasril, Chandrasekar Ramasamy, Nazarudin Bujang & Yahya Nur Ifriza

To cite this article: Muhammad Sam'An, Yosza Dasril, Chandrasekar Ramasamy, Nazarudin Bujang & Yahya Nur Ifriza (2023) Improved Segregated Advancement (I-SA): a new method for solving full triangular fuzzy transportation problems, International Journal of Management Science and Engineering Management, 18:1, 65-75, DOI: <u>10.1080/17509653.2022.2118885</u>

To link to this article: <u>https://doi.org/10.1080/17509653.2022.2118885</u>



Published online: 25 Sep 2022.

(Ø,

Submit your article to this journal 🗹

Article views: 44



View related articles 🗹

🌔 View Crossmark data 🗹



Improved Segregated Advancement (I-SA): a new method for solving full triangular fuzzy transportation problems

Muhammad Sam'An D^a, Yosza Dasril^b, Chandrasekar Ramasamy^b, Nazarudin Bujang^b and Yahya Nur Ifriza^c

^aDepartement of Informatics, Universitas Muhammadiyah Semarang, Semarang, Indonesia; ^bFaculty of Technology Management and Business, Universiti Tun Hussein Onn Malaysia (UTHM), Batu Pahat, Malaysia; ^cDepartment of Computer Science, Universitas Negeri Semarang, Gunungpati, Semarang, Indonesia

ABSTRACT

In this paper, triangular fuzzy numbers (TFN) are used to represent the uncertainty of data in the transportation problems (TP), which are referred to as fuzzy transportation problems (FTP). The main issues in the FTP are the lack of information, error of the basic trading rules, incomplete fuzzy data, and the limitations of the existing fuzzy-ranking functions which is failed to compare two TFN. Segregated advancement (SA) is a separation approach where the TFN represented by low, middle, and upper are solved part by part. The flaw of SA is that it uses the classical algorithms which are NWC, LCM, and VAM. Therefore, we proposed the improvement based on the combination of total ratio cost matrix and total difference method by column without using classical ranking functions. The first and second examples illustrate the existing methods without using the SA approach. The results show that the proposed method obtained the optimal solution of FTP, whereas the existing methods along with the SA approach. The results show that the proposed method is capable in solving the third example whereas the existing approach failed to solve the said example.

ARTICLE HISTORY

Received 8 June 2021 Accepted 18 August 2022

KEYWORDS

Full triangular fuzzy transportation problems; segregated method; total ratio cost matrix; total difference method; optimal solution

1. Introduction

The globalization era has caused intense global market competition. Companies must have good systematic planning and accuracy in making decision systems, especially in the commodity distribution process. According to Hitchcock (1941), the optimum solution of transportation problems (TP) is modeled by linear programming (LP) to achieve the most cost-effective transportation routes and network. TP deals with the distribution of the commodities and sources from suppliers and demands from customers, where ideally a balanced state reflects total supplies equal to the total demands.

In the real-life situation, the TP parameters (supply, demand, and transportation cost) sometimes have uncertain values. The transportation cost depends on fuel prices, congested routes, and weather, while supply may be interrupted due to reduced quantities of raw materials, machine breakdowns, and production failures. In addition, volatile market situations create uncertainty in demand. Therefore, Zadeh (1965) introduced TP with the numbers of supply, demand, and transportation cost represented by fuzzy number, which is called Fuzzy transportation problem (FTP). Many studies use triangular fuzzy number to represent TP parameters because it is easy and simple to make estimates, especially in FTP. Kundu, Kar, and Manoranjan (2013) introduced multiobjective multi-items solid FTP with fuzzification method based on fuzzy LP and proposed fixed charge type-2 FTP critical value (CV)-based reduction approach.

This paper divides the methodology in solving of FTP into three approaches. In the first approach, the determination of the fuzzy optimal solution of FTP is done without the defuzzification process. Lotfi, Allahviranloo, Alimardani, and Alizadeh (2009) proposed the lexicography method to find a symmetric triangular fuzzy approximate solution of balanced FTP. Hatami-Marbini, Agrell, Tavana, and Emrouznejad (2013) developed a stepwise fuzzy LP model based on the fuzzy constraint relation between the value of possibility and necessity unconsidering fuzzy objective function. Ezzati, Khorram, and Enayati (2015) stated that the lexicographic approach proposed by Lotfi et al. (2009) is not reliable as the optimal solution results are inexact. They proposed the conversion of the fuzzy LP model into multiobjective LP (MOLP) model based on lexicographic method. In the same year, this model was denied by Bhardwaj and Kumar (2015) through two numerical examples given regarding a fuzzy LP with inequality constraints to be transformed into fuzzy LP with equality constraints.

Ebrahimnejad (2015) presented a two-step method, i.e. converting FTP with distribution cost under fuzzy data into LP model by using standard fuzzy operation and transforming the LP model into three crisp bounded TP by using decomposition technique. Ebrahimnejad (2016a) also divided the FTP under left and right (LR) fuzzy numbers into four crisp LP models and solved it by using the simplex method. Similarly, Ebrahimnejad (2017) divided the FTP under triangular fuzzy number. Ebrahimnejad and Verdegay introduced a new approach for solving intuitionistic FTP. Srivastava and Bisht (2020) proposed a segregated scheme method without ranking function with the minimum demand-supply method and stepping stone technique.

The second approach is to determine the fuzzy optimal solution of FTP based on Initial Basic feasible solution (IBFS)

under fuzzy data while considering the fuzzification process. Kumar and Kaur (2011a) used Yanger's ranking index for the defuzzification process, the classical transportation methods i.e. North-West corner (NWC), least-cost method (LCM), or Vogel's Approximation Method (VAM) to find IBFS and Modified Distribution (MODI) or Stepping Stone to determine optimal solutions of FFTPs under LR fuzzy data. Kumar and Kaur (2011b) also used Jai Mata (Mehar) Di or JMD ranking, the classical transportation methods, and MODI method to solve FFTPs under trapezoidal fuzzy number. Kumar, Kaur, and Singh (2011) presented ranking function and fully fuzzy LP model to obtain optimal solution under triangular fuzzy numbers. Kaur and Kumar (2011c), Kumar & Singh (2012) introduced generalized ranking function, classical transportation methods i.e. G-NWC, G-LCM, G-VAM, and MODI method to solve of FTP under fuzzy data. Kumar and Murugesan (2012) modified revised simplex method under fuzzy parametric data. Saberi Najafi and Edalatpanah (2013) corrected the calculation of Kumar et. al. method in determining non-negative fuzzy approximation solution. Ebrahimnejad (2018) simplified the generalized ranking function proposed by Kaur and Kumar (2011c, 2012) based on Liou and Wang's ranking and applied the G-LCM of classical transportation method and G-MODI to solve FTP under trapezoidal fuzzy data. Rani and Gulati (2014) used ranking function, VAM, and MODI method to solve unbalanced FTP under trapezoidal fuzzy numbers. Chandran and Kandaswamy (2016) proposed ranking scores method, modified LCM, and MODI to solve FTP under fuzzy data. Ebrahimnejad (2016a) corrected the calculation of Chandran and Kandaswamy's method in obtaining nonnegative fuzzy approximation solution.

Chakraborty, Jana, and Roy (2016) introduced generalized Hukuhara differences ranking functions and applied those to classical transportation algorithm and MODI in determining triangular fuzzy optimal solution of FTP. Mathur, Srivastava, and Paul (2016) discussed ranking method and minimum demand–supply method under trapezoidal fuzzy number. Rani and Gulati (2017) presented preference index-based integral value and VAM based on generalized LR fuzzy numbers. Hunwisai and Kumam (2017) employed a robust ranking function to rank trapezoidal fuzzy numbers, then allocation table method (ATM) to find IBFS, and MODI method to obtain optimal solution of FTP.

Sam'an, Farikhin, Hariyanto, and Surarso (2018a) discussed the total integral ranking-based interpolation and LCM under trapezoidal fuzzy numbers. Saini, Sangal, and Prakash (2018) employed generalized integral ranking under triangular-trapezoidal fuzzy numbers that is implemented with minimum row-column method. Sam'an, Farikhin, Surarso, and Zaki (2018b) presented a ranking score method, then modified LCM-based Simple Additive Weighting (SAW) and MODI under trapezoidal fuzzy numbers. Kumar, Edalatpanah, Jha, and Singh (2019) introduced a new ranking function based on Pythagorean fuzzy numbers and VAM. Farikhin, Sam'an, Surarso, and Bambang (2019) proposed a new generalized total integral value-based LR membership function of triangular fuzzy numbers, then applied the modification in the LCM based on SAW.

Bisht and Srivastava (2020) introduced trisectional fuzzification approach to create a trapezoidal fuzzy numbers and the ranking technique based on concept of the triangle incircle center and applied to classical transportation algorithm and MODI in solving FTP. Mathur and Srivastava (2020) used classical ranking function and minimum demand-supply method under generalized trapezoidal fuzzy numbers. Muthuperumal, Titus, and Venkatachalapathy (2020) solved unbalanced FTP under triangular fuzzy numbers without adding a dummy to supply and demand and also used a classical ranking function. Sam'an, an, and Farikhin (2021) proposed a novel on total integral value based on inversed ranking function, then applied it to the general classical transportation algorithm and generalized MODI method.

The third approach is the direct approach in solving FTP such as the zero-point method to obtain optimal solution under trapezoidal fuzzy numbers was introduced by Pandian and Natarajan (2010). Basirzadeh (2011) used the concept of measuring the trapezoidal fuzzy numbers based on the average area of right and left sides of trapezoidal in converting fuzzy parameters to crisp, then applied the zero-point algorithm to determine the optimal solution FTP. Fegade (2012) applied robust ranking to rank the triangular fuzzy numbers and zero suffix method to obtain optimal solution of FTPs. Fuzzy dual matrix to obtain the optimal solution of FTP was proposed by Edward Samuel and Venkatachalapathy (2012); Selvakumari and Sathya Geetha (2020). The improvement of the zero-point method was proposed Edward Samuel (2012); Edward Samuel and Venkatachalapathy (2013, 2014), Karthy and Ganesan (2016) revised by Karthy and Ganesan (2019). Zero point modified was proposed by Akilbasha, Natarajan, and Pandian (2016) and also a mid-width method for solving interval FTP in pharmaceutical logistics was proposed by Akilbasha, et al. (2018). Particle Swarm Optimization algorithm (PSO) with fuzzy constraint and conjugate constraint was proposed by Baykasoglu and Subulan (2019). A multiobjective nonlinear transportation problem based on neutrosophic compromise programming approach with intuitionistic fuzzy parameter (Firoz and Adhami 2019). A MOLP-based Phytagorean based Interactive Pythagoreanhesitant fuzzy parameter (Adhami & Ahmad, 2020).

The aforementioned studies illustrate that standard fuzzy operations and ranking functions in the defuzzification process are crucial to determine the optimal solution under fuzzy environment on FTP. Standard fuzzy operations are widely used to rank fuzzy numbers and to determine the non-negative fuzzy approximate solution of FTP. However, as expressed by Srivastava and Bisht (2020), fuzzy approximation solution and minimum fuzzy optimal solution obtained by Kumar and Kaur (2011a,c) have negative fuzzy numbers, which is contradictory to the fuzzy constrains on the model of FTP.

The first issue was appointed by Sam'an et al. (2021), who stated that the classical ranking function proposed by Kumar et al. (2011); Ebrahimnejad and Verdegay () failed to convert two equal fuzzy numbers. The ranking function also ignores the fuzziness of fuzzy numbers; for instances, fuzzy inputoutput is disconnected FTP's in a real-life problem. The second issue is with the method of Ezzati et al. (2015), where the total allocations of the non-negative initial fuzzy approximation solution did not achieve the non-degeneracy criterion, which is m + n - 1 solutions. The shortcoming of Ebrahimnejad (2015) method is that the utilization of the simplex method requires a long process and is not friendly to numerical computations.

Therefore, the main contribution of this paper is to overcome the issues in standard fuzzy operation and ranking function generated by improving the SA based on the total ratio cost matrix and total difference method by column, which is improved segregated advancement (I-SA).

2. Triangular fuzzy transportation problem formulation

Table 1 represents all the FTP parameters, where the first column shows the destination represented by the vector D_j , (j = 1, ..., n) and the first row shows the source represented by the vector $S_i(i = 1, ..., m)$. The matrices \tilde{c}_{ij} and \tilde{x}_{ij} , (i = 1, ..., m; j = 1, ..., n) represent the cost for each cell and the number of commodities to be transported from source S_i to destination D_j , respectively. Meanwhile, vectors \tilde{d}_j and \tilde{s}_i , (i = 1, ..., m; j = 1, ..., n) represent demand and supply, respectively. The TP is balanced if $(\tilde{d}_j = \tilde{s}_i)$ otherwise it is unbalanced.

The FTP can be written as

$$\min \tilde{T} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}$$
(1)

subject to

$$\sum_{i=1}^{n} \tilde{x}_{ij} \le \tilde{s}_i \tag{2}$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \le \tilde{d}_j \tag{3}$$

and

$$\tilde{x}_{ij} \ge 0$$
 $i, j \sim (i = 1, 2, \dots, m; j = 1, 2, \dots, n).$ (4)

where m represents total supply, n represents total demand, $\tilde{s}_i = (s_i^a, s_i^b, s_i^c)$ is i^{th} fuzzy supply, $\tilde{d}_j = (d_j^a, d_j^b, d_j^c)$ is j^{th} fuzzy demand, $\tilde{c}_{ij} = (c_{ij}^a, c_{ij}^b, c_{ij}^c)$ is fuzzy distribution cost from i^{th} fuzzy supply to j^{th} fuzzy demand, $\tilde{x}_{ij} = (x_{ij}^a, x_{ij}^b, x_{ij}^c)$ is the number of fuzzy approximation unit to assign from i^{th} fuzzy supply to j^{th} fuzzy demand, min T is minimized total

Table 1	 The fully 	triangular	fuzzy	transportation	problem	table
				Dectination		

	Destination										
Source	C) 1	D	2		D) _q		D	n	<i>š</i> i
S ₁	<i>č</i> ₁₁		<i>c</i> ₁₂			\tilde{c}_{1q}			ĩc₁n		ŝ ₁
		<i>x</i> ₁₁		<i>x</i> ₁₂			\tilde{x}_{1q}			\tilde{x}_{1n}	
S ₂	<i>c</i> ₂₁		<i>c</i> ₂₂			\tilde{c}_{2q}		• • •	ĩc₂n		ŝ ₂
		<i>x</i> ₂₁		<i>X</i> ₂₂			<i>х</i> _{2q}			χ _{2n}	
	÷		÷			÷		• • •	÷		÷
S_p	\tilde{c}_{p1}		\tilde{c}_{p2}			\tilde{c}_{pq}			<i>c̃</i> pn		ĩ₅p
		х _{р1}		х _{р2}			х _{рq}			х _{рп}	
	÷		÷		• • •	÷		• • •	÷		÷
S _m	č _{m1}		č _{m₂}			<i>c̃mq</i>			<i>c̃</i> mn		ĩs _m
		<i>x̃</i> m₁		ĩx _{m2}			\widetilde{x}_{mq}			\tilde{x}_{mn}	
<i>d</i> j	∂1		∂̃2			\tilde{d}_q			\tilde{d}_n		

fuzzy distribution cost and the fuzzy number is represented by the fuzzy triangular number.

3. The new proposed method for solving FTP

In this section, the I-SA (Improved Segregated Advancement) method is proposed to determine the fuzzy optimal solution of FTP that is represented by triangular fuzzy numbers. The I-SA consists of a segregated advancement. It is a separation approach where the triangular fuzzy numbers represented by low, middle, and upper are solved part by part. Subsequently, IBFS is determined by using the combination of total ratio cost and total difference method, and MODI is used to obtain fuzzy optimal solution based on IBFS. The flowchart for this proposed model shown in Figure 1.

3.1. A segregated scheme

The segregated method is based on the possibility of optimized output concept of balanced FTP, when TP parameters under triangular fuzzy numbers with the corresponding demand and supply are partitioned part by part. This scheme consists of point-wise segregation of each triangular fuzzy parameters such that the first element of each triangular fuzzy parameters is defined first as a segregated transportation problem and denoted by SA_1 . Similarly, the second and third elements of each triangular fuzzy parameters are denoted as SA_2 and SA_3 , respectively.

By using SA method, Equations (1) and (2) can be transformed into three SA_t , t = 1, 2, 3 as follows.

(a) SA₁

$$\min Z_1^a = \sum_{i=1}^m \sum_{j=1}^n c_{ij}^a x_{ij}^a$$
(5)

$$\sum_{j=1}^{n} x_{ij}^{a} \leq s_{i}^{a} \quad i = 1, 2, \dots, m,$$

$$\sum_{i=1}^{m} x_{ij}^{a} \leq d_{j}^{a} \quad j = i = 1, 2, \dots, n$$

$$x_{ij}^{a} \geq 0 \qquad i, j$$
(6)

(b) SA_2

$$\min Z_2^b = \sum_{i=1}^m \sum_{j=1}^n c_{ij}^b x_{ij}^b$$
(7)

$$\sum_{j=1}^{n} x_{ij}^{b} \leq s_{i}^{b} \quad i = 1, 2, \dots, m,$$

$$\sum_{i=1}^{m} x_{ij}^{b} \leq d_{j}^{b} \quad j = i = 1, 2, \dots, n \quad (8)$$

$$x_{ij}^{b} \geq 0 \qquad i, j$$

(c) SA_3

$$\min Z_3^c = \sum_{i=1}^m \sum_{j=1}^n c_{ij}^c x_{ij}^c$$
(9)

$$\sum_{i=1}^{n} x_{ij}^{c} \leq s_{i}^{c} \quad i = 1, 2, \dots, m,$$

$$\sum_{i=1}^{m} x_{ij}^{c} \leq d_{j}^{c} \quad j = i = 1, 2, \dots, n \quad (10)$$

$$x_{ij}^{c} \geq 0 \qquad i, j$$

3.2. The combination of total ratio cost and total difference method by column

In order to obtain a balanced of TP with the corresponding distribution cost c_{ij}^r , where r = a, b, c the transportation cost



Figure 1. Flowchart of this proposed model.

matrix of order (m,n) having equal supply s_i^r , i = 1, 2, ..., m; and demand d_i^r , j = 1, 2, ..., n.

The combination of total ratio cost and total difference method (TDM) by column is obtained by calculating Total Ratio Cost Matrix (TRCM). As for TRCM, the process is calculation of row ratio matrix (α_{ij}^r) and column ratio matrix (β_{ij}^r) by using Equations (11) and (12), respectively. Then, the sum of α_{ij}^t and β_{ij}^t is determined.

$$\alpha_{ij}^r = \frac{c_{ij}^r}{\theta_i^r}, i = 1, 2, \dots, m; j = 1, 2, \dots, n; r = a, b, c.$$
 (11)

$$\beta_{ij}^{r} = \frac{c_{ij}}{\theta_{j}^{r}}, i = 1, 2, \dots, m; j = 1, 2, \dots, n; ; r = a, b, c.$$
(12)

where $\theta_i^r = \min(c_{ij}^r) = (c_{i1}^r, c_{i2}^r, \dots, c_{in}^r)$ and $\theta_j^r = \min(c_{ij}^r) = (c_{1j}^r, c_{2j}^r, \dots, c_{mj}^r)$

The combination of TRCM and TDM by column in detail is shown in Algorithm 1.

3.3. Modified distribution method

The MODI is used to obtain the optimal solution of balanced FTP stated in the following Algorithm 2.

3.4. I-SA to optimized balanced FTP

The treatment of balanced FTP under triangular fuzzy number. The proposed algorithm in detail is given in the following Algorithm 3.

4. Numerical simulations

This section shows the validity of the new proposed method for solving FTP. This paper provided three FTP, of which two were taken from reputable journals and the other was created to illustrate the capability of the proposed method. The first FTP, Example 4.1, was taken from Ezzati et al. (2015) regarding the case study of a leading beverage company, Dali, in Taiwan. The second FTP, Example 4.2, was adapted from Ebrahimnejad (2015). It is related to the optiAlgorithm 1: The combination of TRCM and TDM by column

Data: Initialization: Number of rows is m, number of column is n, supply (s_i^r) , demand $(d^{r}_{i}),$ distribution cost $(c^{r}_{ij}),$ the number of approximation unit (x_{ij}^t) Result: min T by Eq. (1) Calculate α_{ij}^r by Eq. (11) and β_{ij}^r by Eq. (12); Calculate the TRCM which is the entries are the sum of the row and column ratio matrix; TRCM is denoted by ω_{ij}^r .; repeat {Produce a IBFS}; for i=1 to n do Find the penalty (F_j^r) for each j^{th} column by $F_j^r = \sum_{i=1}^m (\omega_{ij}^r - \min(\omega_{ij}^r));$ Select highest of F_j^r (HP) by HP = max (F_j^r) ; In case of a break-even (i.e. equal HP); (a) select HP with the smallest ω_{ij}^r ; (b) if (a) is equal, then select F_j^r with the greatest total of TRCM by $T\omega_j^r = \sum_{i=1}^m \omega_{ij}^r;$ (c) if (b) is equal, then select penalty with the max allocation of x_{ij}^r . Select the least ω_{ij}^r of HP. If tie, then select least ω_{ij}^r with max x_{ij}^r ; Allocate the x_{ij}^r to it; There may arise the following three cases; $\begin{array}{l} \mathbf{if} \ \min(s_i^r, d_j^r) = s_i^r \ \mathbf{then} \\ \mid \ x_{ij}^r = s_i^r, \ d_j^r = d_j^r - s_i^r, \ s_i^r = 0, \ \mathrm{cross} \ \mathrm{out} \ \mathrm{of} \ s_i^r \end{array}$ \mathbf{end} $\begin{array}{l} \mathbf{if} \ \min(s_i^r, d_j^r) = d_j^r \ \mathbf{then} \\ \big| \ x_{ij}^r = d_j^r, \ s_i^r = s_i^t - d_j^r, \ d_j^r = 0, \ \text{cross out of} \ d_j^r \end{array}$ end $\begin{array}{l} \mathbf{if} \ s_i^r = d_j^r \ \mathbf{then} \\ \mid \ s_i^r = 0, \ d_j^r = 0, \ \mathrm{cross} \ \mathrm{out} \ \mathrm{of} \ s_i^r \ \mathrm{and} \ d_j^r \end{array}$ \mathbf{end} end Recalculate the penalty without considering the cross out rows and columns until $\{\sum_{i=1}^m s_i^r = \sum_{j=1}^n d_j^r\};$

Algorithm 2: Modified-Distribution Method

 Step 1: The IBFS obtained of FTP by using Algorithm 1 Step 2: Introduce u^r_i and v^r_j as variable convenient for every ith and jth, respectively. In front of ith write u^r_i in row and at v^r_j the under of jth in column. Let u^r_i = 0 is maximum number of allocations row; Step 3: Determine λ^r_{i,j} and v^r_j by using c^t_{i,j} = u^r_i + v^r_j for base of cell, then determine λ^r_{i,j} = c^r_{i,j} - (u^r_i + v^r_j) of non-base of cells. Next, two possibilities as follows. (a) If λ^r_{i,j} ≥ 0, ∀i, j, then the result of IBFS is obtained. In other words, an optimal solution has been satisfied; (b) Otherwise, ∃λ^r_{i,j}, then the result of IBFS is not obtained yet. In other words, the fuzzy optimal solution is not optimal. Therefore, an optimal solution is chosen a cell of (i, j)th in which λ^r_{i,j} is the smallest negative. Next, make a horizontal and vertical closed path that starts from an unchosen base of cell of
 Step 2: Introduce u^r_i and v^r_j as variable convenient for every ith and jth, respectively. In front of ith write u^r_i in row and at v^r_j the under of jth in column. Let u^r_i = 0 is maximum number of allocations row; Step 3: Determine λ^r_{i,j} and v^r_j by using c^t_{ij} = u^r_i + v^r_j for base of cell, then determine λ^r_{i,j} = c^r_{ij} - (u^r_i + v^r_j) of non-base of cells. Next, two possibilities as follows. (a) If λ^r_{i,j} ≥ 0, ∀i, j, then the result of IBFS is obtained. In other words, an optimal solution has been satisfied; (b) Otherwise, ∃λ^r_{i,j}, then the result of IBFS is not obtained yet. In other words, the fuzzy optimal solution is not optimal. Therefore, an optimal solution is chosen a cell of (i, j)th in which λ^r_{i,j} is the smallest negative. Next, make a horizontal and vertical closed path that starts from an unchosen base of cell of
 respectively. In front of ith write u_i^r in row and at v_j^r the under of jth in column. Let u_i^r = 0 is maximum number of allocations row; Step 3: Determine λ_{i,j}^r and v_j^r by using c_{ij}^t = u_i^r + v_j^r for base of cell, then determine λ_{i,j}^r = c_{ij}^r - (u_i^r + v_j^r) of non-base of cells. Next, two possibilities as follows. (a) If λ_{i,j}^r ≥ 0, ∀i, j, then the result of IBFS is obtained. In other words, an optimal solution has been satisfied; (b) Otherwise, ∃λ_{i,j}^r, then the result of IBFS is not obtained yet. In other words, the fuzzy optimal solution is not optimal. Therefore, an optimal solution is chosen a cell of (i, j)th in which λ_{i,j}^r is the smallest negative. Next, make a horizontal and vertical closed path that starts from an unchosen base of cell of
 column. Let u_i^r = 0 is maximum number of allocations row; Step 3: Determine λ_{i,j}^r and v_j^r by using c_{ij}^t = u_i^r + v_j^r for base of cell, then determine λ_{i,j}^r = c_{ij}^r - (u_i^r + v_j^r) of non-base of cells. Next, two possibilities as follows. (a) If λ_{i,j}^r ≥ 0, ∀i, j, then the result of IBFS is obtained. In other words, an optimal solution has been satisfied; (b) Otherwise, ∃λ_{i,j}^r, then the result of IBFS is not obtained yet. In other words, the fuzzy optimal solution is not optimal. Therefore, an optimal solution is chosen a cell of (i, j)th in which λ_{i,j}^r is the smallest negative. Next, make a horizontal and vertical closed path that starts from an unchosen base of cell of
 Step 3: Determine λ^r_{i,j} and v^r_j by using c^t_{ij} = u^r_i + v^r_j for base of cell, then determine λ^r_{i,j} = c^r_{ij} - (u^r_i + v^r_j) of non-base of cells. Next, two possibilities as follows. (a) If λ^r_{i,j} ≥ 0, ∀i, j, then the result of IBFS is obtained. In other words, an optimal solution has been satisfied; (b) Otherwise, ∃λ^r_{i,j}, then the result of IBFS is not obtained yet. In other words, the fuzzy optimal solution is not optimal. Therefore, an optimal solution is chosen a cell of (i, j)th in which λ^r_{i,j} is the smallest negative. Next, make a horizontal and vertical closed path that starts from an unchosen base of cell of
 determine λ^r_{i,j} = č^r_{i,j} - (u^r_i + v^r_j) of non-base of cells. Next, two possibilities as follows. (a) If λ^r_{i,j} ≥ 0, ∀i, j, then the result of IBFS is obtained. In other words, an optimal solution has been satisfied; (b) Otherwise, ∃λ^r_{i,j}, then the result of IBFS is not obtained yet. In other words, the fuzzy optimal solution is not optimal. Therefore, an optimal solution is chosen a cell of (i, j)th in which λ^r_{i,j} is the smallest negative. Next, make a horizontal and vertical closed path that starts from an unchosen base of cell of
 follows. (a) If λ^r_{i,j} ≥ 0, ∀i, j, then the result of IBFS is obtained. In other words, an optimal solution has been satisfied; (b) Otherwise, ∃λ^r_{i,j}, then the result of IBFS is not obtained yet. In other words, the fuzzy optimal solution is not optimal. Therefore, an optimal solution is chosen a cell of (i, j)th in which λ^r_{i,j} is the smallest negative. Next, make a horizontal and vertical closed path that starts from an unchosen base of cell of
 (a) If λ_{i,j}^r ≥ 0, ∀i, j, then the result of IBFS is obtained. In other words, an optimal solution has been satisfied; (b) Otherwise, ∃λ_{i,j}^r, then the result of IBFS is not obtained yet. In other words, the fuzzy optimal solution is not optimal. Therefore, an optimal solution is chosen a cell of (i, j)th in which λ_{i,j}^r is the smallest negative. Next, make a horizontal and vertical closed path that starts from an unchosen base of cell of
 solution has been satisfied; (b) Otherwise, ∃λ^r_{i,j}, then the result of IBFS is not obtained yet. In other words, the fuzzy optimal solution is not optimal. Therefore, an optimal solution is chosen a cell of (i, j)th in which λ^r_{i,j} is the smallest negative. Next, make a horizontal and vertical closed path that starts from an unchosen base of cell of
(b) Otherwise, $\exists \lambda_{i,j}^r$, then the result of IBFS is not obtained yet. In other words, the fuzzy optimal solution is not optimal. Therefore, an optimal solution is chosen a cell of $(i, j)^{th}$ in which $\lambda_{i,j}^r$ is the smallest negative. Next, make a horizontal and vertical closed path that starts from an unchosen base of cell of
the fuzzy optimal solution is not optimal. Therefore, an optimal solution is chosen a cell of $(i, j)^{th}$ in which $\lambda_{i,j}^r$ is the smallest negative. Next, make a horizontal and vertical closed path that starts from an unchosen base of cell of
chosen a cell of $(i, j)^{th}$ in which $\lambda_{i,j}^r$ is the smallest negative. Next, make a horizontal and vertical closed path that starts from an unchosen base of cell of
horizontal and vertical closed path that starts from an unchosen base of cell of
$(i, j)^{th}$. The path can only replace to angle on base of cell $(i, j)^{th}$ and the path
is chosen must pass through base and non-base cell of $(i, j)^{th}$;
Step 4 : Give sign $(+)$ and $(-)$ for closed loop started with $(+)$ for chosen
non-base cells. After that, determine fuzzy quantity on cells with signs (+)
and $(-)$. Consequently, new TP table is obtained.
Step 5 : Repeat of steps 2, 3 and 4 for TP table until $\lambda^r > 0, \forall i, j$
Step 6 : Obtain a new improved solution by allocating units to the unfilled cell
according step 5 and calculate the new TP.
Step 7 : Determine the value of fuzzy optimal solution or objective function
min Z_t

Algorithm 3: I-SA method

- **Step 1** Imply the segregated scheme discussed in Section 3.1 to determine the corresponding three $SA_t, t = 1, 2, 3$.
- **Step 2** Imply the combination of TRCM and TDM by column in Algorithm 1 to each SA_t to find IBFS.
- **Step 3** Imply the MODI method in Algorithm 2 to each of the three SA_t to obtain the optimal solutions.
- Step 4 Determine a fuzzy optimal solution of balanced FTP as a combination of the optimal solutions obtained in Step 3.

 Table 2. Supply, demand, and the cost of transportation in terms of triangular

 fuzzy number for Example 4.1.

	Triangular fuzzy approximately v				
Parameter		Least (a)	Most (b)	Greatest(c)	
Transportation Costa	c ₁₁	8.0	10.0	10.8	
	<i>c</i> ₁₂	20.4	22.0	24.0	
	<i>c</i> ₁₃	8.0	10.0	10.6	
	C ₁₄	18.8	20.0	22.0	
	<i>c</i> ₂₁	14.0	15.0	16.0	
	<i>c</i> ₂₂	18.2	20.0	22.0	
	<i>c</i> ₂₃	10.0	12.0	13.0	
	C ₂₄	6.0	8.0	8.8	
	C ₃₁	18.4	20.0	21.0	
	<i>c</i> ₃₂	9.6	12.0	13.0	
	<i>c</i> ₃₃	7.8	10.0	10.8	
	C ₃₄	14.0	15.0	16.0	
Supply in dozens	s ₁	7.2	8.0	8.8	
	s ₂	12.0	14.0	16.0	
	S 3	10.2	12.0	13.8	
Demand in dozens	d_1	6.2	7.0	7.8	
	d_2	8.9	10.0	11.1	
	d ₃	6.2	7.0	7.8	
	d_4	7.8	9.0	10.2	
^a in dollar (\$)					

mization of product transportation costs at trading companies. The third FTP was created to counter the method given by Srivastava and Bisht (2020).

Example 4.1. A Dali company of Taiwan supplies soft drink from three sources i.e. Changhua (S_1) , Touliu (S_2) , and Hsinchu (S_3) of Taiwan to four destinations situated at Taichung (D_1) , Chiayi (D_2) , Kaohsiung (D_3) , and Taipei (D_4) . The main goal is to minimize the distribution cost. Table 2 summarizes the supply available from three sources, the demand from the four destination centers, and the unit distribution cost for each route by used the soft drink company. The environmental coefficients and related parameters are imprecise in real-life due to incompleteness or unavailable information.

Solution:

To obtain the solution for Example 4.1, Algorithm 3 was used.

Step 1 Implementing the Segregated scheme discussed in Section 3.1, the three crisp TP i.e. SA_1, SA_2, SA_3 were obtained as follows:

(a) for SA_1 by using Equations (5) and (6)

$$\min Z_1^a = 8x_{11}^a + 20.4x_{12}^a + 8x_{13}^a + 18.8x_{14}^a + 14x_{21}^a + 18.2x_{22}^a + 10x_{23}^a + 6x_{24}^a + 18.4x_{31}^a + 9.6x_{32}^a + 7.8x_{33}^a + 14x_{34}^a$$
(13)

subject to

$$\begin{aligned} x_{11}^{a} + x_{12}^{a} + x_{13}^{a} + x_{14}^{a} &= 7.2 \\ x_{21}^{a} + x_{22}^{a} + x_{23}^{a} + x_{24}^{a} &= 12.0 \\ x_{31}^{a} + x_{32}^{a} + x_{33}^{a} + x_{34}^{a} &= 10.2 \\ x_{11}^{a} + x_{21}^{a} + x_{31}^{a} &= 6.2 \\ x_{12}^{a} + x_{22}^{a} + x_{32}^{a} &= 8.9 \\ x_{13}^{a} + x_{23}^{a} + x_{33}^{a} &= 6.5 \\ x_{14}^{a} + x_{24}^{a} + x_{34}^{a} &= 7.8 \end{aligned}$$
(14)

(b) for SA_2 by using Equations (7) and (8)

$$\min Z_{2}^{b} = 10x_{11}^{b} + 22x_{12}^{b} + 10x_{13}^{b} + 20x_{14}^{b} + 15x_{21}^{b} + 20x_{22}^{b} + 12x_{23}^{b} + 8x_{24}^{b} + 20x_{31}^{b} + 12x_{32}^{b} + 10x_{33}^{b} + 15x_{34}^{b}$$
(15)

subject to

$$\begin{aligned} x_{11}^{\mu} + x_{12}^{b} + x_{13}^{b} + x_{14}^{b} &= 8.0 \\ x_{21}^{b} + x_{22}^{b} + x_{23}^{b} + x_{24}^{b} &= 14.0 \\ x_{31}^{b} + x_{32}^{b} + x_{33}^{b} + x_{34}^{b} &= 12.0 \\ x_{11}^{\mu} + x_{21}^{b} + x_{31}^{b} &= 7.0 \\ x_{12}^{\mu} + x_{22}^{b} + x_{32}^{b} &= 10.0 \\ x_{12}^{\mu} + x_{23}^{b} + x_{33}^{b} &= 8.0 \\ x_{14}^{\mu} + x_{24}^{b} + x_{34}^{b} &= 9.0 \end{aligned}$$
(16)

(c) for SA_3 by using Equations (9) and (10)

$$\min Z_{3}^{c} = 10.8x_{11}^{c} + 24x_{12}^{c} + 10.6x_{13}^{c} + 22x_{14}^{c} + 16x_{21}^{c} + 22x_{22}^{c} + 13x_{23}^{c} + 8.8x_{24}^{c} + 21x_{31}^{c} + 10.8x_{32}^{c} + 16x_{33}^{c} + 16x_{34}^{c}$$
(17)

subject to

$$\begin{aligned} x_{11}^{c} + x_{12}^{c} + x_{13}^{c} + x_{14}^{c} &= 8.8\\ x_{21}^{c} + x_{22}^{c} + x_{23}^{c} + x_{24}^{c} &= 16.0\\ x_{31}^{c} + x_{32}^{c} + x_{33}^{c} + x_{34}^{c} &= 13.8\\ x_{11}^{c} + x_{21}^{c} + x_{31}^{c} &= 7.8\\ x_{12}^{c} + x_{22}^{c} + x_{32}^{c} &= 11.1\\ x_{13}^{c} + x_{23}^{c} + x_{33}^{c} &= 9.5\\ x_{14}^{c} + x_{24}^{c} + x_{34}^{c} &= 10.2 \end{aligned}$$

$$(18)$$

Step 2 The combination of TRCM and TDM by column in Algorithm 1 was applied,

(a) For SA₁, α_{ij}^a , β_{ij}^a and ω_{ij}^a , *i*=1,2,3; ~ *j*=1,2,3,4 were obtained as follows.

$$\alpha_{ij}^{a} = \begin{pmatrix} 1.00 & 2.55 & 1.00 & 2.25 \\ 2.33 & 3.03 & 1.67 & 1.00 \\ 2.35 & 1.23 & 1.00 & 1.79 \end{pmatrix}$$

and $\beta_{ij}^{a} = \begin{pmatrix} 1.00 & 2.21 & 1.02 & 3.00 \\ 1.75 & 1.89 & 1.28 & 1.00 \\ 2.30 & 1.00 & 1.00 & 1.87 \\ 2.00 & 4.67 & 2.02 & 5.25 \\ 4.08 & 4.92 & 2.94 & 2.00 \\ 4.65 & 2.23 & 2.00 & 4.12 \end{pmatrix}$

Thus, the penalty values are $F_1 = 4.74$, $F_2 = 5.14$, $F_3 = 0.97$, and $F_4 = 5.37$, such that the IBFS obtained

$$egin{array}{rcl} x^a_{11} = 6.20 & x^a_{12} = 0.00 & x^a_{13} = 1.00 & x^a_{14} = 0.00 \ x^a_{21} = 0.00 & x^a_{22} = 0.00 & x^a_{23} = 4.20 & x^a_{24} = 7.80 \ x^a_{31} = 0.00 & x^a_{32} = 8.90 & x^a_{33} = 1.30 & x^a_{34} = 0.00 \end{array}$$

(b) For SA_2 , α_{ij}^b , β_{ij}^b and ω_{ij}^b , i=1,2,3; j=1,2,3,4 were obtained as follows.

$$\alpha_{ij}^{b} = \begin{pmatrix} 1.00 & 2.20 & 1.00 & 2.00 \\ 1.87 & 2.50 & 1.50 & 1.00 \\ 2.00 & 1.20 & 1.00 & 1.50 \end{pmatrix}$$

and $\beta_{ij}^{b} = \begin{pmatrix} 1.00 & 1.83 & 1.00 & 2.50 \\ 1.50 & 1.67 & 1.20 & 1.00 \\ 2.00 & 1.00 & 1.00 & 1.87 \end{pmatrix}$
$$\omega_{ij}^{b} = \begin{pmatrix} 2.00 & 4.03 & 2.00 & 4.50 \\ 3.37 & 4.16 & 2.70 & 2.00 \\ 4.00 & 2.20 & 2.00 & 3.37 \end{pmatrix}$$

Thus, the penalty values ares $F_1 = 3.37$, $F_2 = 3.80$, $F_3 = 0.70$ and $F_4 = 3.87$ such that the IBFS obtained

(c) For SA₃, α_{ij}^c , β_{ij}^c and ω_{ij}^c , *i*=1,2,3; *j* =1,2,3,4 are obtained as bellows.

$$\alpha_{ij}^{c} = \begin{pmatrix} 1.01 & 2.26 & 1.00 & 2.07 \\ 1.82 & 2.50 & 1.47 & 1.00 \\ 1.94 & 1.20 & 1.00 & 1.48 \end{pmatrix}$$

and $\beta_{ij}^{c} = \begin{pmatrix} 1.00 & 1.84 & 1.00 & 2.50 \\ 1.48 & 1.69 & 1.22 & 1.00 \\ 1.94 & 1.00 & 1.01 & 1.81 \end{pmatrix}$
$$\omega_{ij}^{3} = \begin{pmatrix} 2.01 & 4.11 & 2.00 & 4.57 \\ 3.29 & 4.19 & 2.70 & 2.00 \\ 3.88 & 2.20 & 2.01 & 3.29 \end{pmatrix}$$

Thus, the penalty values ares $F_1 = 3.15$, $F_2 = 3.89$, $F_3 = 0.72$ and $F_4 = 3.87$ such that the IBFS obtained

$x_{11}^b = 7.80$	$x_{12}^b = 0.00$	$x_{13}^b = 1.00$	$x_{14}^b = 0.00$
$x_{21}^b = 0.00$	$x_{22}^b = 0.00$	$x_{23}^b = 5.80$	$x_{24}^b = 10.2$
$x_{31}^b = 0.00$	$x_{32}^b = 11.1$	$x_{33}^{b} = 2.7$	$x_{34}^b = 0.00$

Step 3 Algorithm 2 was used to obtain the optimal solution based on the IBFS value of all three SA_t , t = 1, 2, 3 and by using Equation (1). The resulting minimal transportation cost obtained were $SA_1 = 241.98$, $SA_2 = 352.00$, and $SA_3 = 433.46$.

Step 4 Combining the minimal transportation cost of all three SA_t value, the fuzzy optimal solution of balanced FTP for Example 4.1 is (241.98, 352.00, 433.46). The result shows that the most possible amount of minimum total transportation cost is \$352.00, and the least amount of minimum total transportation cost is \$241.98. Meanwhile, the greatest amount of minimum total transportation is \$433.46.

Discussion:

This paper also solved Example 4.1 by using the existing algorithms by Kumar and Kaur (Kaur & Kumar, 2011c);

Kumar and Singh (2012); Kumar et al. (2011); Ezzati et al. (2015); Chakraborty et al. (2016); Ebrahimnejad (2017); Srivastava and Bisht (2020). The comparison for the results of minimal fuzzy total distribution cost and fuzzy optimal solution between the existing algorithms and the new proposed algorithm is shown in Figure 2.

Figure 2, shows that the existing method proposed by Kaur and Kumar (2011c) obtained two approximate solutions that produced two minimal total transportation cost. This is because the method by Kaur and Kumar (2011c) uses a classical ranking function to rank triangular fuzzy number i.e $\tilde{c}_{1,3} = (\$8, ,\$10,$ (10.6) = 9.65, which has equal ranking to the ranking value of triangular fuzzy number i.e $\tilde{c}_{3,3} = (\$7.88, ,\$10,$ (10.8) = 9.65. Meanwhile, the minimal total transportation cost obtained by I-SA is less than the two minimal total transportation costs obtained by method of Kaur and Kumar (2011c). Meanwhile, the fuzzy approximate solution and minimal total transportation cost obtained form the methods of Kumar et al. (2011) and Srivastava and Bisht (2020) is equal to that obtained by I-SA. However, Kumar et al. (2011) used the FLP technique and classical-ranking function without considering conditions in real life.

The least amount, most possible amount, and the greatest amount from fuzzy minimal total transportation cost are result obtained by Kumar and Singh (2012) method were less than to those obtained from the I-SA method, and are displayed in Figure 2. Kumar and Singh (2012). Also employed the FLP technique and classical-ranking function based on parametric value without considering the external factors in solving FTP such that it appear equal ranking value. Meanwhile, the fuzzy approximate solution and fuzzy minimal total transportation cost determined by Chakraborty et al. (2016) are equal to those from Kumar



Figure 2. The comparison of the existing method and the new proposed method.

and Singh (2012). The difference is that Chakraborty et al. (2016) proposed a modification of the classical fuzzy transportation algorithm on operation fuzzy numbers.

Figure 2 also displays that Ezzati et al. (2015) generated the amounts of least and most possible minimal fuzzy total transportation cost that are equal to those obtained from the I-SA method, while the greatest of fuzzy minimal total transportation cost more compared to that obtained by the I-SA method. Ezzati et al. (2015) declared that the result obtained for fuzzy minimal total transportation cost is more effective than that obtained by the I-SA method. However, in solving the FTP for Example 4.1, the method of Ezzati et al. (2015) does not consider external factors in real-life, where it used the definition of ranking to rank the variables of FTP and also employed fuzzy LP model based on lexicography method with MOLP. Meanwhile, Ebrahimnejad (2017) generated the amounts of least, most possible and greatest from fuzzy minimal total transportation cost that are equal to Ezzati et al.'s (2015)method. The difference is Ebrahimnejad (2017) method presents fuzzy LP model based on the lexicography method without ranking fuzzy number.

Example 4.2. A great company wants to make decisions regarding the number of products that must be shipped from each warehouse to each destination in order to obtain minimal transportation costs and maximum profit. The company has two warehouses and three distribution center destinations. All the parameter data of TP are presented under the triangular fuzzy number and the detail is shown in Table 3.

Solution:

In order to solve the problem in Example 4.2 steps similar to those used in Example 4.1 were adopted. After segregation scheme and implementing Algorithm 1 to the segregated TPs, the following were obtained: The IBFS obtained for SA_1 is $x_{11}^a = 35, x_{12}^a = 25, x_{13}^a = 15, x_{21}^a = 0, x_{22}^a = 0$ and $x_{23}^a = 45$. The IBFS obtained for SA_2 is $x_{11}^b = 45, x_{12}^b = 35, x_{13}^b = 15, x_{21}^b = 0, x_{22}^b = 0$ and $x_{23}^b = 65$. The IBFS obtained for SA_3 is $x_{11}^c = 65, x_{12}^c = 45, x_{13}^c = 15, x_{21}^c = 0, x_{22}^c = 0$ and $x_{23}^u = 95$. The MODI in Algorithm 2 was used to obtain the optimal solution of all three SA_t values. The minimal transportation costs obtained were: $SA_1 = 4525, SA_2 = 7425, \text{ and } SA_3 = 1425$

Furthermore, combining the minimal transportation cost of all three SGA_t values, the fuzzy optimal solution of balanced FTP obtained was (4525, 7425, 12,425), which represents that the most possible amount of minimum total transportation cost is \$7425 but the least amount of

 Table 3. Supply, demand, and the cost of transportation in terms of triangular fuzzy number for Example 4.2.

		Triangular fuzzy approximately value				
Parameter		Least (a)	Most (b)	Greatest(c)		
Transportation Costa	c ₁₁	15	25	35		
	c ₁₂	55	65	85		
	c ₁₃	85	95	105		
	c ₂₁	65	75	85		
	c ₂₂	80	90	110		
	c ₂₃	30	40	50		
Supply in units	S 1	75	95	125		
	s ₂	45	65	95		
Demand in units	d_1	35	45	65		
	<i>d</i> ₂	25	35	45		
	d ₃	60	80	110		

minimum total transportation cost is \$4525. Meanwhile, if things are not in favor of the decision maker, the greatest possible amount of minimum total transportation is \$12,425.

Discussion:

This paper also solved Example 4.2 using the method proposed by Ebrahimnejad (2015) and Srivastava and Bisht (2020). The result of fuzzy approximate solution and fuzzy minimal transportation cost is equal to results from I-SA method. Therefore, for this example, the I-SA method performs in producing an approximate solution and a minimum optimal solution under a triangular fuzzy number equivalent to the existing methods (Ebrahimnejad (2015); Srivastava and Bisht (2020))

Example 4.3. The full fuzzy transportation problem on delivery of cement product at PT. ABC with four consumer stores i.e. D_1, D_2, D_3 , and D_4 has quantity of demands under triangular fuzzy numbers in thousand units (8,10,12), (6,10,10), (8,10,12) and (9,10,14), respectively. PT. ABC has warehouse located at four areas, S_1, S_2, S_3 , and S_4 . Each warehouse has quantity of supplies under triangular fuzzy data in thousand units (9,10,14), (7,10,10), (8,10,14) and (7,10,10), respectively The transportation cost on sack unit in thousands under triangular fuzzy data of distribution of PT. ABC from warehouse to consumer's store is listed in Table 4. Based on these condition, determined four allocations for the warehouses to distribute cement products to four consumer stores to obtain that the minimum total transportation cost under fuzzy.

Solution:

To solve the problem in Example 4.3, the steps similar to those used for Examples 4.1 and 4.2 were adopted. After segregation scheme and implementing Algorithm 1 to the segregated TPs, the following was obtained: The IBFS obtained for SA_1 is $x_{11}^a = 1, x_{12}^a = 0, x_{13}^a = 8, x_{14}^a = 0, x_{21}^a = 0, x_{22}^a = 0, x_{23}^a = 0, x_{24}^a = 7, x_{31}^a = 0, x_{32}^a = 6, x_{33}^a = 0, x_{34}^a = 2, x_{41}^a = 7, x_{42}^a = 0, x_{43}^a = 0$ and $x_{44}^a = 0$. The IBFS obtained

 Table 4. Supply, demand, and the cost of transportation in terms of triangular fuzzy number for Example 4.3.

		Triangular fuzzy approximately value				
Parameter		Least (a)	Most (b)	Greatest(c)		
Transportation Costa	C ₁₁	2	5	8		
	C ₁₂	1	3	5		
	C ₁₃	0.5	2	2.5		
	C ₁₄	2	5	7		
	c ₂₁	2	3	4		
	c ₂₂	3	5	7		
	C ₂₃	0.5	3	5.5		
	c ₂₄	0.5	2	3		
	<i>c</i> ₃₁	4	5	6		
	C ₃₂	1	2	3		
	C ₃₃	1	2	3		
	C ₃₄	1	3	4		
	<i>c</i> ₄₁	0.5	3	5.5		
	c ₄₂	1	3	5		
	C43	1	3	5		
	C ₄₄	4	5	8		
Supply in units	s ₁	9	10	14		
	s ₂	7	10	10		
	s ₃	8	10	14		
	<i>s</i> ₄	7	10	10		
Demand in units	d_1	8	10	12		
	<i>d</i> ₂	6	10	10		
	d_3	8	10	12		
	<i>d</i> ₄	9	10	14		

^ain dollar (\$)

for SA_2 is SGA^2 is $x_{11}^b = 0, x_{12}^b = 0, x_{13}^b = 10, x_{14}^b = 0, x_{21}^b = 0, x_{22}^b = 0, x_{23}^b = 0, x_{24}^b = 10, x_{31}^b = 0, x_{32}^b = 10, x_{33}^b = 0, x_{34}^b = 0, x_{41}^b = 10, x_{42}^b = 0, x_{43}^b = 0$ and $x_{44}^b = 0$. The IBFS obtained for SA_3 is SGA^3 is $x_{11}^c = 2, x_{12}^c = 0, x_{13}^c = 12, x_{14}^c = 0, x_{21}^c = 0, x_{22}^c = 0, x_{23}^c = 0, x_{24}^c = 10, x_{31}^c = 0, x_{32}^c = 10, x_{33}^c = 0, x_{34}^c = 4, x_{41}^c = 10, x_{42}^c = 0, x_{43}^c = 0$ and $x_{44}^c = 0$. The MODI in Algorithm 2 was used to obtain the optimal solution of all three SA_t values and the minimal transportation costs were obtained: $SA_1 = 21, SA_2 = 90$, and $SA_3 = 177$

In addition, combining the minimal transportation cost of all three SA_t values, the fuzzy optimal solution obtained for balanced FTP was (21, 90, 177) which shows that most possible amount of minimum total transportation cost is \$90 but the least amount of minimum total transportation cost is \$90. Meanwhile, if things are not in favor of the decision maker, the greatest amount of minimum total transportation is \$177.

Discussion:

Example 4.3 was provided to illustrate the shortcoming of Srivastava and Bisht (2020) method. Example 4.3 presents the TP parameters where the transportation cost matrix, demand, and supply have equal values. As such, that the method proposed by Srivastava and Bisht (2020) cannot solve this problem because of there is no complete rule for selecting the equal cost-minimum cell when the supply and demand are equal. This causes Srivastava and Bisht (2020) method to fail to solve special problems such as example 4.3. Therefore, I-SA is proposed to overcome the shortcoming of Srivastava and Bisht (2020)'s method by completing the rules for allocating an approximation solution with the equal cost cell case where supply and demand are equal.

5. The advantages of the I-SA method

The advantages of our proposed method over existing methods are as follows:

(1) In Kumar et al. (2011); Kumar and Kaur (2011a),c, Kumar & Singh (2012), the classical-ranking function was used and the interpretation of the objective function is not considered in such a way that there is inconsistency with the fuzzy number conversion. This paper overcomes this issue without considering a ranking function and consistent with the fuzzy number conversion.

(2) The approximate solution and the minimum total transportation cost under triangular fuzzy number generated by Kaur and Kumar (2011c) method resulted in negative fuzzy numbers. This contradicts the constraint rule of FTP that fuzzy number must be positive. Therefore, this I-SA approach is able to overcome this problem and the results obeyed all the restrictions of FTP. This has been shown on Subsection 4, Example 4.1

(3) In solving of Example 4.1, from the results for Ezzati et al. (2015) method, the total number of non-negative allocations did not satisfy the criteria for a non-degenerate solution, whereas I-SA fulfilled the non-degeneration criteria.

(4) Ebrahimnejad (2015, 2017) solved FTP in multi-step and used the simplex method which required high computational effort. Meanwhile, the I-SA approach employed the classical TP rules, took less computation effort compared to the simplex method. (5) In solving Example 4.3, Srivastava and Bisht (2020) method could not be used to solve this problem because the rules are incomplete, while I-SA has complete rules and can be applied to specific problems like this.

6. Conclusion

In this study, the optimization of FTP under triangular fuzzy numbers was investigated and reported. The Improved segregated advancement (I-SA) method consists of a segregated scheme to partition triangular fuzzy numbers into three parts: lower, middle, and upper; the use of the combination of total ratio cost and total difference method by column to find the Initial Basic Feasible Solution (IBFS) and together with Modified Distribution method (MODI) to obtain the fuzzy optimal solution. The justification of I-SA is illustrated based on three test problems taken from various reputable journals. The comparison of the results obtained in this study with those obtained by the existing SA methods showed that the allocations for Example 4.1 were equivalent to the result obtain by Kumar et al. (2019); Srivastava and Bisht (2020), and better than the results obtained by Kaur and Kumar (2011c); Kumar and Singh (2012); Chakraborty et al. (2016). All the optimal solutions produced by I-SA method were positive fuzzy numbers, which means that the results are feasible. In contrast, one of the optimal solutions produced by Kumar and Kaur (2011a,c) was negative and violated the constraints of the FTP model. As for Example 4.2, the results obtained were equivalent to those recorded by using the methods of of Srivastava and Bisht (2020); Ebrahimnejad (2015), and were better than the results obtained by Ezzati et al. (2015) method. Finally, for Example 4.3, the results obtained were better than those achieved by using the method proposed by Srivastava and Bisht (2020). This comparison validates the applicability, robustness, and unambiguity of I-SA. The new proposed method can help in decision-making problems that fall under triangular fuzzy data and is also applicable for unbalanced FTP. The drawback of the proposed approach lies in the form of fuzzy numbers used, where if the form of fuzzy numbers used is more than three points, such as quadrangular fuzzy numbers, five, and so on, or the dimensions are high, then the calculation to find the optimal value would be very complex. For future studies, the proposed method could be improved in order to solve the fuzzy transportation problem where one of the components of the triangular fuzzy number is negative or zero. In addition, the proposed method could be considered hesitant fuzzy number (sets).

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This work was supported by the Universiti Tun Hussein Onn Malaysia (UTHM) Research Management Center [research grant TIER 1 (H777)].

ORCID

Muhammad Sam'An i http://orcid.org/0000-0001-5408-5562

References

- Adhami, A. Y., & Ahmad, F. (2020). Interactive Pythagorean-hesitant fuzzy computational algorithm for multiobjective transportation problem under uncertainty. *International Journal of Management Science and Engineering Management*, 15(4), 288–297.
- Akilbasha, A., Natarajan, G., & Pandian, P. (2016). A new approach for solving transportation problems in fuzzy nature. *International Journal of Applied Engineering Research*, 11(1), 498–502.
- Akilbasha, A., Pandian, P., & Natarajan, G. (2018). An innovative exact method for solving fully interval integer transportation problems. *Informatics in Medicine Unlocked*, 11, 95–99.
- Basirzadeh, H. (2011). An approach for solving fuzzy transportation problem. Applied Mathematical Sciences, 5(32), 1549–1566.
- Baykasoglu, A., & Subulan, K. (2019). A direct solution approach based on constrained fuzzy arithmetic and metaheuristic for fuzzy transportation problems. *Soft Computing*, 23(5), 1667–1698.
- Bhardwaj, B., & Kumar, A. (2015). A note on "A new algorithm to solve fully fuzzy linear programming problems using the MOLP problem. *Applied Mathematical Modelling*, 39(19), 5982–5985.
- Bisht, D. C. S., & Srivastava, P. K. (2020). Trisectional fuzzy trapezoidal approach to optimize interval data based transportation problem. *Journal of King Saud University Science*, 32(1), 195–199.
- Chakraborty, D., Jana, D. K., & Roy, T. K. (2016). A new approach to solve fully fuzzy transportation problem using triangular fuzzy number. *International Journal of Operational Research*, 26(2), 153–179.
- Chandran, S., & Kandaswamy, G. (2016). A fuzzy approach to transport optimization problem. Optimization and Engineering, 17(4), 965–980.
- Ebrahimnejad, A. (2014). A simplified new approach for solving fuzzy transportation problems with generalized trapezoidal fuzzy numbers. *Applied Soft Computing*, *19*, 171–176.
- Ebrahimnejad, A. (2015). An improved approach for solving fuzzy transportation problem with triangular fuzzy numbers. *Journal of Intelligent and Fuzzy Systems*, 29(2), 963–974.
- Ebrahimnejad, A. (2016a). New method for solving Fuzzy transportation problems with LR flat fuzzy numbers. *Information Sciences*, 357, 108–124.
- Ebrahimnejad, A. (2016b). Note on "A fuzzy approach to transport optimization problem. Optimization and Engineering, 17(4), 981–985.
- Ebrahimnejad, A. (2017). A lexicographic ordering-based approach for solving fuzzy transportation problems with triangular fuzzy numbers. *Nternational Journal of Management and Decision Making*, 16(4), 346–374.
- Ebrahimnejad, A., & Verdegay, J. L. (2018). A new approach for solving fully intuitionistic fuzzy transportation problems. *Fuzzy Optimization and Decision Making*, *17*(4), 447–474.
- Edward Samuel, A. (2012). Improved zero point method (IZPM) for the transportation problems. *Applied Mathematical Sciences*, 6(109), 5421–5426.
- Edward Samuel, A., & Venkatachalapathy, M. (2012). A new dual based approach for the unbalanced fuzzy transportation problem. *Applied Mathematical Sciences*, 6(89–92), 4443–4455.
- Edward Samuel, A., & Venkatachalapathy, M. (2013). Improved zero point method for solving fuzzy transportation problems using ranking function. *Far East Journal of Mathematical Sciences*, 75(1), 85–100.
- Edward Samuel, A., & Venkatachalapathy, M. (2014). IZPM for unbalanced fuzzy transportation problems. *International Journal of Pure and Applied Mathematics*, 94(4), 419–424.
- Ezzati, R., Khorram, E., & Enayati, R. (2015). A new algorithm to solve fully fuzzy linear programming problems using the MOLP problem. *Applied Mathematical Modelling*, 39(12), 3183–3193.
- Farikhin, Sam'an, M., Surarso, B., & Bambang, I. (2019). The fuzzy optimal solution of fuzzy transport problems using a new fuzzy least cost method. Proceedings of the Mathematics, Informatics, Science, and Education International Conference (MISEIC 2019) September 28, 2019 – September 29, 2019 Surabaya, 48–53. doi:10.2991/miseic-19.2019.18
- Fegade, M. R. (2012). Solving fuzzy transportation problem using zero suffix and robust ranking methodology. *IOSR Journal of Engineering*, 2(7), 36–39.
- Firoz, A., & Adhami, A. Y. (2019). Neutrosophic programming approach to multiobjective nonlinear transportation problem with fuzzy parameters. *International Journal of Management Science and Engineering Management*, 14(3), 218–229.

- Firoz, F.A., & Adhami, A. Y. (2019). Neutrosophic programming approach to multiobjective nonlinear transportation problem with fuzzy parameters. *International Journal of Management Science and Engineering Management*, 14(3), 218–229.
- Hatami-Marbini, A., Agrell, P. J., Tavana, M., & Emrouznejad, A. (2013). A stepwise fuzzy linear programming model with possibility and necessity relations. *Journal of Intelligent and Fuzzy Systems*, 25(1), 81–93.
- Hitchcock, F. L. (1941). The distribution of a product from several sources to numerous localities. *Journal of Mathematics and Physics*, 20(1-4), 224-230.
- Hunwisai, D., & Kumam, P. (2017). A method for solving a fuzzy transportation problem via Robust ranking technique and ATM. *Cogent Mathematics*, 4(1), 1283730.
- Karthy, T., & Ganesan, K. (2016). Improved zero point method for the fuzzy optimal solution to fuzzy transportation problems. *Global Journal of Pure and Applied Mathematics*, 12(1), 255–260.
- Karthy, T., & Ganesan, K. (2019). Revised improved zero point method for the trapezoidal fuzzy transportation problems. *AIP Conference Proceedings* 11–12 January 2019, Chennai, India, 2112, 20063.
- Kaur, A., & Kumar, A. (2011c). A new method for solving fuzzy transportation problems using ranking function. Applied Mathematical Modelling, 35(12), 5652–5661.
- Kaur, A., & Kumar, A. (2012). A new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers. *Applied Soft Computing*, 12(3), 1201–1213.
- Kumar, R., Edalatpanah, S. A., Jha, S., & Singh, R. (2019). A Pythagorean fuzzy approach to the transportation problem. *Complex & Intelligent Systems*, 5(2), 255–263.
- Kumar, A., & Kaur, A. (2011a). Application of classical transportation methods to find the fuzzy optimal solution of fuzzy transportation problems. *Fuzzy Information and Engineering*, 3(1), 81–99.
- Kumar, A., & Kaur, A. (2011b). Application of classical transportation methods for solving fuzzy transportation problems. *Journal of Transportation Systems Engineering and Information Technology*, 11(5), 68–80.
- Kumar, A., Kaur, J., & Singh, P. (2011). A new method for solving fully fuzzy linear programming problems. *Applied Mathematical Modelling*, 35(2), 817–823.
- Kumar, B. R., & Murugesan, S. (2012). on Fuzzy transportation problem using triangular fuzzy numbers with modified revised simplex method. *International Journal of Engineering Science and Technology*, 4(1), 285–294.
- Kumar, A., & Singh, P. (2012). A new method for solving fully fuzzy linear programming problems. Annals of Fuzzy Mathematic and Information, 3, 103–118.
- Kundu, P., Kar, S., & Manoranjan, M. (2013). Multi-objective multi-item solid transportation problem in fuzzy environment. *Applied Mathematical Modelling*, 37(4), 2028–2038.
- Lotfi, F. H., Allahviranloo, F., Alimardani, M. A., & Alizadeh, L. (2009). Solving a full fuzzy linear programming using lexicography method and fuzzy approximate solution. *Applied Mathematical Modelling*, 33 (7), 3151–3156.
- Mathur, N., & Srivastava, P. K. (2020). An inventive approach to optimize fuzzy transportation problem. *International Journal of Mathematical, Engineering and Management Sciences*, 5(5), 985–994.
- Mathur, N., Srivastava, P. K., & Paul, A. (2016). Trapezoidal fuzzy model to optimize transportation problem. *International Journal of Modeling, Simulation, and Scientific Computing*, 7(3), 1650028.
- Muthuperumal, S., Titus, P., & Venkatachalapathy, M. (2020). An algorithmic approach to solve unbalanced triangular fuzzy transportation problems. *Soft Computing*, 24(24), 18689–18698.
- Pandian, P., & Natarajan, G. (2010). A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems. *Applied Mathematical Sciences*, 4(2), 79–90.
- Rani, D., & Gulati, T. R. (2014). A new approach to solve unbalanced transportation problems in imprecise environment. *Journal of Transportation Security*, 7(3), 277–287.
- Rani, D., & Gulati, T. R. (2017). Time optimization in totally uncertain transportation problems. *International Journal of Fuzzy Systems*, 19 (3), 739–750.
- Saberi Najafi, H., & Edalatpanah, S. A. (2013). A note on "A new method for solving fully fuzzy linear programming problems. *Applied Mathematical Modelling*, 37(14–15), 7865–7867.

- Saini, R. K., Sangal, A., & Prakash, O. (2018). Fuzzy transportation problem with generalized triangular-trapezoidal fuzzy number. Advances in Intelligent Systems and Computing, 583, 723–734.
- Sam'an, M., & Farikhin. (2021). A new fuzzy transportation algorithm for finding fuzzy optimal solution. *International Journal of Mathematical Modelling and Numerical Optimisation*, 11(1), 1–19.
- Sam'an, M., Farikhin, Hariyanto, S., & Surarso, B. (2018a). Optimal solution of full fuzzy transportation problems using total integral ranking. *Journal of Physics Conference Series*, 1(983), 12075.
- Sam'an, M., Farikhin, Surarso, B., & Zaki, S. (2018b). A modified algorithm for full fuzzy transportation problem with simple additive

weighting. 2018 International Conference on Information and Communications Technology (ICOIACT) Yogyakarta, Indonesia, 684–688. doi:10.1109/ICOIACT.2018.8350745

- Selvakumari, K., & Sathya Geetha, S. (2020). A new approach for solving intuitionistic fuzzy transportation problem. *Journal of Advanced Research in Dynamical and Control Systems*, 12(5), 956–963.
- Srivastava, P. K., & Bisht, D. C. S. (2020). A segregated advancement in the solution of triangular fuzzy transportation problems. *American Journal of Mathematical and Management Sciences*, 1–11. doi:10. 1080/01966324.2020.1854137
- Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8(3), 338-353.